Lecture 23

- HW 5 posted
- Solutions for HW 3 posted
- HW 3 graded
- Reminder: Email me your presentation topics

Chern insulator (with inversion symmetry)

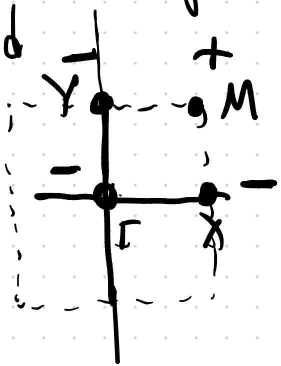
$$h(\vec{k}) = (\Delta + \Delta \cos k_y + t_1 \cos k_x)\sigma_z + t_2 \sin k_x \sigma_y + t_2 \sin k_y \sigma_x$$

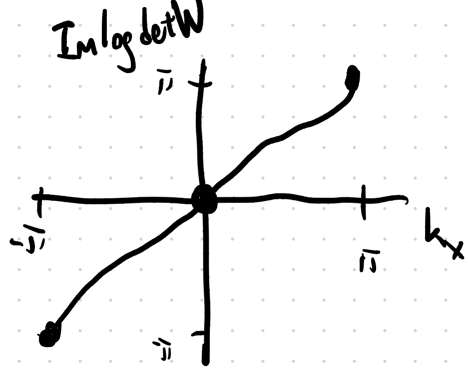
$$B(I) = \sigma_z$$

Two phases

① $|2\Delta| > t_1$, $\Delta, t_1, t_2 > 0$

Inversion eigenvalues for occupied band



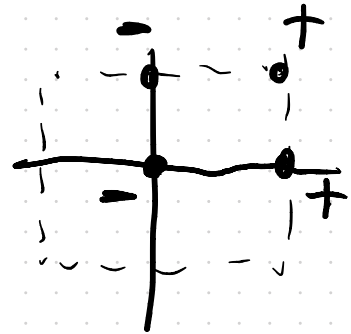


Chern number $C = \frac{1}{2\pi}\int_{-\pi}^{\pi} dk_x \frac{\partial}{\partial k_x} \mathrm{Im}\log\det W_{2\pi\leftarrow 0}(k_x)$

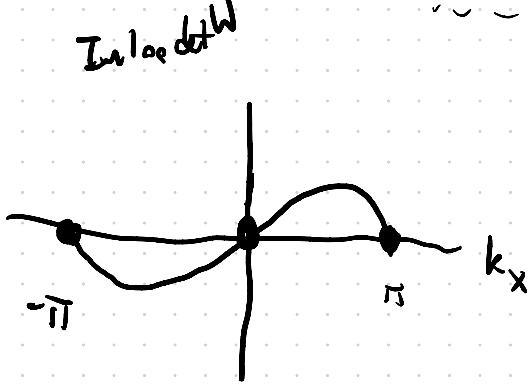
$= \frac{1}{2\pi}\oiint_{B.Z} d^2k\, \mathrm{tr}(\Omega)$ Berry curvature

$\in \mathbb{Z}$ $\quad$ $C = 1$

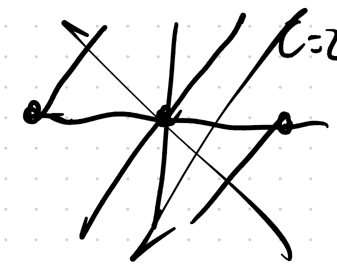
② $2\Delta < t_1$, $\Delta, t_1 > 0$



$$\varepsilon_\pm = \pm\sqrt{(\Delta + \Delta\cos k_y + t_1 \cos k_x)^2 + (t_2 \sin k_x)^2 + (t_2 \sin k_y)^2}$$



$C = 0$



$$(-1)^C = \prod_{\substack{\text{Inversion} \\ \text{invariant Momenta} \\ k_*}} B^{--}_{k_*}(I) \leftarrow$$

Symmetry indicator formula

$C \neq 0$: Nonzero Hall conductivity

- No exponentially localized Wannier functions - occupied band is topologically nontrivial)

$$\det W(k_\perp) = e^{i\gamma(k_\perp)}$$

$$\det W(k_\perp + \vec{b}_\perp) = \det W(k_\perp)$$

$$\gamma(k_\perp + \vec{b}_\perp) = \gamma(k_\perp) + 2\pi C$$

Now: Time-reversal symmetry: $\Rightarrow C = 0$

Time-reversal symmetry

$$W^{nm}_{2\pi\leftarrow 0}(k_x) = \langle u_{n k_x, 2\pi} | \prod_{k_y'}^{2\pi\leftarrow 0} P(k_x, k_y') | u_{m k_x, 0} \rangle$$

