Adiabatic processes

Lecture # 9
PHYS 598 AQC
Fall 2017

We saw last time that we can manipulate the state of a two-level system w/ "pulses" of a "coupling" field, such as w/ an oscillating transverse field for spins / pseudospins.

In this case, our control of the final state (in multi-pulse scenarios) can depend crucially on parameter control $(\Omega, \delta, \varphi)$ and on precise timing. Is there an alternative approach?

Yes! Instead of using "pulses," which are fast (some would say that they are "Heisenberg-limited" w.r.t. time-energy uncertainty), we can adiabatically prepare chosen states.

Basic idea - we control the "form" or "structure" of our dressed eigenstates through an applied field

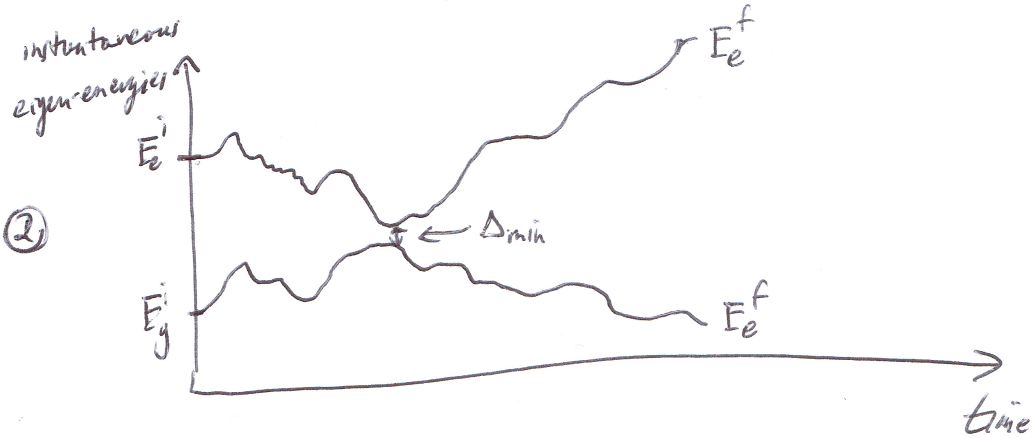
recall

$$|e\rangle = \cos\frac{\theta}{2}|\tilde{\uparrow}\rangle + \sin\frac{\theta}{2}e^{i\varphi}|\tilde{\downarrow}\rangle$$
$$|g\rangle = -\sin\frac{\theta}{2}e^{-i\varphi}|\tilde{\uparrow}\rangle + \cos\frac{\theta}{2}|\tilde{\downarrow}\rangle$$

w/ $\tan\theta = -\dfrac{2\Omega}{\delta}$

for oscillating $B_\perp$

→ we can prepare an eigenstate for some initial parameter values $(\Omega_i, \delta_i, \varphi_i)$, and if we slowly vary these parameters, our state at any time will follow the instantaneous eigenstate to some final condition $(\Omega_f, \delta_f, \varphi_f)$

② 

how "slow" we have to vary the parameters will depend on the energy gap + off-diagonal matrix elements between available states

What are the general conditions for adiabaticity?

Let's say that the instantaneous eigenstates @ time t are given by

$$\mathcal{H}(t)|\mathcal{E}_k(t)\rangle = E_k(t)|\mathcal{E}_k(t)\rangle \quad \text{w/ energies } E_k(t)$$

we project some initial state onto these eigenstates @ time $t=0$. In our "adiabatic" preparation, we'll assume that $|\Psi_{initial}\rangle$ fully projects onto a single eigenstate $|\mathcal{E}_k(t=0)\rangle$ of $\mathcal{H}(t=0)$. More generally, our state

$$|\Psi(t)\rangle = \sum_l c_l(t)|\mathcal{E}_l(t)\rangle.$$

Plugging in this initial state to $i\hbar|\dot{\Psi}(t)\rangle = \mathcal{H}(t)|\Psi(t)\rangle$, we find

$$i\hbar\sum_l\left[\dot{c}_l(t)|\mathcal{E}_l(t)\rangle + c_l(t)\frac{d}{dt}|\mathcal{E}_l(t)\rangle\right] = \sum_l c_l(t)E_l(t)|\mathcal{E}_l(t)\rangle$$

let's the multiply both sides from the left by $\langle\mathcal{E}_k(t)|$

③ this yields the expression

(Eq A)

$$\underbrace{i\hbar\, \dot{c}_K(t) = E_K\, c_K(t)}_{\text{this first part}} - \underbrace{i\hbar \sum_{\ell} c_\ell(t) \langle \mathcal{E}_K(t) | \frac{d}{dt} | \mathcal{E}_\ell(t) \rangle}$$

this first part represents the traditional evolution in the eigenbasis $|\mathcal{E}_K\rangle$ when H is time-independent

- this second part introduces mixing between the eigenstate due to non-adiabatic transitions.

