$$H = H_{hfs} + H_B = hA_{hfs}\frac{\vec{I}\cdot\vec{J}}{\hbar^2} + \frac{\mu_B}{\hbar}(g_J J + g_I I)B_z$$

(a) [3 pts] In general, finding the eigenstates and energies in this system requires a numerical solution (diagonalization of this Hamiltonian). However, we stated in class that for states with J = 1/2, as in the $S_{1/2}$ ground states of hydrogen and the alkalis, the energies can be described by the Breit-Rabi formula

$$E = -\frac{\Delta E_{hfs}}{4F^+} - g_I m \mu_B B_z \pm \frac{\Delta E_{hfs}}{2}\sqrt{1 + \frac{4mx}{2F^+} + x^2}$$

Here, $F^+ = (I + 1/2)$, $\frac{\Delta E_{hfs}}{h} = A_{hfs}F^+$, $m = m_I \pm m_J$ is the *z* component of the total angular momentum, and the term *x* is given by $x = (g_J + g_I)\mu_B B_z / \Delta E_{hfs}$.

Show that the state energies follow this Breit-Rabi formula for a general field value B_z.

L7: 2-level systems & classical resonance

Foot 1.6, 1.8

Examples of “2-level” systems

- spin-1/2 particle (electron/proton spin)
- (pseudo)spin-1/2 particle (hyperfine, etc.)

L7: 2-level systems & classical resonance

Foot 1.6, 1.8

Examples of "2-level" systems

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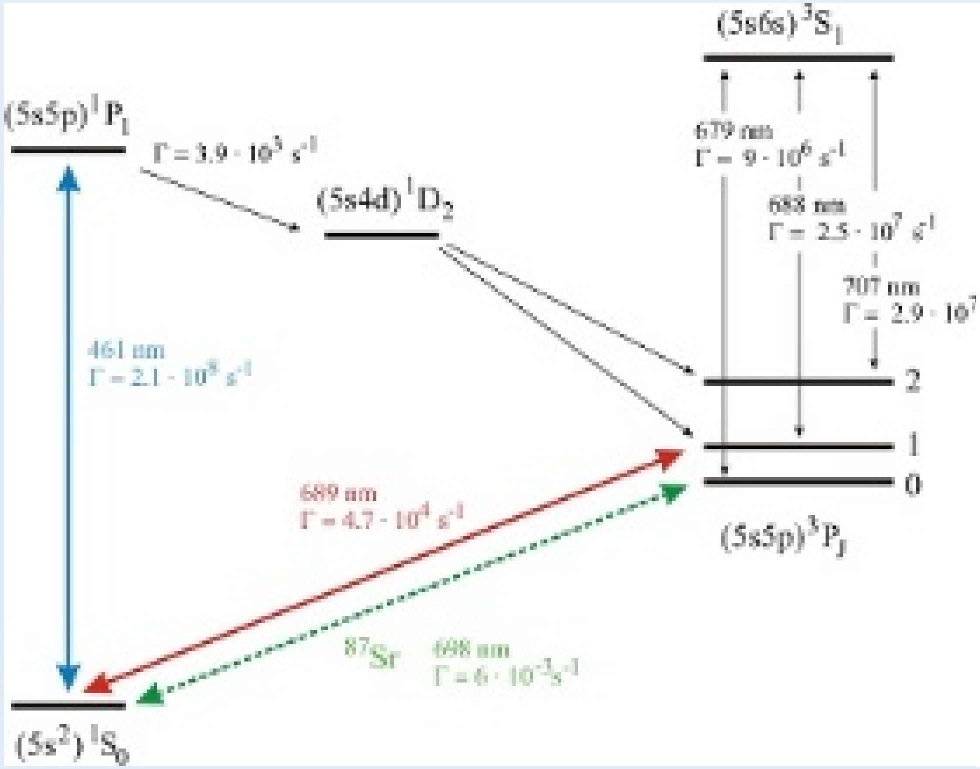
- (pseudo)spin-1/2 particle (hyperfine, etc.)
    - optical excitations



