P598NEU 2019 Lecture Notes

Weak Interaction

(Please read Chapter 1 of Giunti and Kim. You might also wish to read Chapter 2, Sects. 2.1 to 2.9 of Giunti and Kim for a nice review of Dirac field.)

1. Historical Overview

Radioactivity was discovered by Becquerel in 1896 and β-ray was soon identified as electrons. In 1914 Chadwick observed that the energy of electrons emitted in β-decay is continuous. This puzzling discovery suggested violation of energy and angular momentum conservation. In the early 1930s, Pauli proposed the existence of a weakly interacting fermion, neutrino, as a solution to this puzzle. Soon afterwards, Fermi put forward his theory of nuclear β-decay in 1933.

Although the study of weak interaction was once limited to nuclear β decays, we now know that weak interaction is present for all quarks and leptons, although the weak interaction is often masked by the much stronger electromagnetic and strong interactions.

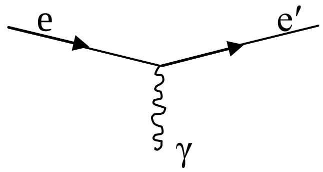
The Lagrangian density ($\mathscr{L}$) for electromagnetic interaction can be written as

$$\mathscr{L}_{Int}^{EM} = -e\bar{\psi}\gamma_\mu \psi A^\mu = -J_\mu A^\mu \quad (\text{note } \mathscr{L} = T - V) \tag{1}$$

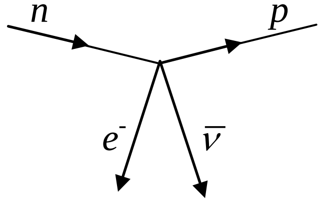
where

$$J_\mu = e\bar{\psi}\gamma_\mu \psi$$

graphically,



Fermi assumed that nucleon β-decay is represented by a similar diagram where the EM field is replaced by the $\bar{\nu}e^-$ current:



Fermi suggested the following substitutions to go from EM interaction to weak interaction:

$$\begin{aligned} \bar{\psi}_e \gamma_\mu \psi_e &\rightarrow \bar{\psi}_p \gamma_\mu \psi_n \\ A^\mu &\rightarrow \bar{\psi}_e \gamma^\mu \psi_\nu \\ e &\rightarrow G_F \big/ \sqrt{2} \end{aligned} \tag{2}$$

The Fermi coupling constant, G_F, was to be determined by experiment and was found to be

$$G_F = 1.03 \times 10^{-5} \left(m_p\right)^{-2} \tag{3}$$

The Lagrangian density for β-decay was therefore given as

$$\mathscr{L}_\beta = -\frac{G_F}{\sqrt{2}} \bar{\psi}_p \gamma_\mu \psi_n \bar{\psi}_e \gamma^\mu \psi_{\nu e} \tag{4}$$

In 1934 β^+ decay was observed by Curie and Joliot. Later, Alvarez observed electron capture in 1938. Since Equation 4 only describes an emission of e- (or an absorption of e+), in order to describe β^+ decay and electron capture the Lagrangian density needs to be generalized to

$$\mathscr{L}_\beta = -\frac{G_F}{\sqrt{2}} \left[\bar{\psi}_p \gamma_\mu \psi_n \bar{\psi}_e \gamma^\mu \psi_{\nu e} + \bar{\psi}_n \gamma_\mu \psi_p \bar{\psi}_{\nu e} \gamma^\mu \psi_e \right] \tag{5}$$

In other words, the Hermitian conjugate of Equation 4 is added to the Lagrangian density.

Note that $\mathscr{L}_\beta$ is a sum of scalar product of two vectors, and is invariant with respect to Lorentz transformation and spatial inversion.

In 1936 Gamow and Teller pointed out that $\mathscr{L}_\beta$ can contain other terms too without violating parity and $\mathscr{L}_\beta$ was generalized to

$$\mathscr{L}_\beta = \sum_j \left(-\frac{G_F}{\sqrt{2}}\right)\left(\overline{\psi}_p 0_j \psi_n \overline{\psi}_e 0_j \psi_{\nu e} + h.c.\right) \tag{6}$$

where

$$0_j = 1,\ \gamma^5,\ \gamma^\mu,\ \sigma^{\mu\nu},\ \gamma^\mu\gamma^5 \tag{7}$$

representing scalar, pseudoscalar, vector, tensor, axial-vector (*S*, *P*, *V*, *T*, *A*) interaction respectively.

For nuclear β-decays, momentum and energy transfers are very small, and one can use non-relativistic wave function for the nucleons:

$$\psi = \begin{pmatrix} \phi \\ 0 \end{pmatrix} \tag{8}$$

where the relativistic 'small' component is set to zero.