2-level system

→ ok, let's apply this to the case

$$H = \hbar \begin{pmatrix} -\delta/2 & \Omega e^{-i\varphi} \\ \Omega e^{i\varphi} & \delta/2 \end{pmatrix}$$

and assume $\varphi = 0$

this was our RWA expression for coupled levels $|+\rangle$ & $|-\rangle$,

w/ $\tan\theta = \frac{-2\Omega}{\delta}$ and $|e\rangle = \cos\frac{\theta}{2}|+\rangle + \sin\frac{\theta}{2}|-\rangle$

$|g\rangle = -\sin\frac{\theta}{2}|+\rangle + \cos\frac{\theta}{2}|-\rangle$

Let's ~~~~ now let Ω and δ be functions of time, $\Omega(t)$ and $\delta(t)$, w/

$$\theta(t) = \arctan\left(\frac{-2\Omega(t)}{\delta(t)}\right)$$

If we consider the relevance of Eq A as applied to two levels, ℓ and k (w/ $\ell \neq k$), and reset the energies such that $E_k \to 0$ and $E_\ell \to E_\ell - E_k$, we see that if $C_k = 1$ @ $t = 0$ and $C_\ell = 0$,

$$i\hbar\, \dot{C}_\ell = (E_\ell - E_k)\, \underset{0}{C_\ell} - i\hbar \left\langle \mathcal{E}_\ell \middle| \left(\frac{d}{dt} |\mathcal{E}_k\rangle\right)\right. C_k$$

in first-order perturbation theory, the time-dependence of H and thus $|\mathcal{E}$ will lead to a modified eigenstate

$$|\psi_k'\rangle = |\mathcal{E}_k\rangle \underbrace{- i\hbar \frac{\langle \mathcal{E}_\ell | \dot{\mathcal{E}}_k \rangle}{E_k - E_\ell} |\mathcal{E}_\ell\rangle}$$

to keep this term small, i.e. to maintain the instantaneous eigenstates, we require

$$\boxed{\hbar\, |\langle \mathcal{E}_\ell | \dot{\mathcal{E}}_k \rangle| \ll |E_k - E_\ell|}$$

Let's apply this to our simple 2-level system

⑤ we want ~~it~~ to stay in $|g(t)\rangle$ if starting in $|g(t=0)\rangle$,

and this requires that

$$\hbar \, |\langle e|\dot{g}\rangle| \ll |E_e - E_g|$$ for all t throughout the process

$$\langle e| = \cos\frac{\theta}{2}\langle +| + \sin\frac{\theta}{2}\langle -|$$

$$|g\rangle = -\sin\frac{\theta}{2}|+\rangle + \cos\frac{\theta}{2}|-\rangle$$

$$\cancel{\langle e|\dot{g}\rangle = \langle e|\frac{d}{dt}|g\rangle}$$

$$\langle e(t)|\left(\frac{d}{dt}|g(t)\rangle\right) = \cos\frac{\theta(t)}{2}\left[\frac{d}{dt}\left(-\sin\frac{\theta(t)}{2}\right)\right] + \sin\frac{\theta}{2}\left[\frac{d}{dt}\left(\cos\frac{\theta(t)}{2}\right)\right]$$

$$= -\cos^2\frac{\theta(t)}{2} \times \frac{\dot{\theta}}{2} - \sin^2\frac{\theta(t)}{2} \times \frac{\dot{\theta}}{2} = -\frac{\dot{\theta}}{2}$$

$$\theta(t) = \arctan\left(\frac{-2\Omega(t)}{\delta(t)}\right)$$

$$\dot{\theta} = \frac{1}{1+\left(-\frac{2\Omega}{\delta}\right)^2}\,\frac{d}{dt}\left(\frac{-2\Omega(t)}{\delta(t)}\right) = \frac{\cancel{\delta^2}}{\delta^2+4\Omega^2}\left[-2\right]\left[\frac{\delta\dot{\Omega}-\Omega\dot{\delta}}{\cancel{\delta^2}}\right]$$