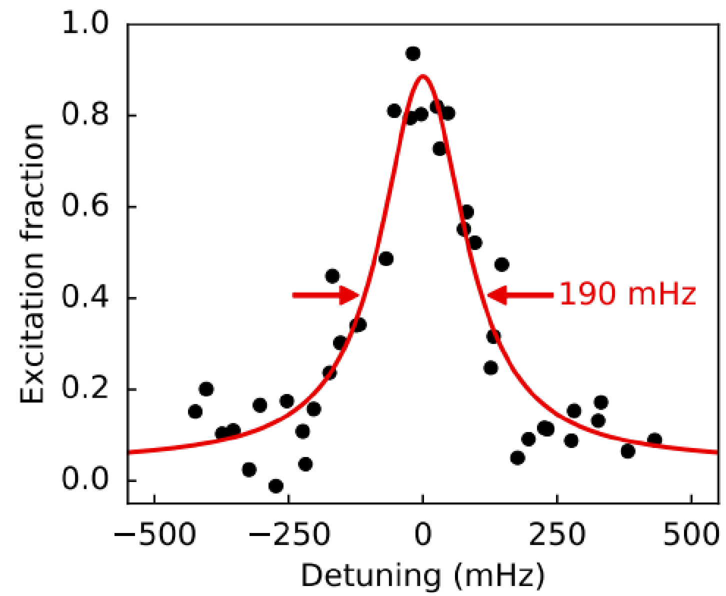


Fig. S6. Narrow-line Rabi spectroscopy. Rabi spectroscopy data for a 4 s π-pulse time, showing a 190(20) mHz Fourier-limited linewidth, taken with $m_F = 9/2$ and rescaled by the relative spin population.

L7: 2-level systems & classical resonance

Foot 1.6, 1.8

Examples of “2-level” systems

- spin-1/2 particle (electron/proton spin)
- (pseudo)spin-1/2 particle (hyperfine, etc.)
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    - energy state of anharmonic oscillator
    - rot/vib states of molecules
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L7: 2-level systems & classical resonance

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Examples of “2-level” systems

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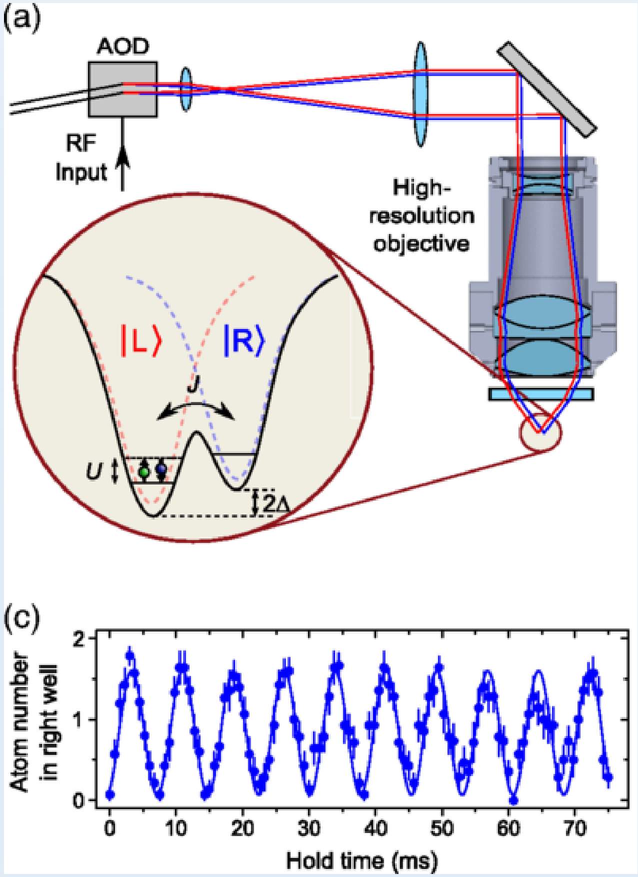
- double-well site index



L7: 2-level systems & classical resonance

Foot 1.6, 1.8

Examples of "2-level" systems

- spin-1/2 particle (electron/proton spin)
- (pseudo)spin-1/2 particle (hyperfine, etc.)
  - optical excitations
  - energy state of anharmonic oscillator
  - rot/vib states of molecules
  - etc.
- double-well site index
- interferometer path
- photon polarization / other d.o.f.
- states of a macroscopic object
- field occupation number
- etc.

