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Observation of Quantum Effects and Quantum Ground State for Microscale and Smaller Oscillators

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1 Introduction

Quantum vacuum zero-point energy (ZPE) has been one of the cornerstones of modern quantum mechanics, quantum field theory, and quantum gravitation for quite some time. Historically, the first suggestion of ZPE existence was made by A. Einstein and O. Stern. [1] Specifically, they improved the theoretical model of the specific heat of hydrogen at low temperature by adjusting Planck's formula for the radiating energy of a vibrating atom by a temperature independent value $\frac{h\nu}{2}$:

$$\varepsilon = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1} + \frac{h\nu}{2}.$$

This adjustment to better fit the experimental data was later proven to actually have physical significance when the solution of the Schrödinger's equation for a quantum harmonic oscillator showed that the ground state has non-zero energy relative to the bottom of the harmonic potential well: $E = \hbar\omega(n + \frac{1}{2})$. This result on its own may not seem too important. However, most physical potentials can be approximated as harmonic near their equilibrium points, so the conclusion has a large range of possible applications. Moreover, when quantum field theory started to be developed, it became evident that (since all the fields can be represented as an infinity of quantum harmonic oscillators at every point of the space) the ground state of the field would also have non-zero energy, i.e. vacuum energy. The problem arises when one tries to calculate an integral of the zero-point energy of quantum harmonic oscillators over all the possible modes, as this integral is clearly divergent. It was figured out how to avoid the problem of infinite ground state energy by "renormalizing" by infinity and by actually dealing only with energy differences (just like in the case of classical potential energy). This seemed to work for quantum mechanics and quantum field theory applications, but created problems for quantum gravity, where one cannot neglect the absolute value of the vacuum energy.

So far there is no completely certain measurement of a physical effect following from the existence of vacuum energy of the fields. Vacuum energy is one of the possible explanations for the cosmological constant, the Casimir effect, and the Lamb shift, but so far there exist other plausible explanations not involving the vacuum energy. For example, with the cosmological constant, even if we try to create an argument that maybe we should make a Planck scale cut-off of the modes of the field that we integrate over, the calculated ZPE density is many orders larger than the experimental value of the cosmological constant.

At present, this is a very active area of both theoretical and experimental research and speculation. First, the ground state energy of single particles (atoms, ions) were of interest. Several decades ago, the first experiments were conducted in which measurements were taken on ions and neutral atoms cooled down to their vibrational ground state [2–4], which was also important in the development of quantum computer. Then the question arose: is it possible to measure the quantum ground state energy on a larger system?

Several attempts were made to cool down a nanomechanical resonator (e.g. [5–7]) to bring the quantum mechanics zero-point energy effects to the mesoscopic level. This might make possible the demonstration of a quantum mechanical effect on a mechanical system, which could actually be seen by a naked eye. For example, in [7] a microwave-frequency mechanical oscillator was cooled to its vibrational ground state while coupled to a superconducting qubit. Another method of measuring properties of the quantum state of a nanomechanical oscillator is cavity and circuit optomechanics, but the results in this area [5, 6] seem controversial. It is still not clear to some researchers if the demonstrated regime of the resonator is classical or quantum, or if it is possible to implement the precise control of a quantum state of this type of resonator.

This paper concentrates on the experimental techniques [5, 6] and theoretical background (e.g. [8, 9]) needed to bring a nanomechanical resonator to its vibrational ground state with appropriate phonon occupancy or at least detect its quantum motion. We discuss several controversial arguments and try to conclude if several recent experiments [5] can be considered enough of a proof that a nanomechanical resonator has indeed been cooled down to its quantum state using cavity and circuit optomechanics. We also discuss the scientific importance of those results and possible straightforward applications.