$$|\langle e|\dot{g}\rangle| = \left|\frac{-\dot{\theta}}{2}\right| = \left|\frac{\delta\dot{\Omega}-\Omega\dot{\delta}}{\delta^2+4\Omega^2}\right| \ll |E_e-E_g| = 2\tilde{\Omega} = 2\sqrt{\Omega^2+(\delta/2)^2} = \sqrt{4\Omega^2+\delta^2}$$

~~[illegible]~~

⑥ This simple two-level system let's us explore the general idea of adiabatic rapid passage (ARP)

timescales

Let's say @ $t_i = 0$ we start w/ $|g(t_i)\rangle \approx |-\rangle$

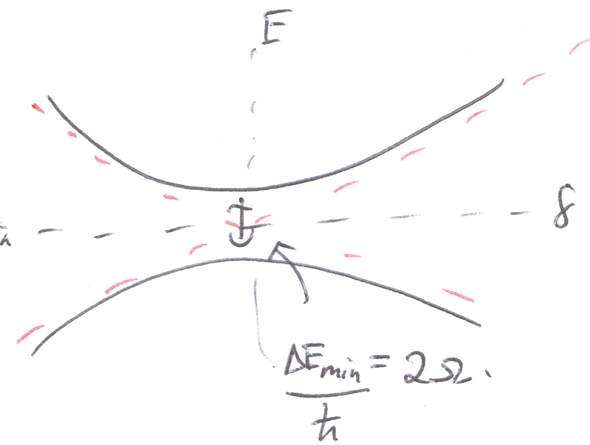
and we want to smoothly modify the system such that

at time $t_f = \tau$, $|g(t_f)\rangle \approx |+\rangle$

this requires a change in θ of $\delta\theta = \frac{\pi}{4} \approx O(1)$

the adiabaticity condition required

$$\hbar\dot{\theta} \ll \underbrace{|E_e(t) - E_g(t)|}_{\text{hardest to follow @ } \Delta E_{min}}$$



So, if we satisfy

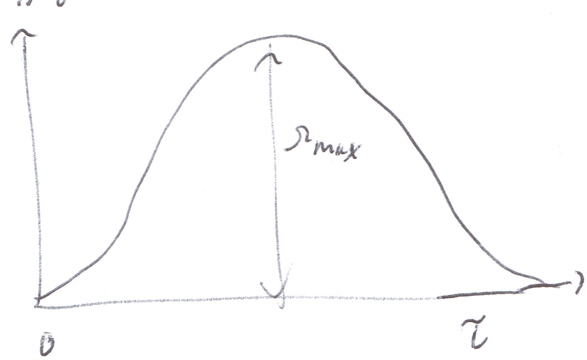
$\dot{\theta} \ll 2\Omega$, we'll be adiabatic @ all times

we can relate the total change in θ to Ω and τ by

$$\delta\theta = \int \dot{\theta}\, dt \ll \int 2\Omega(t)\, dt = 2A$$ where A is the pulse area