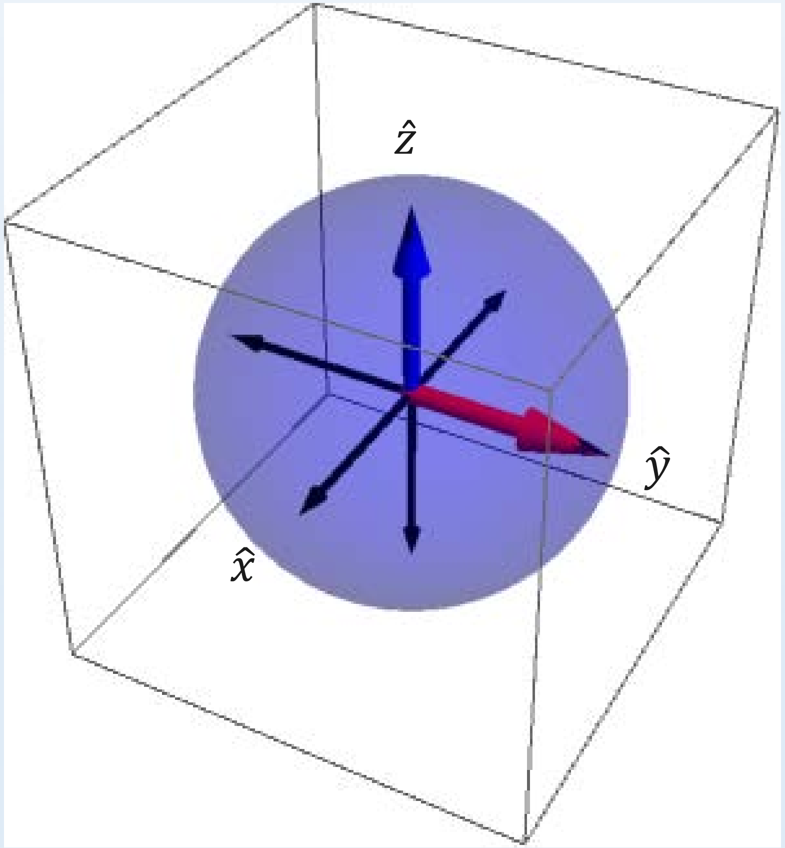
Ignoring intricacies,
all these 2-level systems are
described by a similar formalism