2 Existing experimental techniques

2.1 Creating and detecting ground state of ions and atoms

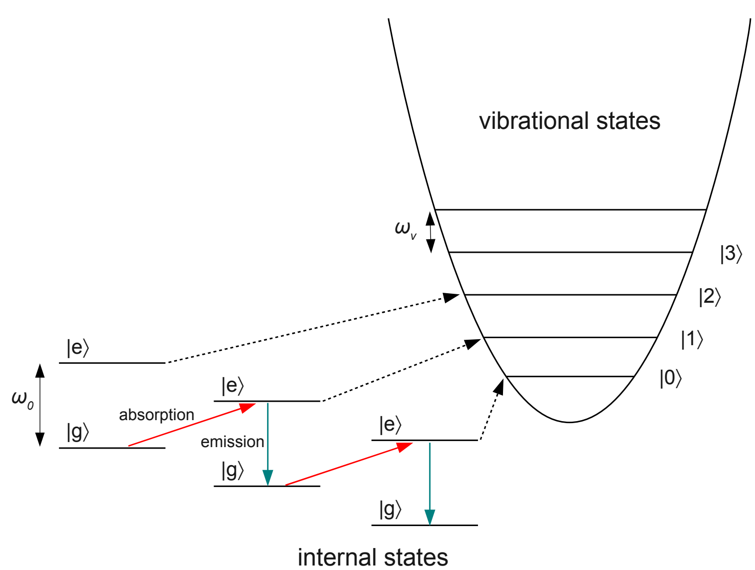


Figure 1. Schematic of sideband laser cooling with the laser frequency $\hbar(\omega_0 - \omega_v)$.

The first experiments dealing with vibrational ground states of atoms or ions were carried out in the late 1980s and early 1990s [2–4]. Ions are usually trapped using the Paul RF trap, while atoms are trapped with the magneto-optical trap (e.g. [10]). Particles are cooled down using different variations of laser cooling techniques. The first cooling stage is usually achieved with the optical molasses technique until the Doppler cooling limit is reached, or even beyond that using Sisyphus cooling with orthogonally polarized counter-propagating laser beams. In Doppler cooling, particle behavior can be considered classically, and cooling is possible under red-tuned laser irradiation because of the existing disbalance in absorption between photons moving towards the motion of an ion or atom and photons moving in the same direction. The higher probability of absorbing photons with higher frequency in the particle's frame of reference, along with the random emission following the absorption, provides an overall cooling of an ion or atom cloud until the Doppler limit is reached, when $k_B T = \frac{\hbar\Gamma}{2}$. [10]

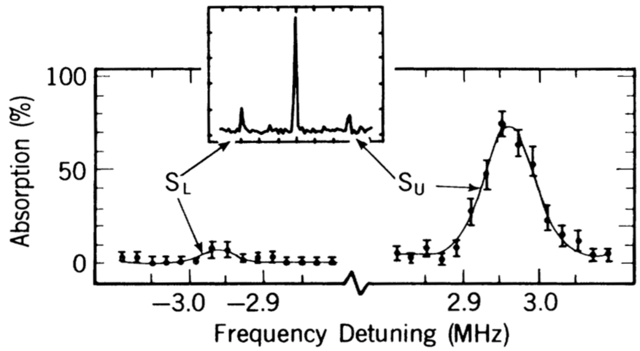


Figure 2. Example of the difference in the absorption spectrum for a trapped $^{198}Hg^{+}$ ion. The inset spectrum shows the carrier signal at ω_0 and two first sidebands at frequencies $\omega_0 - \omega_v$ and $\omega_0 + \omega_v$ with similar intensity before sideband cooling is applied. The enlarged spectrum shows differences in intensities of sidebands after sideband cooling. [2]

In order to cool ions or atoms down to discrete quantum vibrational states which can be detected, resolved sideband cooling is usually used (Figure 1). If we consider a particle to be a simple quantum harmonic oscillator, the spacing between every pair of closest vibrational levels is constant, and is equal to $\hbar\omega_v$. If the chosen internal transition of the ion/atom has energy of $\hbar\omega_0$, a laser tuned to the first red sideband at $\hbar(\omega_0 - \omega_v)$ would allow transitions to the excited state $|e\rangle$ but of the next lower vibrational level. From this state the emission to the ground state $|g\rangle$ of the same vibrational level is much more probable than back to the ground state of the higher vibrational level.