$= 2\Omega_{avg}\tau$

if $\Omega(t)$ is of the form



such that $2A = \Omega_{max}\tau$,

then $\delta\theta \approx 1 \ll \Omega_{max}\tau$

$\boxed{\text{and } \tau \gg \frac{1}{\Omega_{max}}}$

Note: fo

Rabi π-pulse

$\tau = \frac{\pi}{4\Omega}$

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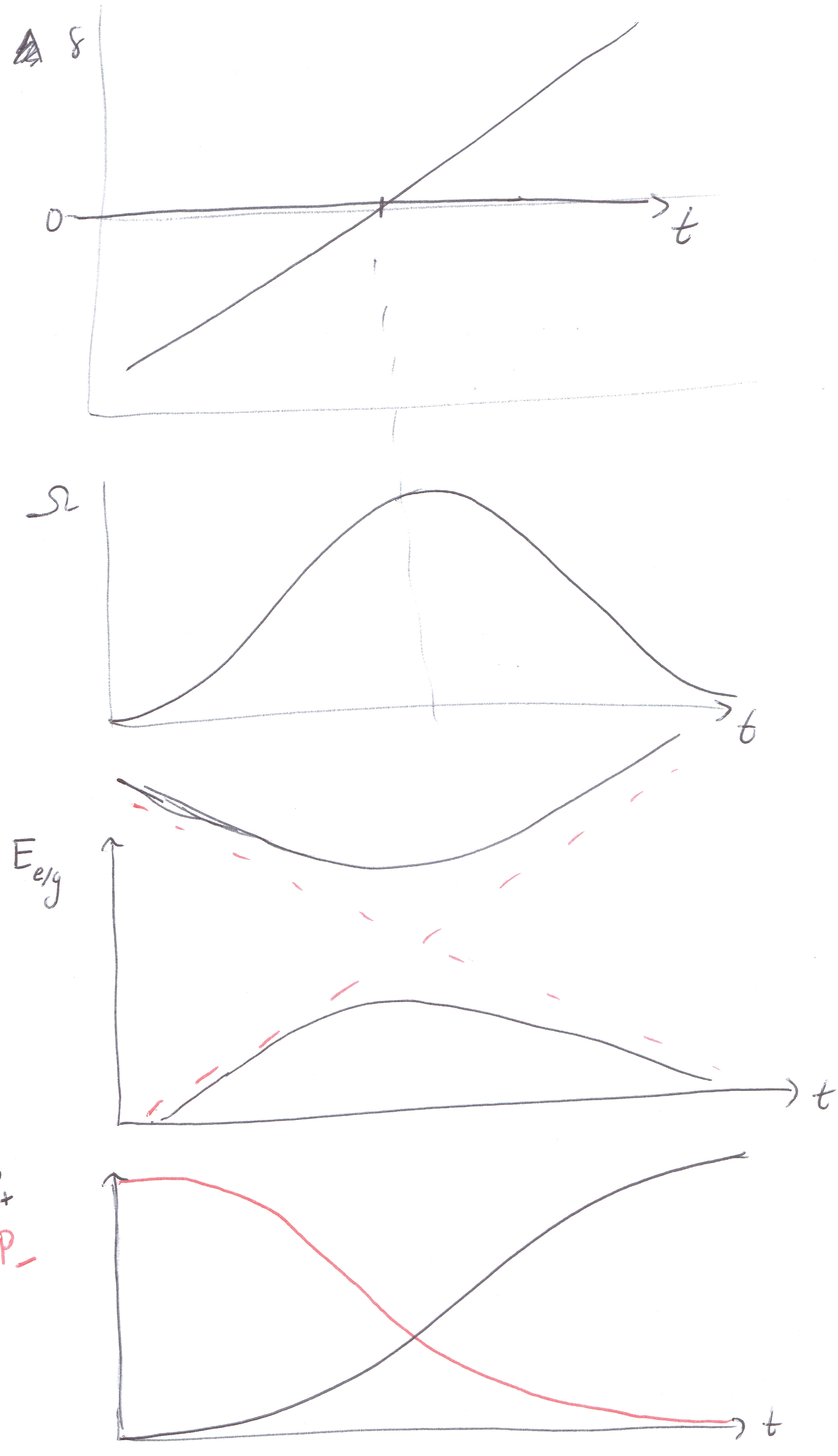
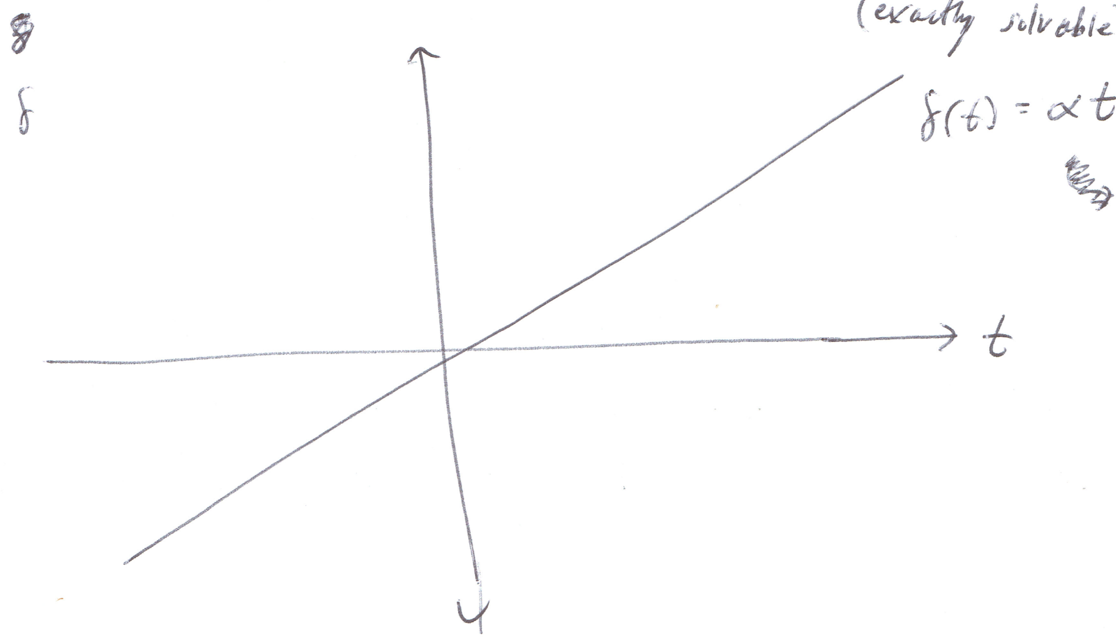
ARP



