PHYS598 AQG, Fall 2017
Homework Set #1, due by 4:30 pm on Friday 9/8

Q1) [1 pts] Foot 2.4 – penetration of the electron wave function into the nucleus

Q2) [3.5 total pts] **Effects of penetration (from Q1) and finite size of the proton**

Assume that the proton can be described as a sphere of radius r_p having uniform charge density ρ_p, where $\rho(r) = \rho_p$ for $r < r_p$, and 0 outside.

a) [0.75 pts] Derive the electric potential $V_p(r)$ due to this proton charge distribution
b) [1.75 pts] Let's assume that the proton has a radius of 1 fm. Use first order perturbation theory and this modified potential to estimate the shift in energy of the hydrogen 1s state (as compared to a point-like proton charge).
c) [1 pts] Perform a similar calculation as in (b) for the 2s state. Now consider measurement of the hydrogen 1s-2s transition energy. What fractional accuracy would be necessary to allow one to discern a 1% variation in the proton radius (1.00 fm to 1.01 fm)?

Note: Foot Tables 2.1 and 2.2 can be used to determine the 1s and 2s wave functions.

Q3) [2.5 total pts] **Helium ground state**

a) [1 pt] Foot 3.5a
b) [1.5 pts] Adding this positive interaction energy between electrons in the $1s^2$ configuration to the "non-interacting" ground state energy of -109 eV yields a revised estimate of -75 eV for the ground state of helium. This is still quite a bit off from the measured ground state energy of -79 eV (see discussion on Foot page 46).

   This simple estimate assumes that the 1s wave functions are not modified by the Coulomb repulsion between the two electrons. A better estimate of the ground state energy may be gained through a variational approach, where one assumes that the atomic number Z is effectively modified by the electron-electron interactions, taking a value Z'.

   Use this variational approach, i.e. plugging a modified value atomic number Z' into the form of the 1s wave functions, and find the value of Z' that minimizes the ground state energy, and this minimum energy value.

Q4) [1 pt] Foot 4.3 – quantum defect

Q5) [1 pt] Foot 4.4 – quantum defect

Q6) [1 pt] Foot 4.7 – fine structure & quantum defect