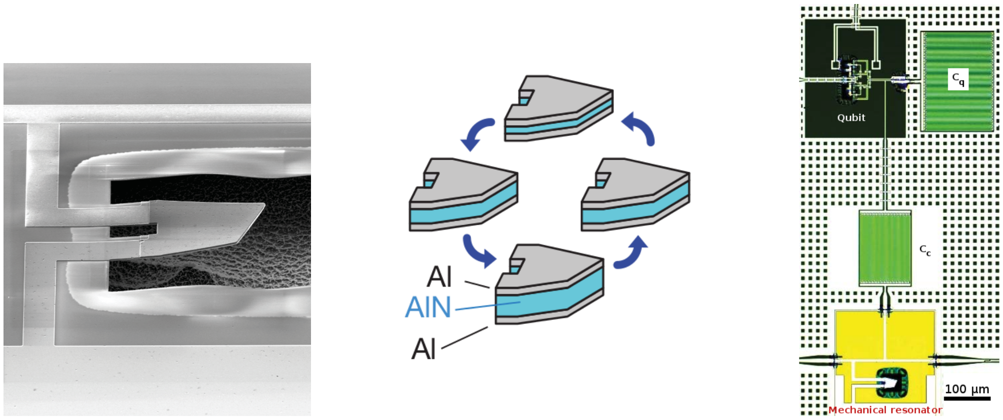
This means that on the average every photon absorbed by the atom in its ground state on some vibrational level reduces the atom's energy by $\hbar\omega_v$. If n_v is a vibrational quantum number, it is reduced by 1 on average after one photon absorption. In the end, most of the time ($\geq 95\%$ [2]) an ion/atom can be confined in its ground state with $\langle n_v \rangle \ll 1$.

Feedback on this cooling process is carried out by measuring the absorption spectrum of the trapped ion/atom. For example, in [2] a $^{198}Hg^{+}$ ion was studied. The strength of absorption on the first red sideband is proportional to $\langle n_v \rangle$, so when the ion is in the ground state, this absorption cannot happen. The absorption strength on the first blue sideband, however, is proportional to $\langle n_v \rangle + 1$. Upon reaching the quantum ground state of the harmonic oscillator, the red sideband absorption approaches zero. If a significant drop in the red sideband absorption signal is detected, it means that the ion is mostly in its ground state (Fig. 2). This idea of the ground state not being able to supply energy is to some extent used in the recent study of the nanomechanical resonator close to its ground state. [5]

Another version of sideband cooling is the resolved sideband Raman cooling with stimulated Raman transitions (between, for example, two hyperfine states). [4] In this case two pumping laser frequencies are used, exciting an atom to a virtual level and then de-exciting it with the second frequency to a lower level. This approach gives more flexibility and variation of the cooling linewidth.

2.2 Detecting quantum ground state of a mechanical resonator by coupling with a superconducting qubit

One of the pioneering works of great significance towards the achievement of a quantum ground state and control of a mechanical resonator is the work by A.D. O'Connell et al. [7] Their research showed that quantum mechanics effects may be demonstrated not only on the atomic



(a) (b) (c)

Figure 3. (a) SEM image of a suspended resonator. (b) An example of an oscillation cycle of the dilatational resonator. (c) Schematic of the experimental setup. [7]

scale, but also on the nano- and microscopic scale. In their experiment, a mechanical mode was cooled to its quantum ground state by using conventional dilution refrigeration (by mixing of ^{3}He with ^{4}He). The temperature needed should satisfy the equation $k_B T \ll \hbar\omega_0$, where ω_0 is the mode frequency. A micromechanical bulk dilatational resonator ('quantum drum') was engineered for this experiment. The active element in the resonator is the sandwiched piezoelectric material (AlN and Al) (Fig. 3(a)): an example of the oscillation cycle is shown in Fig. 3(b). The resonance frequency of this resonator is $\omega_0/2\pi = 6G$Hz, and hence the quantum ground state can be reached at $T \lesssim 0.1$K.