(8) Linear sweep, constant Ω $\rightarrow$ Landau-Zener (...) problem

(exactly solvable)



starting in $|-\rangle$ @ $t = -\infty$, it was shown that at long times

$$\boxed{P_+^{t\to\infty} = 1 - e^{-2\pi|\Omega_0|^2/|\alpha|} = P_{\text{diabatic}}}$$

for $\delta = 4\left(\frac{t}{T}\right)$

(9) Can we do better?

- faster transfer?
- still impervious to variations of parameters?

superadiabatic or counterdiabatic protocols

for $H(t)$ non-adiabatic, design $H'(t) = H(t) + H_s(t)$

that corrects for non-adiabatic terms of $H(t)$

for $H(t) = \Omega_0 \hat{\sigma}_x + \alpha t \, \hat{\sigma}_z$

correction needed is of the form

$$\boxed{H_s(t) \propto \dot{\phi} \, \hat{\sigma}_y} \quad \text{w/} \quad \tan\phi = \frac{\Omega}{\delta(t)}$$

Looks a bit like a Lorentz force

"field" along $\hat{\sigma}_x$ @ $\delta = 0$

"velocity" along $\hat{\sigma}_z$

induced "kick" along $\hat{\sigma}_y$

"Geometry and non-adiabatic response in quantum and classical systems"

Phys. Rep. 697 (2017) 1-87

Kolodrubetz, Sels, Mehta, Polkovnikov

$$H \to H - \dot{\lambda} A_\lambda$$

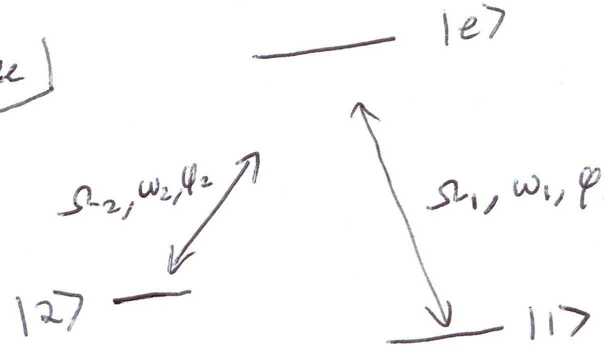
$$\vec{A}_\lambda = i\hbar \langle n(\lambda) | \vec{\nabla}_\lambda | n(\lambda) \rangle$$

Berry connection

$$\gamma_\lambda = \int_C d\vec{r} \cdot \vec{A}(r)$$

Three-level systems + STIRAP

Λ-type)



let $E_{e2}, E_{e1} \gg E_{12}$

and

$-\hbar\Delta_1 = E_{e1} - \hbar\omega_1$,

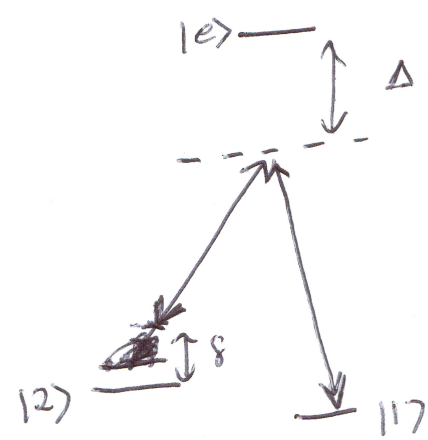
$-\hbar\Delta_2 = E_{e2} - \hbar\omega_2$

