PHYS598 AQG, Fall 2017
Homework Set #3, due by 4:30 pm on Friday 10/6

Q1) [7 total pts – 1 pt for each part] **A driven two-level system**

Consider a pseudo-spin, 2-level system of states $|e\rangle$ and $|g\rangle$, having "bare" energies E_e and E_g, respectively. In the absence of "coupling," the Hamiltonian of the system is given simply by $H_0 = (\hbar\omega_0/2)\hat{\sigma}_z$, where $\hbar\omega_0 = (E_e - E_g)$. A time-dependent off-diagonal "coupling" term of the form

$$H_c = -h_\perp(t)|e\rangle\langle g|e^{-i[\omega t+\varphi(t)]} + h.c.$$

is added to the system, such as through an oscillating transverse magnetic field for the case of coupled spin states. The detuning δ of the frequency of this coupling term from the "bare" resonance frequency is given as $\delta = \omega - \omega_0$.

(a) Writing the initial state as $|\psi\rangle = a_e|e\rangle + a_g|g\rangle$, what are the equations of motion for a_e and a_g (from the time-dependent Schrödinger equation)?

(b) Assuming that $h_\perp$ and φ are constants (with $h_\perp$ a real, positive energy given by, e.g. $h_\perp = 0.1\hbar\omega_0$), solve these equations of motion for the starting conditions $a_e(t=0) = 1$ and $a_g(t=0) = 0$. Plot (over an appropriate range of times) the time-dependent probability $P_g(t)$ to find the state $|g\rangle$ for the cases $\delta = 0$, $\delta = h_\perp/\hbar$, and $\delta = 10h_\perp/\hbar$.

(c) Assume now that you start with an initial state $|\psi(t=0)\rangle = (|g\rangle - i|e\rangle)/\sqrt{2}$ and that the modulation is resonant ($\delta = 0$). For the four different phase values $\varphi = 0$, $\pi/2$, π, and $3\pi/2$, solve for and plot the time-dependent probability to measure the state $|g\rangle$. Again, plot over an appropriate range of times (such as from 0 to $h_\perp t/\hbar = 4\pi$).

(d) Provide a physical explanation (using the ideas of classical magnetic resonance) for any differences in the dynamical evolution observed in the four cases studied in (c).

(e) For the case $\varphi = 0$, compare the numerical solutions of the dynamics as found in (c) to those found for an alternative form of coupling, given by

$$H'_c = -2h_\perp(t)|e\rangle\langle g|\cos(\omega t + \varphi) + h.c.\ .$$

More specifically, show/plot solutions of the dynamics of $P_g(t)$ for these two different forms of coupling (H_c and H'_c) for coupling "strengths" of $h_\perp = 0.01\hbar\omega_0$ and $h_\perp = 0.1\hbar\omega_0$. Describe any observed differences.

(f) Consider again just the H_c form of coupling on resonance ($\delta = 0$). During a first "pulse" (i.e. a period of time in which the parameters δ, $h_\perp$, and φ are fixed), characterized by $\varphi = 0$, the state $|\psi\rangle = (|g\rangle - i|e\rangle)/\sqrt{2}$ is prepared (this "$\pi/2$ pulse" has a duration $\tau_{\pi/2}$). Immediately after this, a second "pulse" is applied, having the same duration and having the same coupling magnitude $h_\perp$, but with a variable phase φ. As a function of φ, determine and plot the population in the state $|g\rangle$ after this second pulse.

(g) Now imagine that the second "$\pi/2$" pulse is applied much later on, after a period of time given by $2\pi \times 2500\hbar/h_\perp$. During the intervening time between the two pulses, a second, separate coupling term of the form $-h'_\perp |e\rangle\langle g| e^{-i[\omega' t+\varphi']} + h.c.$ is applied. This second coupling term is weak ($h'_\perp/h_\perp = 0.01$, with $h_\perp = 0.1\hbar\omega_0$) and off-resonant ($\omega' = 0.8\omega_0$), and we assume to have $\varphi' = 0$. Does the presence of this added coupling term change the final measurement outcome (i.e. the φ-dependence of the $|g\rangle$ state population measured after this second pulse)? If so, how? Does this depend on the magnitude of $h'_\perp$? The phase φ'?

Q2) [3 pts] **Foot Exercise 7.4 –** ***The steady-state excitation rate with radiative broadening***