Photon scattering → scattering force

Each time a photon gets absorbed from a laser beam (for $|g\rangle \to |e\rangle$ transition), the atom gets a momentum "kick" of $\hbar\vec{k}$ (w/ $k = 2\pi/\lambda$ and $\hat{k}$ the direction of beam propagation).

When the atoms decay ($|e\rangle \to |g\rangle$ spontaneous emission), they also get a recoil "kick" of $\sim \hbar k \hat{k}_{random}$, i.e. along some random direction. On average, $\hbar\vec{k}$ is transferred for every scattering event, i.e. $\Delta\vec{p}_{scatt} = \hbar\vec{k}$

We can relate this to a scattering force:

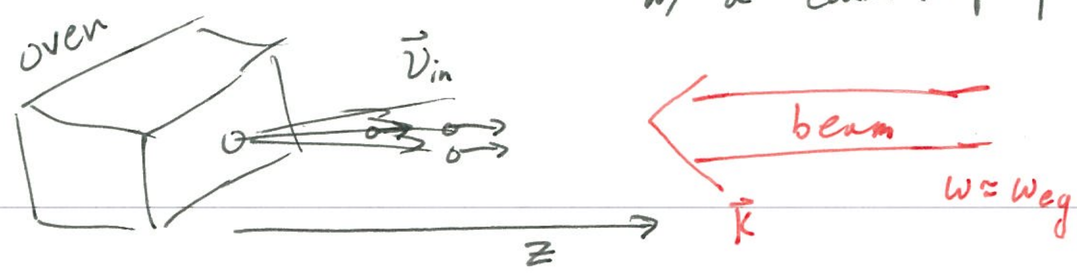
$$\vec{F}_{scatt} = \frac{d\vec{p}}{dt} = \Delta\vec{p}_{scatt} \underbrace{R_{scatt}}_{\text{scattering rate}} = \hbar\vec{k}\,\Gamma\rho_{ee} = \hbar\vec{k}\,\frac{\Gamma}{2}\,\frac{s}{1+s+\left(\frac{2\delta}{\Gamma}\right)^2}$$

for atom at rest, $\underline{\delta^{rest} = \omega - \omega_{eg}}$

if moving, also a Doppler shift $\delta^{v} = \omega - \omega_{eg} + kv$

scattering rate depends on velocity!

One typical scenario: You produce a fast beam of atoms / molecules from an oven, and you'd like to slow them w/ a counter-propagating beam



problem: if you start w/ $\delta^{v_{in}} = 0$, i.e. $\omega = \omega_{eg} - k v_{in}$

to maximize scattering force at the start, the velocity changes and the resonance condition δ^{v} changes as well [gives $\Delta v \sim$ few $\times \frac{\Gamma}{k}$ for $s \sim 1$, few $\frac{m}{s}$]

solution → compensate for expected change in velocity along z (change in velocity as function of t

if $\vec{v}_{in}$ is known and s is known (along w/ Γ), can predict $v(z)$ and $v(t)$ if we assume we can maintain resonance condition (keep $R_{scatt} = \frac{\Gamma}{2} \frac{s}{1+s}$)

one way: if $|g\rangle$ and $|e\rangle$ have different magnetic moments, then ω_{eg} depends on magnetic field, $\vec{B} = B_z \hat{z}$. We can vary the field to compensate chang in $-kv$

Zeeman slower :

for constant deceleration: $v = v_{in}\left(1 - \frac{z}{L_{stop}}\right)$

[L_{stop} is stopping distance

if $\vec{F} = \vec{F}_{max} = \frac{\hbar \vec{k} \Gamma}{2}$, $L_{stop}^{min} = \frac{m v_{in}^2}{2|F_{max}|} = \frac{m v_{in}^2}{\hbar k \Gamma}$]