let $\delta = \Delta_1 - \Delta_2$

this system has the Hamiltonian

$$H = \begin{pmatrix} 0 & 0 & -\Omega_1 e^{-i(\omega_1 t + \varphi_1)} \\ 0 & -\delta & -\Omega_2 e^{-i(\omega_2 t + \varphi_2)} \\ -\Omega_1 e^{i(\omega_1 t + \varphi_1)} & -\Omega_2 e^{i(\omega_2 t + \varphi_2)} & -\Delta_1 \end{pmatrix}$$

there are a few limiting cases of interest in these 3-level systems



One limit) $\Omega_{1,2}, \delta \ll \Delta \sim \Delta_{1,2}$

level $|e\rangle$ can be adiabatically eliminated $\left(P_e^{1,2} \sim \frac{\Omega_{1,2}^2}{(\Omega_{1,2}^2 + (\delta/2)^2)} \ll 1 \right)$

yielding
$$\dot{c}_1 = -i \frac{|\Omega_1|^2}{\Delta} c_1 - i \frac{\Omega_2 \Omega_1^*}{\Delta} e^{i(\varphi_2 - \varphi_1)} c_2$$

in "rotating" frame

$$\dot{c}_2 = i\left(\delta - \frac{|\Omega_2|^2}{\Delta}\right) c_2 - i \frac{\Omega_1 \Omega_2^*}{\Delta} e^{i(\varphi_1 - \varphi_2)} c_1$$

and we can just set $\Omega_1 = \Omega_1^*$
$\Omega_2 = \Omega_2^*$

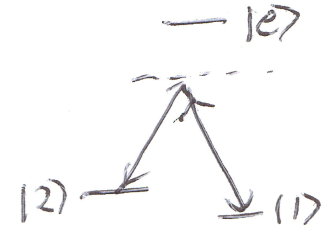
$$\boxed{\Omega_{Rabi}^{eff} = \frac{\Omega_1 \Omega_2}{\Delta}}$$

2-photon transition

effective 2-level system

case of present interest (adiabatic process)

$\underbrace{\delta = 0}_{\Delta_1 = \Delta_2 = \Delta}$ $\rightarrow$ go to rotating frame to get rid of fast time dependence



"2-photon resonance"
$\delta = 0$

$$\tilde{H} = \begin{pmatrix} 0 & 0 & \Omega_1 \\ 0 & 0 & -\Omega_2 \\ -\Omega_1 & -\Omega_2 & -\Delta \end{pmatrix}$$

let $\varphi_1 = \varphi_2 = 0$

$\hookrightarrow$ dressed states

$$|\Phi_+\rangle = \sin\theta \sin\phi |1\rangle + \cos\theta \sin\phi |2\rangle + \cos\phi |e\rangle$$

$$|\Phi_0\rangle = \cos\theta |1\rangle - \sin\theta |2\rangle$$

$$|\phi_-\rangle = \sin\theta \cos\phi |1\rangle + \cos\theta \cos\phi |2\rangle - \sin\phi |e\rangle$$

w/ $E_{+,0,-} = \begin{cases} \frac{1}{2}\left(-\Delta + \sqrt{\Omega^2 + \Delta^2}\right) \\ 0 \\ \frac{1}{2}\left(-\Delta - \sqrt{\Omega^2 + \Delta^2}\right) \end{cases}$

$$\tan\theta = \frac{\Omega_1}{\Omega_2}$$

$$\tan 2\phi = -\frac{\sqrt{|\Omega_1|^2 + |\Omega_2|^2}}{\Delta} = -\frac{\Omega}{\Delta}$$

"Dark" state $\rightarrow$ no $|e\rangle$ component $\rightarrow$ goal, want to populate $|\Phi_0\rangle \approx |1\rangle$ at $t = 0$

$\theta = 0$

$\hookrightarrow \theta = \frac{\pi}{2}$

and adiabatically follow to $|\Phi_0\rangle \approx |2\rangle$ @ t_{final}

(STImulated Raman Adiabatic Passage)