The resonator is coupled to a superconducting Josephson phase qubit via capacitance and superconducting wires. The phase qubit is essentially a Josephson junction with a capacitor and inductor in parallel. It can be approximated as a two-level system with a ground state $|e\rangle$ and an excited state $|g\rangle$, and its transition frequency can be tuned over a range near the resonator's resonance frequency. The electromechanical coupling between the resonator and the qubit is possible due to the piezoelectrical properties of the material of the resonator. The occupancy of the excited state of the qubit is measured with great precision by electrical interrogation, accumulating $\sim 1,000$ single-shot measurements. The schematic of the whole setup can be found in Fig. 3(c).

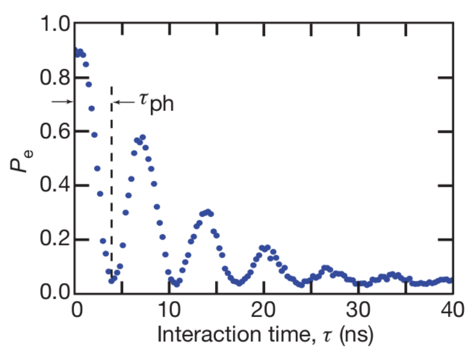


Figure 4. Time dependence of the excited state occupancy probability P_e of the qubit for fixed detuning $\Delta = 73M$Hz. [7]

When the qubit is well tuned to the resonance frequency, an energy exchange will occur between the coupled resonator and the qubit. If the resonator's mode is in its ground state, the occupation probability of the excited state of the qubit will approach zero. Here, the idea introduced above is implemented: an object in its quantum ground state cannot supply energy.

After the resonator was cooled to $25m$K and the qubit prepared in the ground state, measurements were carried out, and the average occupation probability of the ground state of the resonator was measured to be 93% with the corresponding maximum phonon occupancy number $\langle n_v \rangle < 0.07$.

As was also shown in [7], it is not only possible to measure the micromechanical resonator in its ground state, but also to control the state of the resonator by transferring energy from the qubit in its excited state. These impressive results are shown in Fig. 4: after the excitation of the qubit, the excited state is seen 'traveling' back-and-forth between the qubit and the resonator, and the occupation probability of the excited state of the qubit is shown to oscillate in time. Of course, the lifetime of this quantum coherent state was short but the results are nonetheless impressive and promising.

3 Measuring a nanomechanical resonator in its quantum state using cavity and circuit optomechanics

3.1 Experimental techniques

As discussed above, a microscopic mechanical resonator was successfully cooled to its ground state and strongly coupled to a superconducting qubit, allowing detecting and weak control of the quantum state of the resonator. [7] Another interesting approach is to try detecting the quantum motion of a resonator by optomechanical coupling. This is an important fundamental research topic, but also a crucial approach for the development of quantum computing and controlled quantum states using photonic qubits.

Recently, all the systems detecting phonon occupancy of nanomechanical resonators implemented the method of detecting light scattered by the resonator while taking its motion into account. [11] Thus, experiments usually measured a weak signal without the ability to clearly distinguish the classical regime from the quantum regime. If the technique cannot reliably detect quantum effects, then the behavior of the nanomechanical resonator can always be described by an appropriate effective temperature.

Recently A.H. Safavi-Naeini et al. [5] proposed a method to solve this problem based on the resolved sideband cooling technique from cold atoms/ions measurements. As before, the zero-point energy of a resonator cannot be shared but energy supplied by an external source can be absorbed. If two processes with sideband frequencies exist in the system, technically the difference in two signals should be possible to detect if the resonator is close to its ground state. This method has come to be known as motional sideband spectroscopy.