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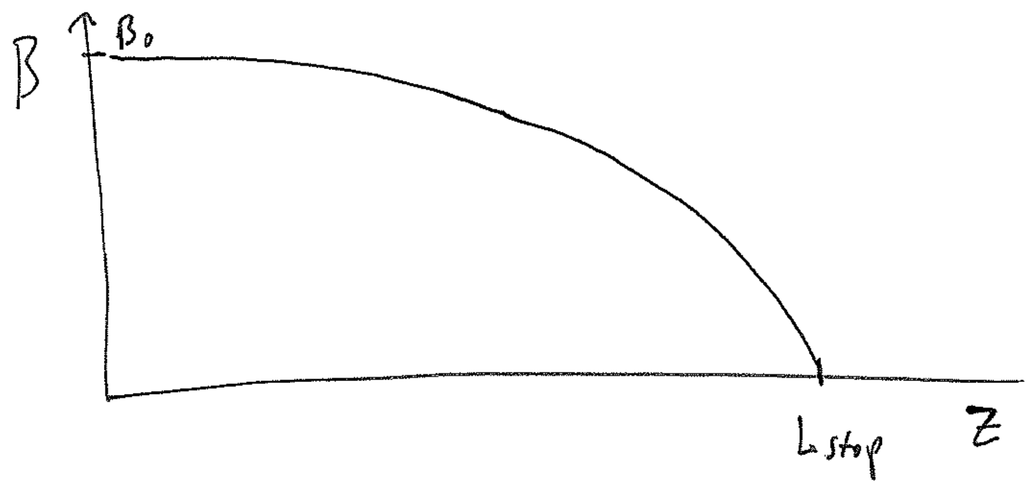
want $\delta = \omega - \left(\omega_{eg} + \frac{\Delta\mu B}{\hbar}\right) + kv = $ constant (pref. 0)

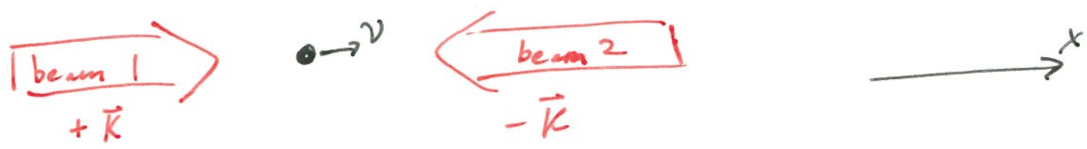
$\left[\frac{\Delta\mu}{\mu_B} = (g_e m_{F,e} - g_g m_{F,g}) \right]$

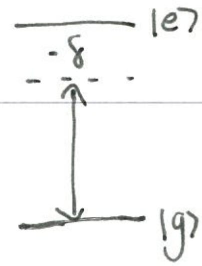
so $B(z) = B_0 \sqrt{1 - \frac{z}{L_{stop}}}$ w/ $B_0 = \frac{\hbar k v_{in}}{\Delta\mu}$



Note: for some systems, it's more convenient to have ω vary in time $\Rightarrow$ "chirped" laser frequency. This gives a pulsed version of this slowing method, but also works.

Optical molasses (1D)





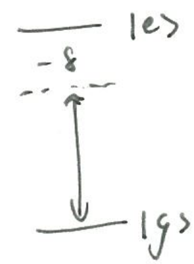
for $v=0$, both lasers detuned by $\delta = \omega - \omega_{eg}$

Atom sees light w/ Doppler shift $\Delta\omega = -\vec{k}\cdot\vec{v}$; different for each beam

Again, assume that Δp_{spont} from spontaneous emission cancels on average, so $|\hbar k|$ imparted on avg. per scattering event, or

$$\vec{F}_{scatt} = \vec{F}_{scatt,1} + \vec{F}_{scatt,2} = +\hbar\vec{k}_1 R_1 + \hbar\vec{k}_2 R_2 = \underline{\hbar k \hat{x}\,(R_1 - R_2)}$$

for atom moving to the left and $\delta < 0$



$\omega < \omega_{eg}$

beam 1 blue-shifted (closer to resonance) $\to$ higher R_1

beam 2 red-shifted (further from resonance) $\to$ lower R_2

$\Rightarrow$ net force in $+\hat{x}$, opposing motion

[exact opposite if atom initially moving to the right!]