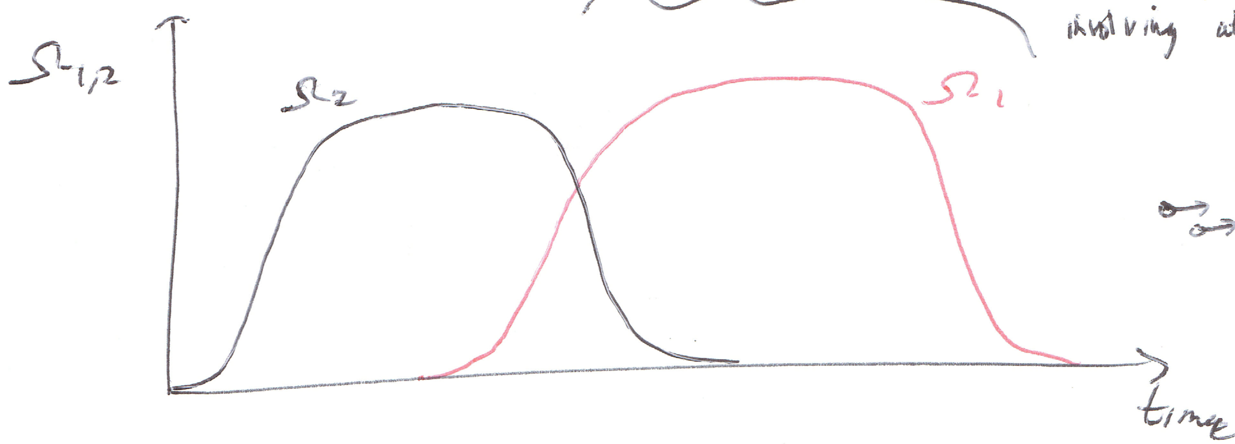
STIRAP

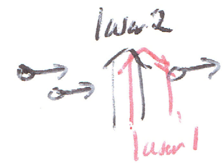
Counter-intuitive approach

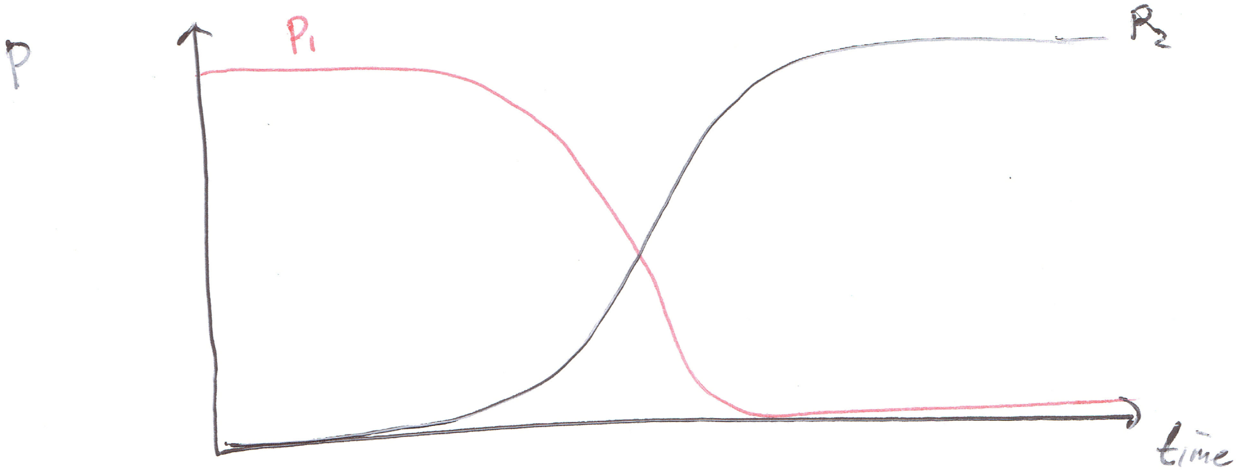
to move from $|1\rangle$ to $|2\rangle$, first "couple" $|2\rangle$ and $|e\rangle$, then "couple" $|1\rangle$ and $|e\rangle$

(crucial: need some overlap in the "couplings")

note highly relevant Ω_α profile w.r.t. experiments involving atomic/molecular beams







retained insensitivity to

$\Omega_1, \Omega_2, \Delta, \varphi_1, \varphi_2$, timing

see RMP 89 015006 (201