Let's review some basic terminology

Field purely along z

$$\vec{B} = B_0 \hat{z}$$

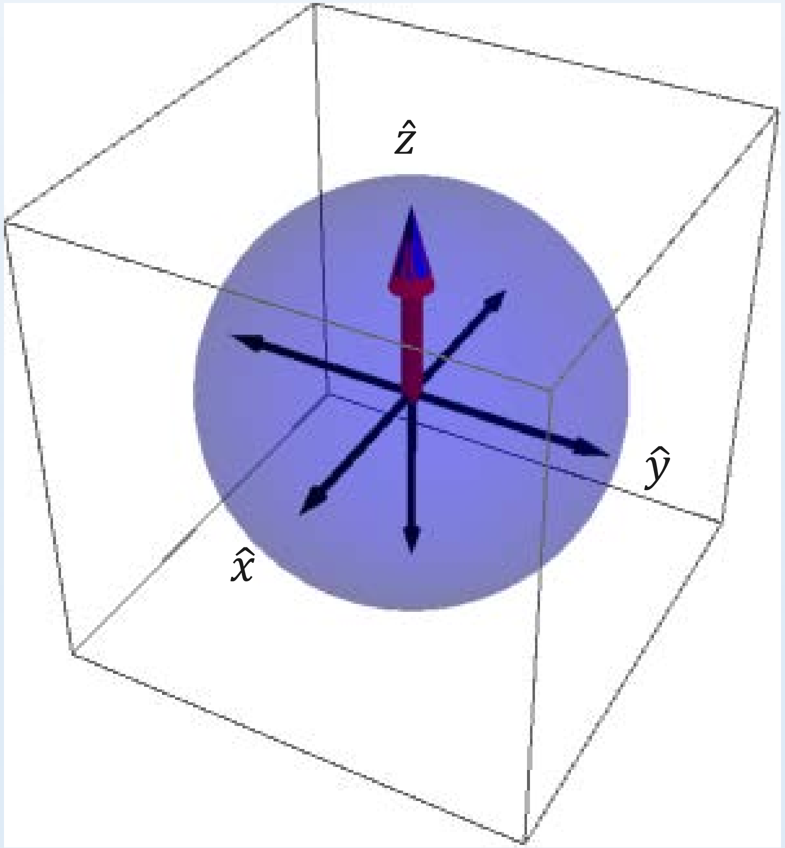
spin in x-y plane



Field purely along z

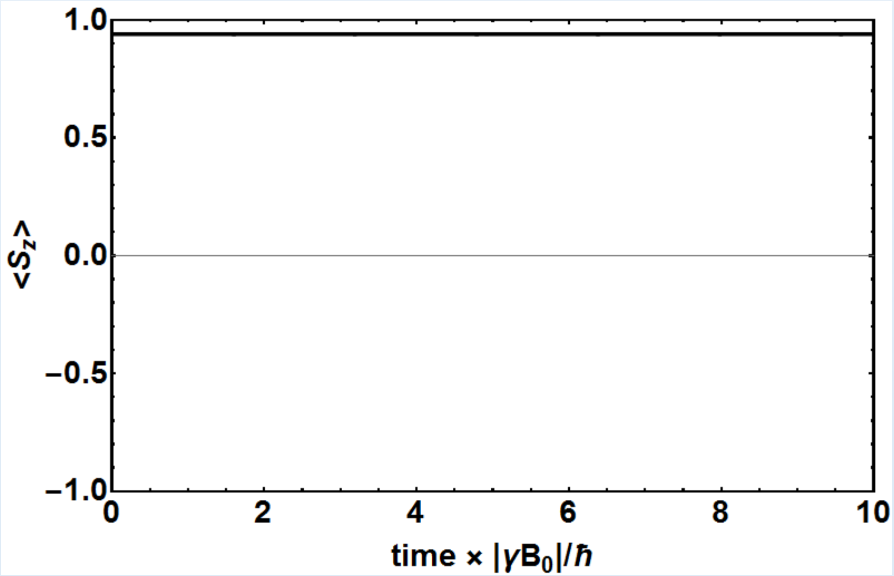
$$\vec{B} = B_0 \hat{z}$$

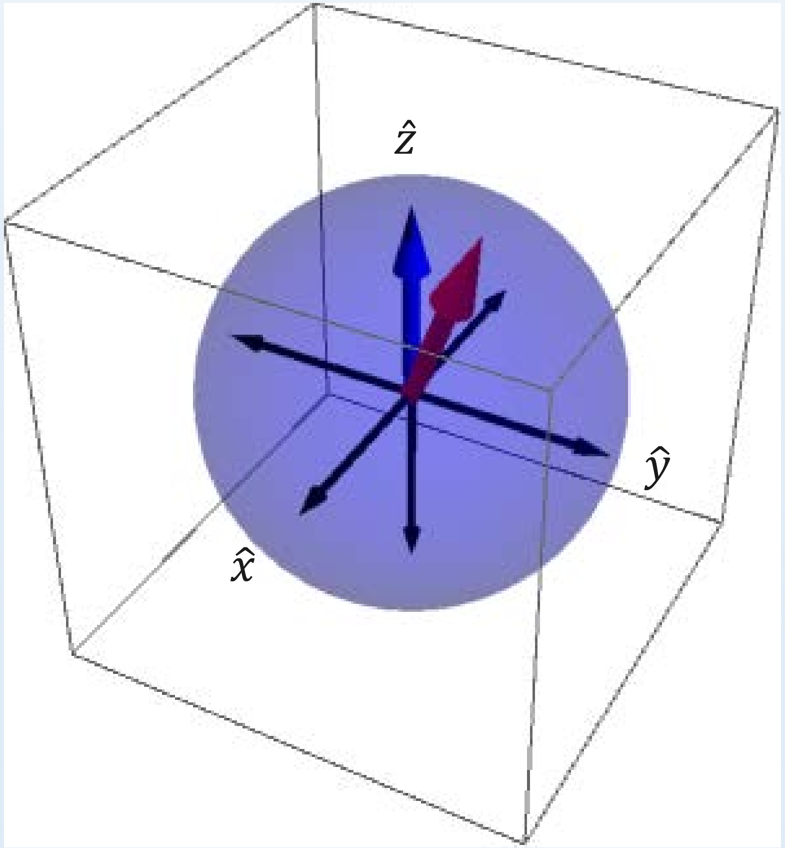
