Q1) [1 pts] Foot 2.4 – penetration of the electron wave function into the nucleus

Q2) [3.5 total pts] Effects of penetration (from Q1) and finite size of the proton

Assume that the proton can be described as a sphere of radius $r_p$ having uniform charge density $\rho_p$, where $\rho(r) = \rho_p$ for $r < r_p$, and 0 outside.

a) [0.75 pts] Derive the electric potential $V_p(r)$ due to this proton charge distribution

b) [1.75 pts] Let’s assume that the proton has a radius of 1 fm. Use first order perturbation theory and this modified potential to estimate the shift in energy of the hydrogen 1s state (as compared to a point-like proton charge).

c) [1 pts] Perform a similar calculation as in (b) for the 2s state. Now consider measurement of the hydrogen 1s-2s transition energy. What fractional accuracy would be necessary to allow one to discern a 1% variation in the proton radius (1.00 fm to 1.01 fm)?

Note: Foot Tables 2.1 and 2.2 can be used to determine the 1s and 2s wave functions.

Q3) [2.5 total pts] Helium ground state

a) [1 pt] Foot 3.5a

b) [1.5 pts] Adding this positive interaction energy between electrons in the $1s^2$ configuration to the "non-interacting" ground state energy of -109 eV yields a revised estimate of -75 eV for the ground state of helium. This is still quite a bit off from the measured ground state energy of -79 eV (see discussion on Foot page 46).

This simple estimate assumes that the 1s wave functions are not modified by the Coulomb repulsion between the two electrons. A better estimate of the ground state energy may be gained through a variational approach, where one assumes that the atomic number $Z$ is effectively modified by the electron-electron interactions, taking a value $Z'$. Use this variational approach, i.e. plugging a modified value atomic number $Z'$ into the form of the 1s wave functions, and find the value of $Z'$ that minimizes the ground state energy, and this minimum energy value.

Q4) [1 pt] Foot 4.3 – quantum defect

Q5) [1 pt] Foot 4.4 – quantum defect

Q6) [1 pt] Foot 4.7 – fine structure & quantum defect