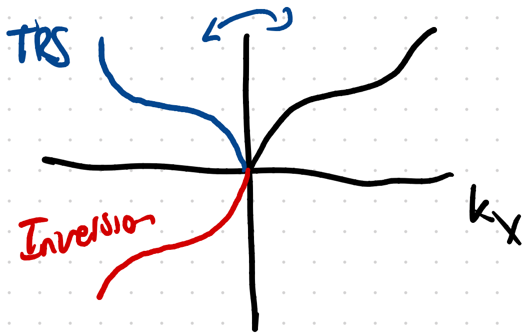
$$B(T)\, W_{2\pi\leftarrow 0}(k_x)\, B^{-1}(T) = W^{\dagger}_{2\pi\leftarrow 0}(-k_x)$$

Since T is represented by an antiunitary operator:

if $$W_{2\pi\leftarrow 0}(k_\perp) |w_{k_x}\rangle = e^{i\gamma(k_x)} |w_{k_x}\rangle$$

$$W^{\dagger}_{2\pi\leftarrow 0}(-k_x)\, U_T |w_{k_x}\rangle = e^{-i\gamma(k_x)}\, U_T |w_{k_x}\rangle$$

Eigenvalues $\left\{ e^{i\gamma_i(k_x)} \right\} = \left\{ e^{i\gamma_i(-k_x)} \right\}$



$$\det W(k_x) = \det W(-k_x) = e^{i\gamma(k_x)}$$

$$C = \frac{1}{2\pi}\int_{-\pi}^{\pi} dk_x \frac{\partial}{\partial k_x}\gamma(k_x) = \frac{1}{2\pi}\int_{-\pi}^{0} dk_x \frac{\partial \gamma}{\partial k_x} + \int_{0}^{\pi} dk_x \frac{\partial \gamma}{\partial k_x} = 0$$

Kane-Mele model:

Space group $P6/mmm1'$ ← time reversal

$1'$ - time-reversal symmetry

Primitive hexagonal

sixfold rotation

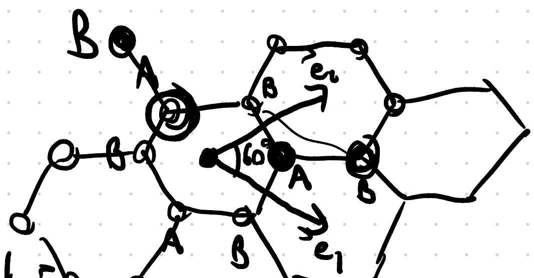
Inversion symmetry

mirror symmetry

Primitive Bravais lattice vectors

$$\vec{e}_1 = \frac{a}{2}\left(\sqrt{3}\,\hat{x} - \hat{y}\right)$$

$$e_2 = \frac{a}{2}\left(\sqrt{3}\,\hat{x} + \hat{y}\right)$$

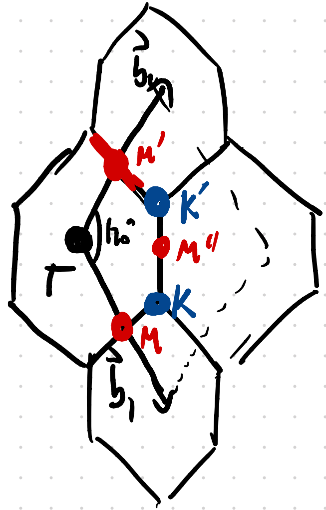


$$r_A = \frac{1}{3}\vec{e}_1 + \frac{1}{3}\vec{e}_2$$

$$r_B = \frac{2}{3}\vec{e}_1 + \frac{2}{3}\vec{e}_2$$

$$\vec{b}_1 = \frac{2\pi}{a}\left(\frac{\hat{x}}{\sqrt{3}} - \hat{y}\right)$$

$$\vec{b}_2 = \frac{2\pi}{a}\left(\frac{\hat{x}}{\sqrt{3}} + \hat{y}\right)$$



$$\Gamma = \vec{0}$$

$$M = \tfrac{1}{2}\vec{b}_1$$

$$M'' = \tfrac{1}{2}\vec{b}_1 + \tfrac{1}{2}\vec{b}_2$$

$$M' = \tfrac{1}{2}\vec{b}_2$$

$$K = \tfrac{2}{3}\vec{b}_1 + \tfrac{1}{3}\vec{b}_2$$

$$K' = \tfrac{1}{3}\vec{b}_1 + \tfrac{2}{3}\vec{b}_2$$

tight binding basis functions

$|\tau \sigma \vec{R}\rangle$

τ: A, B

σ: $\uparrow\downarrow$

$\vec{R}$: lattice site

3fold rotation symmetry

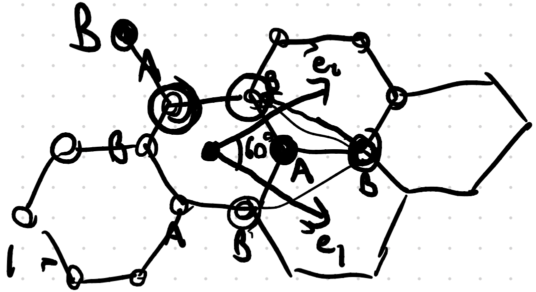
$$U_{C_3}|A\sigma\vec{0}\rangle = e^{\frac{-i\pi\sigma_z}{3}}|A\sigma -\vec{e}_1\rangle$$

$$U_{C_3}|B\sigma\vec{0}\rangle = e^{\frac{-i\pi\sigma_z}{3}}|B\sigma -2\vec{e}_1\rangle$$