spin along z-axis



General Larmor precession

$$\vec{B} = B_0 \hat{z}$$

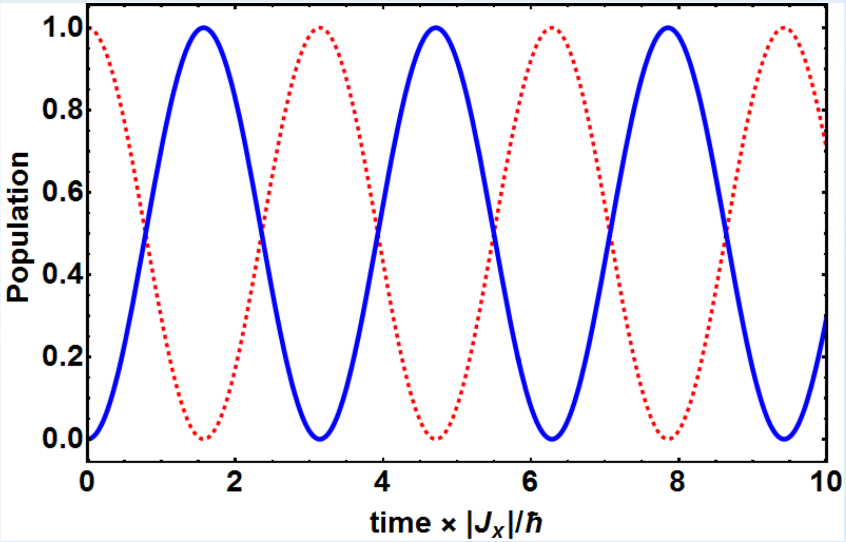


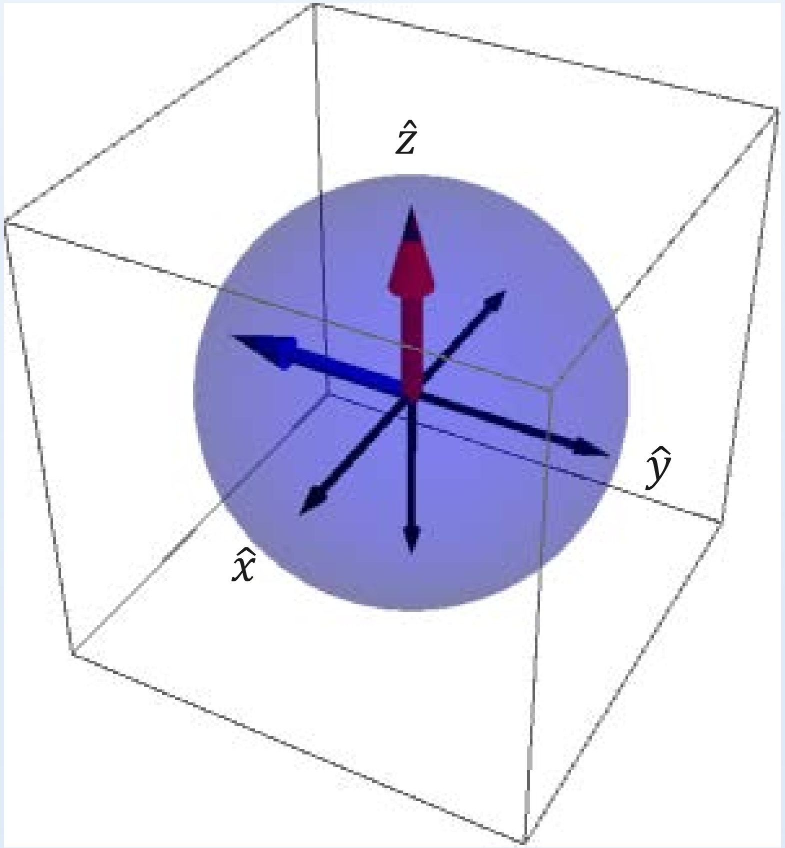


Field purely along x

$$\vec{B} = B_{\perp}\hat{x}$$

spin along z-axis