This gives a velocity-dependent force that decelerates the particle if $\delta < 0$ (just like friction!!!)

so, $\vec{F}_{tot} = \hbar K \hat{x} (R_1 - R_2)$ where $R_{1/2} = \frac{\Gamma}{2} \frac{s}{1 + s + \left(\frac{2(\delta \mp \omega_D)}{\Gamma}\right)^2}$

w/ $\omega_D = K v_{in}$

if $\underline{\omega_D \ll \Gamma}$, i.e. $v_{in} \ll \frac{\Gamma}{K}$,

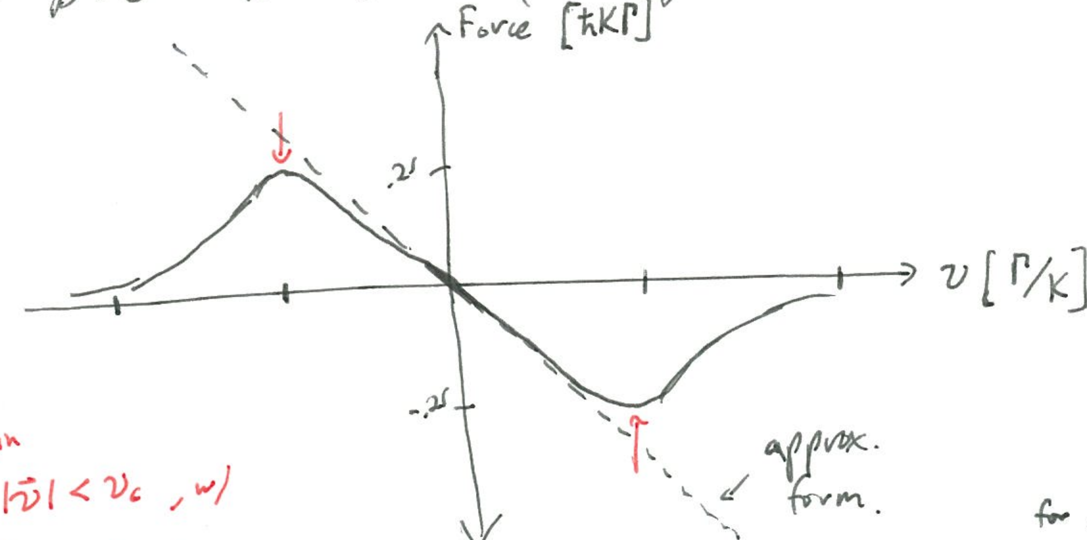
we can approximate this as [ignoring terms $\mathcal{O}\left(\left(\frac{Kv}{\Gamma}\right)^4\right)$]

viscous force called

$$\vec{F}_{OM} \approx \frac{8 \hbar K^2 \delta s_0}{\Gamma \left(1 + s_0 + \frac{4\delta^2}{\Gamma^2}\right)^2} \vec{v} = -\beta \vec{v}$$ optical molasses

where $\beta > 0$ if $\delta < 0$ ($\omega < \omega_{eg}$)

Note if $\delta > 0$ ($\omega > \omega_{eg}$), this accelerates atoms



for $s_0 = 2$, $\delta = -\Gamma$

Works within range of $|\vec{v}| < v_c$, w/ $v_c \approx \frac{\Gamma}{K}$ "capture velocity"

How does this cool? $|\vec{v}|$ gets smaller for $|\vec{v}| \lesssim v_c$, or more directly

$$\dot{E} = \frac{d}{dt}\left(\frac{1}{2} m v^2\right) = m v \dot{v} = v F = -\beta v^2$$

for range of velocities w/ r.m.s. value $\bar{v}$

s.t. $\frac{1}{2} m \bar{v}^2 = \frac{1}{2} k_B T$ (1D)

$$\frac{d}{dt}\left(\frac{1}{2} m \bar{v}^2\right) = k_B \dot{T} = -\beta \bar{v}^2 = -\beta\left(\frac{k_B T}{m}\right)$$

so $\dot{T} = -\frac{\beta}{m} T \Rightarrow$ cooling

Cooling power falls off w/ large Doppler shifts, for $|v| > \Gamma/k \sim$ few m/s in the alkalis

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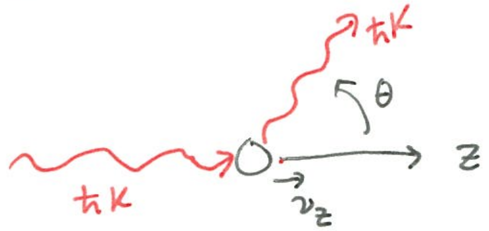
So, how cold can we get? $\dot{T} = -\frac{\beta}{m} T$

cooling to $T = 0$, right?