Inversion: $U_I|A\sigma\vec{0}\rangle = |B\sigma -\vec{e}_1-\vec{e}_2\rangle$

$$U_I|B\sigma\vec{0}\rangle = |A\sigma -2\vec{e}_1-\cancel{2}\vec{e}_2\rangle$$

Fourier space $\boxed{B(I) = \tau_x\sigma_0}$

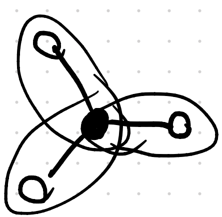


$$\boxed{\begin{aligned} B(C_3) &= \tau_0\, e^{-i\pi\sigma_z/3} \\ B(C_2) &= \tau_x\, e^{-i\frac{\pi}{2}\sigma_z} \end{aligned}}$$

Nearest Neighbor spin independent tight binding model

$$h^{\tau\tau'\sigma\sigma'}(R-R') = \langle \tau\sigma\vec{R}\,|\,H\,|\,\tau'\sigma'\vec{R}'\rangle$$

Nearest neighbors of $|A\sigma\vec{0}\rangle$ are $\left\{\begin{array}{l} |B\sigma\,\vec{0}\rangle \\ |B\sigma\,-\vec{e}_1\rangle \\ |B\sigma\,-\vec{e}_2\rangle \end{array}\right.$



$$t = \langle B \sigma \vec{R} | H | A \sigma \vec{R} \rangle$$

3 fold rotation

$$t = \langle B \sigma R - e_1 | H | A \sigma \vec{R} \rangle = \langle B \sigma \vec{R} - \vec{e}_2 | H | A \sigma \vec{R} \rangle$$

TRS: $t^* = t$

Inversion $t = \langle A \sigma \vec{R} | H | B \sigma \vec{R} \rangle = \langle A \sigma \vec{R} | H | B \sigma \vec{R} - \vec{e}_2 \rangle$

$$= \langle A \sigma \vec{R} | H | B \sigma \vec{R} - \vec{e}_1 \rangle$$

Fourier transforming

$$h^{\tau\tau'\sigma\sigma'}(k) = \sum_{R} e^{-ik\cdot(R + r_{\tau} - r_{\tau'})}\,\delta_{\sigma\sigma'}\,\langle \tau\sigma\vec{R}|H|\tau'\sigma'\vec{0}\rangle$$

$$= \begin{array}{cc} & \tau' \\ \tau & \begin{pmatrix} 0 & q(\vec{k}) \\ q^{*}(\vec{k}) & 0 \end{pmatrix} \end{array} \otimes \sigma_{0}^{\sigma\sigma'}$$

$$q(\vec{k}) = t\left[e^{-\frac{2\pi i}{3}(k_1+k_2)} + e^{\frac{2\pi i}{3}(2k_1-k_2)} + e^{\frac{2\pi i}{3}(2k_2-k_1)}\right]$$

Energies: $\varepsilon_{\pm}(\vec{k}) = \pm|q(k)|$

Three special values:

Γ: $q(\Gamma) = 3t \qquad h(\Gamma) = 3t\,\tau_x\,\sigma_0$

$\varepsilon_{\pm}(\Gamma) = \pm 3t$

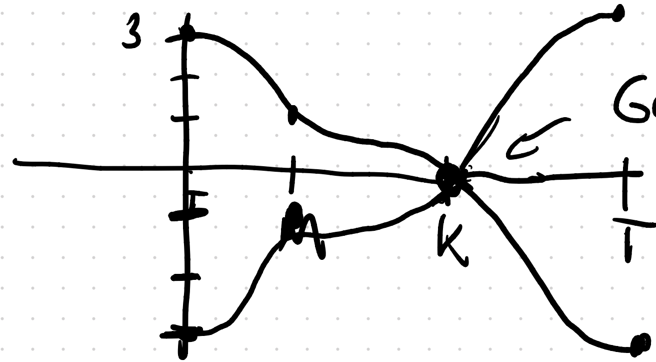
K: $k_1 = \frac{2}{3}$, $k_2 = \frac{1}{3}$ $\qquad q(K) = t\left[e^{-\frac{2\pi i}{3}} + e^{\frac{2\pi i}{3}} + 1\right] = 0$

$\varepsilon_{\pm}(K) = 0$

M: $k_1 = \frac{1}{2}$, $k_2 = 0$ $\qquad q(M) = t\left[\frac{1}{2} - \frac{i\sqrt{3}}{2}\right]$

$$h(M) = t\left(\frac{1}{2}\tau_x + \frac{\sqrt{3}}{2}\tau_y\right)\sigma_0$$

$$\varepsilon_\pm = \pm t$$



Gapless "Dirac cone"