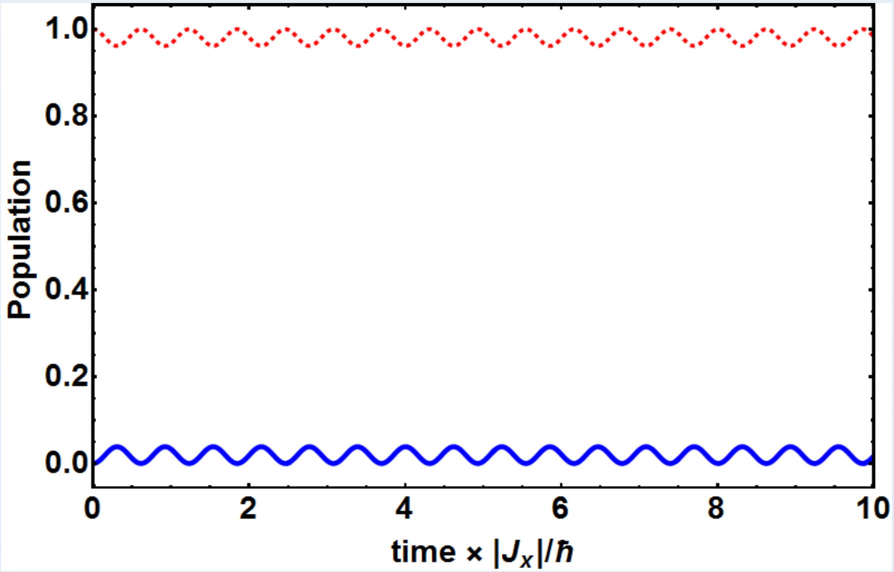


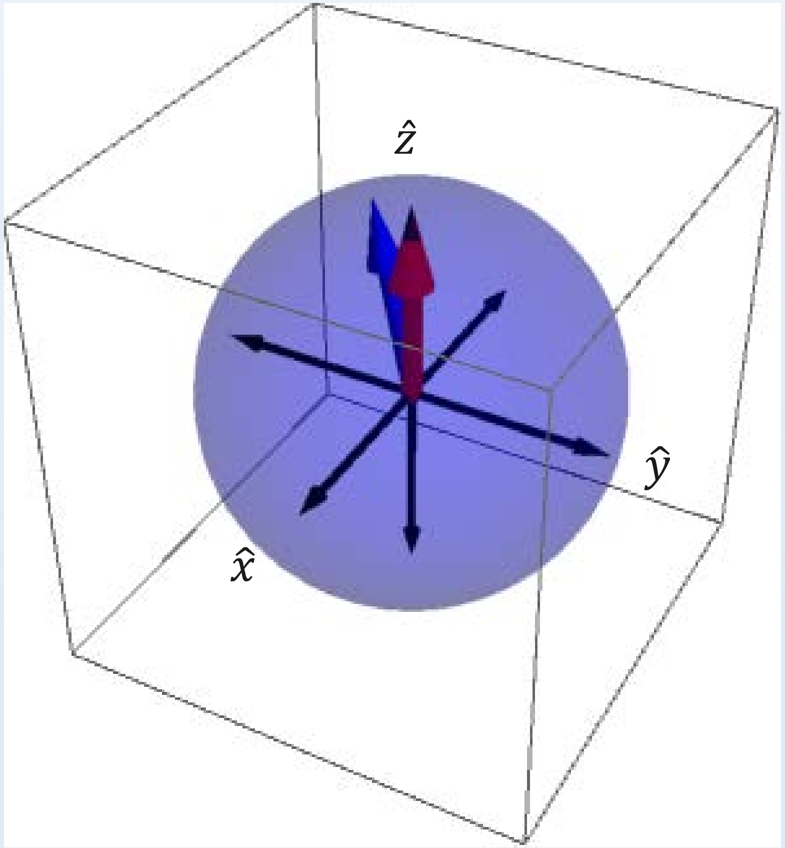


What would the dynamics look like if our spin was initially pointed along x?

Weak transverse field

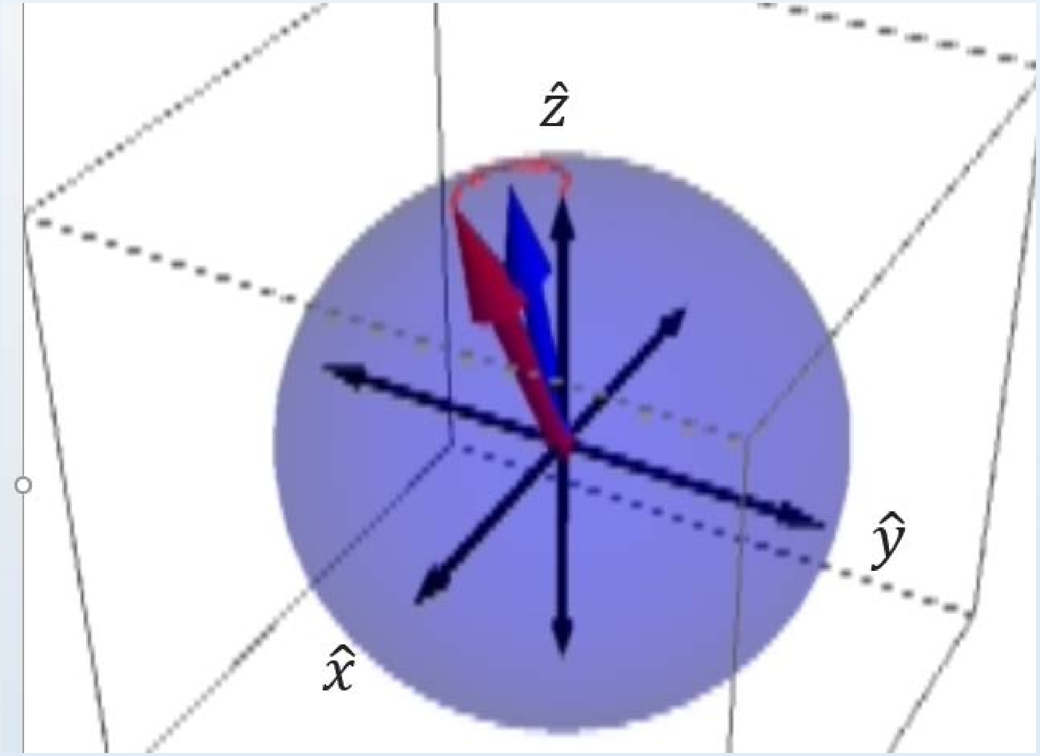
$$\vec{B} = B_0\hat{z} + B_\perp\hat{x}$$