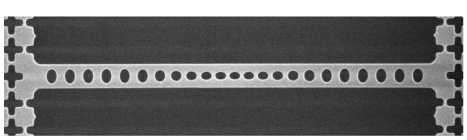


Figure 5. A patterned Si nanobeam used in the experiment. [5]

Just like in the case of ion/atom-trapping techniques, the asymmetry between photon absorption and emission by a resonator arises from the proportionality of those processes to respectively $\langle n_v \rangle$ and $\langle n_v \rangle + 1$. The idea is rather simple and very similar to the resolved sideband cooling of ions/atoms. The ratio of two sidebands amplitudes (blue to red) is equal to $(\langle n_v \rangle + 1)/\langle n_v \rangle$ and is approximately 1 when $\langle n_v \rangle \gg 1$, but increases rapidly with $\langle n_v \rangle$ approaching 0. In this experiment, resolved sideband spectroscopy is achieved by coupling a resonator to a high-Q optical cavity with narrow linewidth allowing separation of the laser frequency from the sidebands.

For this optomechanical coupling, a special device is used: a patterned Si nanobeam designed in such a way as to form an optomechanical crystal (OMC, Fig. 5). [12] In this crystal, mechanical (acoustical) modes are coupled to optical modes, as for example in a Fabry-Pérot cavity (Fig. 6).

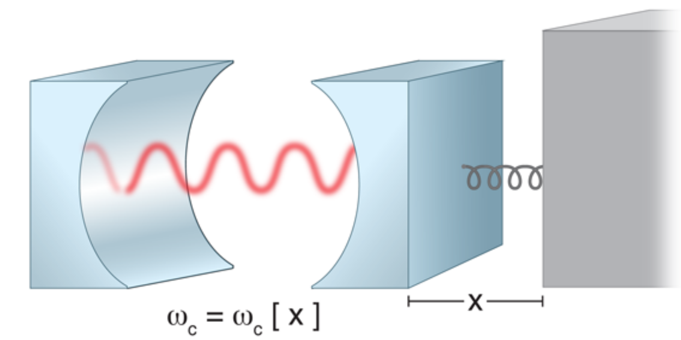


Figure 6. An example of an optomechanical system, a Fabry-Pérot cavity with the resonance light frequency depending on the position of the mirror. [11]

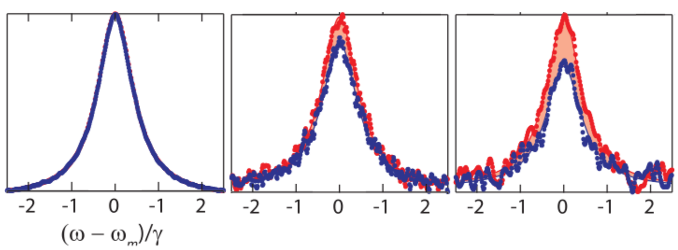


Figure 7. Measured readout beam spectra of Stokes (red) and anti-Stokes (blue) sidebands for respectively $\langle n \rangle = 85, 6.3, 3.2$ phonons from left to right. [5]

If ω is a mechanical resonance frequency of such a nanobeam, then the introduction of laser light of frequency ω_l causes two sidebands $\omega_l - \omega$ (Stokes) and $\omega_l + \omega$ (anti-Stokes) due to the effect of mechanical oscillations on the optical properties of the nanobeam. The design of the nanobeam ensures high-frequency mechanical modes (order of *G*Hz) and hence relatively easy sideband detection. Another technique used to simplify the sideband detection, which was used in the experiments in [5], is laser tuning used to bring each sideband to the mechanical resonance frequency. Hence, each sideband can be selectively filtered. The detection of the sideband signal showed a discrepancy in the amplitudes of the two signals: the maximum asymmetry in the amplitude readings reached 40% while the phonon occupancy $\langle n_v \rangle$ was calculated to be 2.6 ± 0.2. The signal development during cooling of the resonator is shown in Fig. 7 for three different phonon occupation numbers. Thus the quantum nature of the nanomechanical resonator was observed, and it was proven that the quantum behavior of optomechanical systems can be controlled to some extent. This technique may have a chance to be implemented in advanced quantum information analysis, quantum noise physics, etc.

