

## PHYS598 AQG, Fall 2017

### Homework Set #2 [corrected lines underlined in red], due by 4:30 pm on Friday 9/22

Q1) [1.5 pts] Foot Exercise 6.2 – hyperfine structure of lithium

Q2) [5.5 total pts] Atom in a magnetic field

Ignoring quadrupole terms and assuming field shifts that are small compared to the fine-structure splitting (such that  $J$  remains a good quantum number), the Hamiltonian describing an atom in a magnetic field with magnitude  $B_z$  along the  $z$  axis may be written as

$$H = H_{hfs} + H_B = hA_{hfs} \frac{\vec{I} \cdot \vec{J}}{\hbar^2} + \frac{\mu_B}{\hbar} (g_J J + g_I I) B_z$$

- (a) [3 pts] In general, finding the eigenstates and energies in this system requires a numerical solution (diagonalization of this Hamiltonian). However, we stated in class that for states with  $J = 1/2$ , as in the  $S_{1/2}$  ground states of hydrogen and the alkalis, the energies can be described by the Breit-Rabi formula

$$E = -\frac{\Delta E_{hfs}}{4F^+} + g_I m \mu_B B_z \pm \frac{\Delta E_{hfs}}{2} \sqrt{1 + \frac{4mx}{2F^+} + x^2}$$

Here,  $F^+ = (I + 1/2)$ ,  $\frac{\Delta E_{hfs}}{h} = A_{hfs} F^+$ ,  $m = m_I \pm m_J$  is the  $z$  component of the total angular momentum, and the term  $x$  is given by  $x = (g_J - g_I) \mu_B B_z / \Delta E_{hfs}$ .

Show that the state energies follow this Breit-Rabi formula for a general field value  $B_z$ .

At low fields the energy eigenstates are most easily expressed (approximately diagonal) in the  $|F, m_F\rangle$  basis. Express the  $|F, m_F\rangle$  states in terms of the  $|m_I, m_J\rangle$  basis states for arbitrary  $B_z$  field values.

- (b) [1 pt] For the case of  $^{87}\text{Rb}$ , make some nice plots of the energies given by your solutions (i.e. those of the Breit-Rabi formula) for magnetic field values from 0 to 0.2 Tesla (0 to 2000 Gauss). You can find the values of the various constants ( $A_{hfs}$ ,  $g_J$ ,  $g_I$ , etc.) in the [rubidium-87 data document](#) put together by Daniel A. Steck. Label the plotted lines by the relevant quantum numbers at low and high fields. Be careful to note the sign convention and use of  $\mu_B$  with  $g_I$  here.
- (c) [1.5 pts] Many experiments take advantage of the fact that the difference in energy between pairs of these states can be highly insensitive to magnetic field fluctuations for certain values of the parameter  $x$  (certain  $B_z$  values). This allows for accurate determination of the transition energies between such states (as for the  $m_F = 0$  “clock states” used in atomic clocks and Raman atom interferometers), as well as measurements

of small shifts of these transition energies due to other influences (as in the measurement of interaction shifts using “two-photon clock states,” as discussed in class).

At roughly what magnetic field does such an insensitivity in the energy difference between the states  $|F = 1, m_F = -1\rangle$  and  $|2, -1\rangle$  occur? At this field value, what form do the  $|F = 1, m_F = -1\rangle$  and  $|2, -1\rangle$  states take when written in terms of the  $|m_I, m_J\rangle$  basis states? How does this explain the dependence of their energies (and energy difference) near this field value?

Q3) [1.5 pts] **Foot Exercise 1.8 – classical radiative lifetime**

Q4) [1.5 pts] **Foot Exercise 7.2 – Rabi oscillations**