$$B_0 = 10B_\perp$$

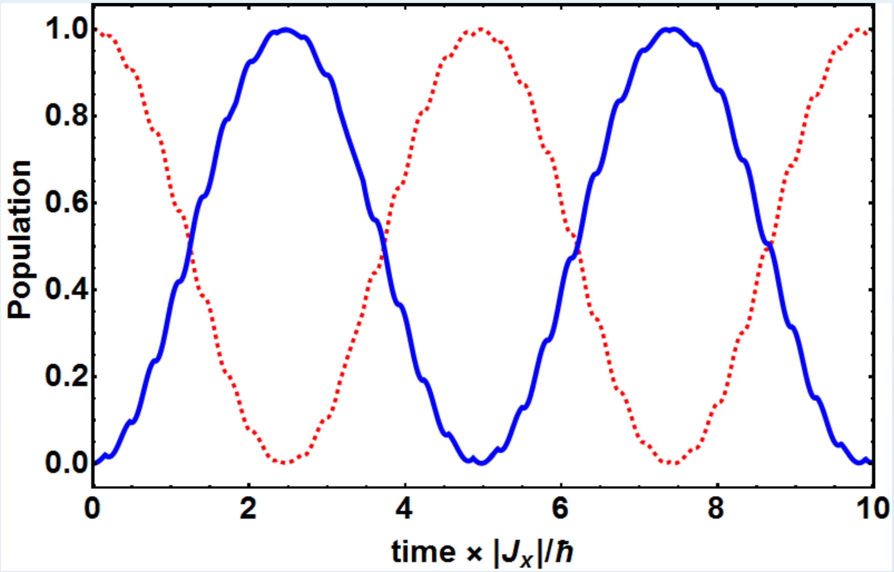
How can we get the spin to keep going towards the "south pole," i.e. -z?