3.2 Discussion

The result obtained by A.H. Safavi-Naeini et al. is interesting and significant if it indeed can be proven that the observed behavior is a consequence of quantum nature of the resonator. Some questions arise from the fact the minimum experimental phonon occupancy was not much less than 1, in fact, it was even greater than 1. Hence, the actual quantum ground state was not detected. Nevertheless, the drop of the anti-Stokes sideband compared to the Stokes sideband should be enough of a proof that the motion of the resonator was quantum but not classical. In the coarse approximation, the intensity of the sideband should be proportional to phonon occupancy $\langle n\rangle$ and $\langle n\rangle+1$ for the anti-Stokes and Stokes sidebands, respectively, as it was discussed earlier. So, the disbalance in the intensities should be observed for phonon occupancy number $\langle n\rangle > 1$. The last question is if 40% of a relative drop is enough to be a proof of quantum motion of the resonator when taking into account possible technical problems that could have arisen during the experiment and created a difference in signal not related to the quantum behavior.

First, let us consider the theoretical formalism used in analysis of the resonator's behavior. The Hamiltonian of the coupled system is given by [5] $\hat{H} = \hbar(\omega_r + g_r\hat{x}/x_{zpf})\hat{a}^\dagger\hat{a} + \hbar(\omega_c + g_c\hat{x}/x_{zpf})\hat{c}^\dagger\hat{c} + \hbar\omega_m\hat{b}^\dagger\hat{b}$, where $\hat{c}$ ($\hat{c}^\dagger$) and $\hat{a}$ ($\hat{a}^\dagger$) are the photon annihilation (creation) operators in two modes of the nanobeam: the cooling mode ($\omega_c/2\pi = 205.3T\text{Hz}$) and the readout mode ($\omega_r/2\pi = 194.1T\text{Hz}$); $\hat{x} \equiv (\hat{b}^\dagger + \hat{b})$ is the displacement operator of the breathing mode of the nanobeam ($\omega_m/2\pi = 3.99G\text{Hz}$); $x_{zpf} \simeq 2.7f\text{m}$ is the breathing mode zero-point fluctuation amplitude; $g_c/2\pi = 960k\text{Hz}$ and $g_r/2\pi = 430k\text{Hz}$ are the zero-point optomechanical coupling rates for respectively the cooling and readout mode, found experimentally. The noise power spectral density (PSD) in this case is [5,8,9,13]:

$$S_{xx}(\omega)/x_{zpf}^2 = \frac{\gamma\langle n\rangle}{(\omega_m+\omega)^2+(\gamma/2)^2} + \frac{\gamma(\langle n\rangle+1)}{(\omega_m-\omega)^2+(\gamma/2)^2}, \tag{1}$$

where γ is the total mechanical damping rate.

For two sidebands the spectrum is described by $S(\omega+\omega_l)$:

$$S(\omega+\omega_l) = \frac{\kappa_{e,r}}{2\pi\kappa_r}\frac{A_-^{(r)}\gamma\langle n\rangle}{(\omega_m-\omega)^2+(\gamma/2)^2} + \frac{\kappa_{e,r}}{2\pi\kappa_r}\frac{A_+^{(r)}\gamma(\langle n\rangle+1)}{(\omega_m+\omega)^2+(\gamma/2)^2}, \tag{2}$$