No! Unfortunately, we've brushed aside the random recoil events.

random recoils lead to diffusion in momentum space $\langle v \rangle_i = 0$, but spread of velocities

Let's look in detail @ absorption / re-emission



to conserve momentum, need

$$v_z \to v_z'$$

w/

$$m v_z + \hbar K = m v_z' + \hbar K \cos\theta$$

in 3D, over Ω

$$\Delta v_z = \frac{\hbar K}{m}(1 - \cos\theta)$$

averaging over θ, direction of scattered photon

$$\langle \Delta v_z^2 \rangle_\Omega = \left(\frac{\hbar K}{m}\right)^2 \int \frac{(1-\cos\theta)^2}{4\pi} d\Omega = \frac{1}{2}\left(\frac{\hbar K}{m}\right)^2 \int_{-1}^{1} (1-x)^2 dx$$

w/ $x = \cos\theta$

$$= \frac{1}{2}\left(\frac{\hbar K}{m}\right)^2 \times \frac{4}{3} = \frac{4}{3}\frac{E_{rec}}{m}$$

w/ $E_{rec} = \frac{p_{rec}^2}{2m} = \frac{\hbar^2 K^2}{2m}$

the recoil energy

for 2 beams (1D optical molasses)

$$\frac{1}{2} m \frac{d}{dt}\langle v_z^2 \rangle_\Omega = \frac{1}{2} m \left(\frac{4}{3}\frac{E_{rec}}{m}\right)(2 R_{scatt}) = \frac{4}{3} E_{rec} R_{scatt}$$

for 6 beams (3D optical molasses)

$$\frac{1}{2} m \frac{d}{dt}\langle v_z^2 \rangle_\Omega = 4 E_{rec} R_{scatt}$$

Combining w/ "cooling"

$$\frac{1}{2} m \frac{d}{dt}\langle \bar{v}_z^2 \rangle = 4 E_{rec} R - \beta \bar{v}_z^2$$

get $$\frac{d}{dt}\langle \bar{v}_z^2 \rangle = \frac{8 E_{rec}}{m} R_{scatt} - \frac{2\beta}{m} \bar{v}_z^2$$

$$\frac{d}{dt}\langle \bar{v}_z^2 \rangle = 0 \quad \text{in equilibrium}$$

giving $$\underbrace{\bar{v}_z^2 = \frac{4 E_{rec} R}{\beta}}$$

ignoring saturation effects (valid if $1 + \left(\frac{2\delta}{\Gamma}\right)^2 \gg s$)

$$R \simeq \frac{\Gamma}{2} s \frac{1}{1+\left(\frac{2\delta}{\Gamma}\right)^2} \qquad \beta \simeq \frac{8 \hbar k^2 \delta s}{\Gamma \left[1+\left(\frac{2\delta}{\Gamma}\right)^2\right]^2}$$

such that

$$\bar{v}_z^2 \simeq \frac{\hbar \Gamma^2}{8 m \delta}\left[1+\left(\frac{2\delta}{\Gamma}\right)^2\right]$$

equipartition

$$\frac{1}{2} m \bar{v}_z^2 = \frac{1}{2} k_B T$$

$$T = \frac{m \bar{v}_z^2}{k_B}$$

$$T = \frac{\hbar\Gamma}{4k_B} \frac{1+\left(\frac{2\delta}{\Gamma}\right)^2}{2\left(\delta/\Gamma\right)} = \frac{\hbar\Gamma}{4k_B}\underbrace{\left(\frac{\Gamma}{2\delta}+\frac{2\delta}{\Gamma}\right)}_{\substack{\min @ \\ 2\delta=\Gamma}} \Longrightarrow \boxed{T_{Dopp} = \frac{\hbar\Gamma}{2k_B}}$$

for alkalis, $T_{Dopp} \sim$ few $\times 100\ \mu K$ "Doppler limit"

$T_{Dopp}^{Na} \approx 235\ \mu K$ w/ $\frac{\Gamma}{2\pi} \approx 9.8$ MHz