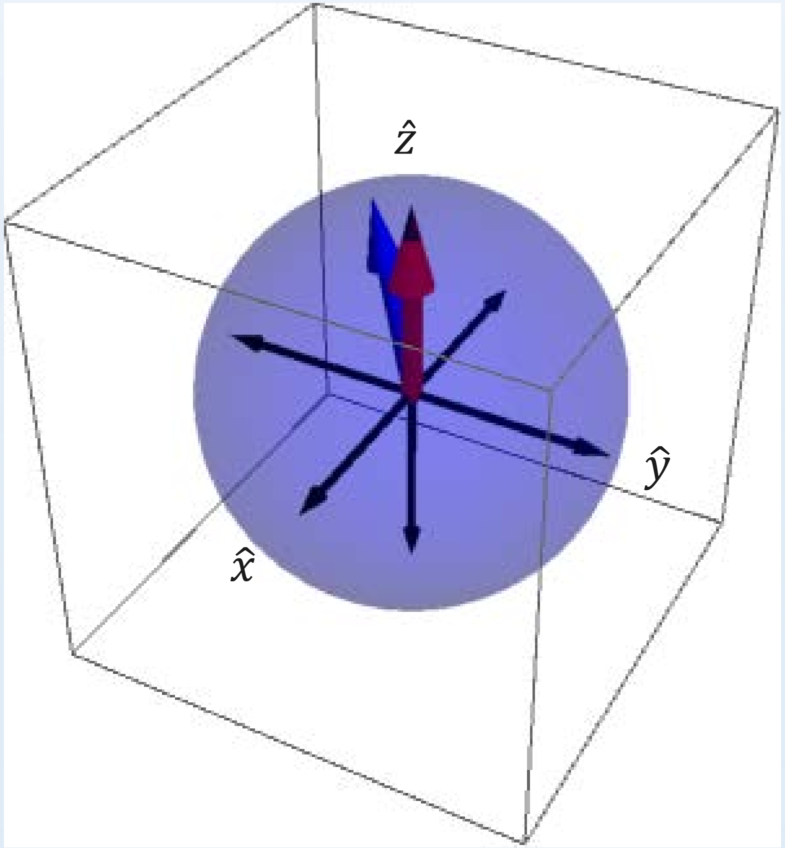
Open response: 5 points towards HW#2

Inverting the field in coordination with the precession

$$\vec{B} = B_0 \hat{z} + B_\perp(t)\hat{x}$$

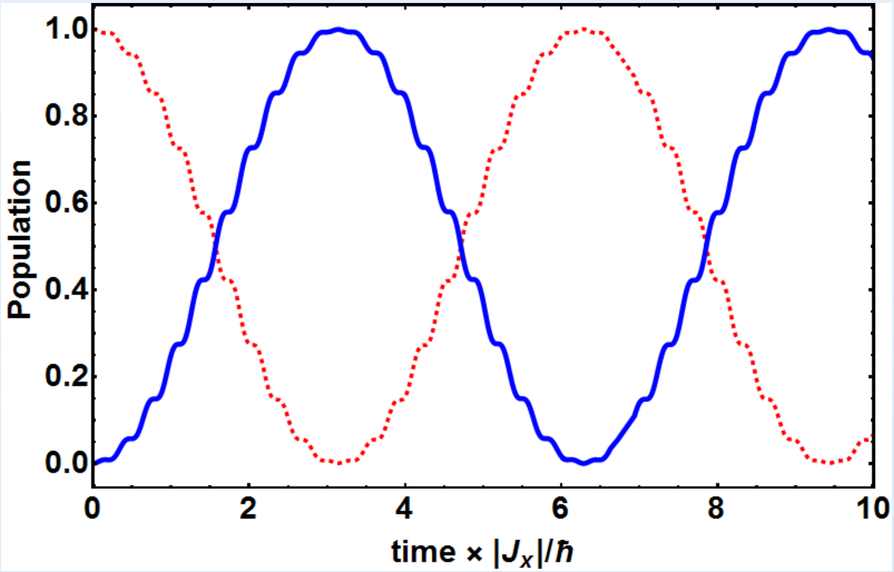


$$B_0 = 10 B_\perp$$

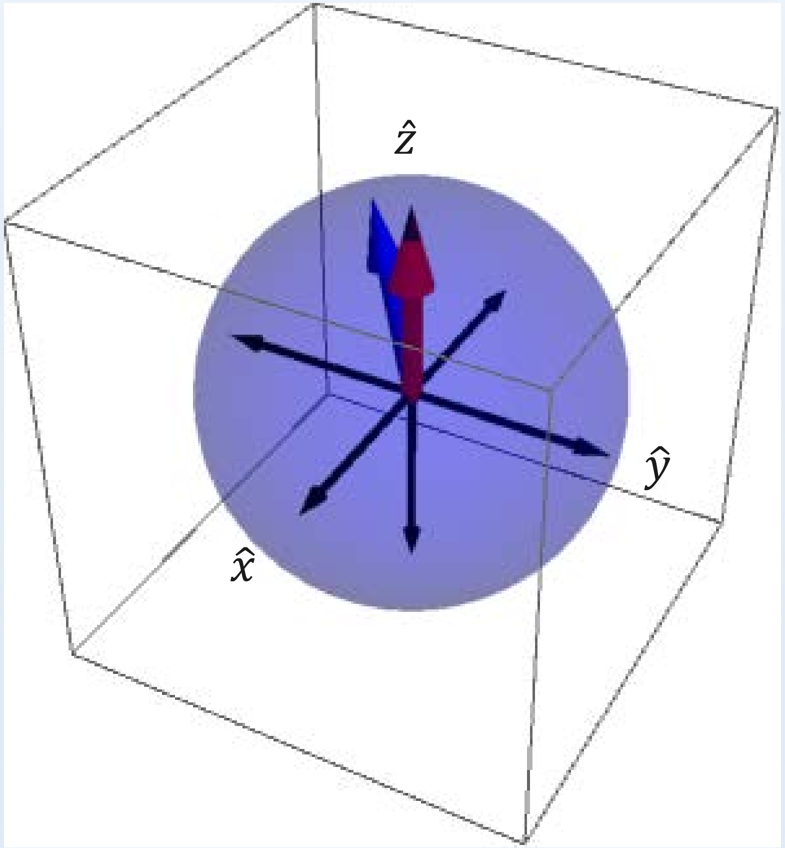


Field oscillating at the Larmor frequency

$$\vec{B} = B_0\hat{z} + B_\perp \cos\left(\mu B_0 t / \hbar\right)\hat{x}$$



$$B_0 = 10 B_\perp$$



many other ways as well…