where $\kappa_{e,r}$ is the coupling rate to the fiber taper waveguide, κ_r is a damping rate for the readout mode resonance, $A_-^{(r)}$ and $A_+^{(r)}$ are the detuning-dependent anti-Stokes and Stokes motional scattering rates of the readout laser respectively. If a detuning is $\Delta = -\omega_m$, $A_-^{(r)} \gg A_+^{(r)}$ and the area of the Stokes sideband signal I_- is proportional to $\langle n\rangle+$, while for a detuning of $\Delta = \omega_m$, $A_-^{(r)} \ll A_+^{(r)}$, resulting in the area of the anti-Stokes sideband I_+ being proportional to $\langle n\rangle$. Hence, their relative change is given by $\eta \equiv = I_-/I_+ - 1 = 1/\langle n\rangle$.

The validity of the paper in question [5] has been recently argued in [14], and the quantum nature of their results was questioned. The first question that was raised concerns the possible presence of classical noise in the system that could give rise to the observed asymmetry in the sideband signal. The argument in favor of the quantum effects states that the researchers in [15] studied the noise properties of the laser (even with the varying power), and did not detect any changes in the sideband intensities ratio. Also, the consistency of the measurements under different parameters proves that even %40 ratio of the intensities is significant when it repeatable. Moreover, in order to satisfy the model suggested in [14] and discussed in [16], the classical noise required for the asymmetry to occur, as was studied thoroughly in [16], should have spectral and amplitude characteristics exactly like the optical vacuum noise and also be extremely stable without being affected by attenuation, varying laser power or spectral filtering. Thus, it seems that even if the classical interpretation of the data presented in [5] is theoretically possible, it is almost surely not applicable to the current experiment.

The second question posed in [14] concerns the interpretation of the results according to the theoretical model. M. Tsang claims that the relative reduction of the sideband signal is not at all connected with mechanical oscillations. On the other hand, A. H. Safavi-Naeini et al. claim that in [14] the Hamiltonian is treated in a specific way by eliminating the system operators and referring only to the bath operators, which in the end leads to the seeming independence of the measurements from the mechanical processes. When addressing the full quantum theory of quantized light and quantized mechanical motion [8, 9], the same result as the one shortly covered in [5] is found: the sideband intensity depends on the phonon occupancy and depends differently for Stokes and anti-Stokes sidebands.

These facts resolve most of the concerns posed by M. Tsang. Apparently, there are only two differences between this experiment and resolved sideband cooling of trapped ions/atoms: (a) the lifetime of an optical resonator is long, so only one sideband can be generated at a time; (b) instead of photon counting, in this experiment the detection of light was implemented through non-linear mixing with a reference frequency.

4 Conclusion

In this work several important physical concepts were discussed along with the corresponding experimental techniques. Studying, detecting and measuring zero-point energies of different systems are advanced modern research topics, both fundamental and practical in their aspect. Fundamentally, ZPE proves some of the existing theories, so further studying it will help to explain controversial experiments not yet obviously connected with the ZPE concept. In mechanical systems, cooling down a larger object brings quantum mechanics to the new spatial level, and some experiments (as we have seen) have already yielded measurements of ground state energies.

Resolved sideband cooling technique applies not only to single atoms and ions, but also larger mechanical objects and successfully helps studying them in their quantum motion. The same idea of the ground state not being able to supply energy runs through several experiments and is successfully used in the development of experimental techniques. Another experimental method discussed earlier is the coupling of a mechanical resonator to the superconducting qubit. By itself strikingly impressive, this result demonstrates non-classical effects not any less clear than the experiment with the optomechanical coupling. This experiment by A. H. Safavi-Naeini et al. can be readily improved to measure the true ground state of the resonator, and is of great importance for demonstrating other quantum phenomena based on optical interaction: Bell's inequalities violations, generation of non-Gaussian states, and use of optomechanics to observe quantum jumps. [11] So, hopefully, this experiment will be helpful for further advancing of the field.

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