Note that

$$\overline{\psi}_p O_j \psi_n = \left(\phi_p^*, 0\right)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} O_j \begin{pmatrix} \phi_n \\ 0 \end{pmatrix} \tag{9}$$

$O_j = 1,\ \gamma^\mu,\ \sigma^{\mu\nu},\ \gamma^\mu\gamma^5$ all have non-vanishing diagonal elements. In contrast, $\gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ has only off-diagonal elements. Hence,

$$\overline{\psi}_p \gamma^5 \psi_n = 0 \tag{10}$$

and the pseudoscalar term can be ignored in nuclear β-decay. From Equation 7.8, we obtain for the various couplings

$$\begin{aligned}
&S: \quad \overline{\psi}_p \psi_n = \phi_p^+ \phi_n \\
&V: \quad \overline{\psi}_p \gamma^\mu \psi_n = \phi_p^+ \phi_n \ \text{ for } u = 0;\ = 0 \text{ for } \mu = 1,\ 2,\ 3 \\
&T: \ \overline{\psi}_p \sigma^{\mu\nu} \psi_n = \phi_p^+ \sigma^k \phi_n \ \text{ for } \mu, \nu = 1,\ 2,\ 3 \text{ cyclic};\ = 0 \text{ otherwise} \\
&A: \overline{\psi}_p \gamma^5 \gamma^\mu \psi_n = -\phi_p^+ \sigma^k \phi_n \ \text{ for } \mu = k = 1,2,3;\ = 0 \text{ for } \mu = 0
\end{aligned} \tag{11}$$

From Equation 11 we conclude that the scalar and vector couplings cannot induce spin-flip transitions. The axial-vector and tensor couplings can cause spin-flip transition. Therefore, we have

$$\begin{aligned} &S,V \text{ couplings}: \Delta J = 0 \ \ \text{(no spin-flip)} \\ &A,T \text{ couplings}: \Delta J = 0,\ \pm 1 \ \ (\text{except } J = 0 \not\Rightarrow J = 0) \end{aligned} \tag{12}$$

Experimentally, $|\Delta J| = 1$ spin-flip transitions were observed in weak decays. Therefore the Axial and/or tensor terms in $\mathscr{L}_\beta$ has to be non-zero. The determination of the magnitudes of C_j had to wait for two decades.

The discovery of muons in the 1930s and their decays as well as the pions, kaons, and hyperons and their decays, suggested that they have similar characteristics and coupling strength. The nuclear β-decay phenomenon was therefore generalized to cover other weak-decay processes. The idea of 'universal charged weak interaction' was put forward to describe various decay processes as simply different manifestation of a general Fermi interaction.

In 1956, the τ/θ puzzle, where

$$\theta \to \pi^+\pi^0 \text{ (even parity)}$$
$$\tau \to \pi^+\pi^+\pi^- \text{ (odd parity)}$$

were observed for two particles, θ and τ, of similar if not identical masses. This prompted Lee and Yang to suggest that parity could be violated in weak decay. In fact, they pointed out that parity conservation was never tested in β-decay experiments.

Lee and Yang's suggestion that parity might not be conserved in weak interaction was soon confirmed by Wu et al., who used polarized $^{60}\text{Co}(5^+)$ and found that the e^- from $^{60}\text{Co} \to {}^{60}N_i^*\left(4^+\right) + e^- + \bar{\nu}_e$ decay were emitted preferentially opposite to the spin orientation of ^{60}Co. This implies that the $\vec{S}\cdot\vec{P}$ term, which is odd in $\vec{\gamma} \to -\vec{\gamma}$ space inversion, is non-vanishing, hence parity is violated.

Subsequent experiments by Garwin, Lederman, Weinrich and by Freidman and Telegdi confirmed that parity was violated in the $\pi^+ \to \mu^+ + \nu_\mu$ and $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$ decays.

The observation of parity violation in weak interaction implies that $\mathscr{L}_\beta$ can have both a scalar component and a pseudoscalar component:

$$\mathscr{L}_\beta = -\frac{G_F}{\sqrt{2}}\sum_j \left[\overline{\psi}_p 0_j \psi_n\right]\left[\overline{\psi}_e 0_j \left(C_j - C_j'\gamma_5\right)\psi_{\nu e}\right] \tag{13}$$

scalar

pseudoscalar

The coefficients C_j and C_j' have to be determined by experiments. They can be complex numbers if time-reversal invariance is violated. Hence, a total of 10 coupling constants (C_j and C_j' for *S*, *P*, *V*, *A*, *T*) with 19 real constants need to be determined (eliminating one arbitrary phase)!

Frauenfelder et al. found that electrons emitted in ^{60}Co β-decay are longitudinally polarized with helicity consistent with $-{}^{v}\!/_{c}$. Similarly, positrons were found to have a helicity of ${}^{v}\!/_{c}$. These results showed that weak interactions result in electrons which are left-handed and positrons of right-handedness in relativistic limit $v \rightarrow c$.

The observed handedness of electrons and positrons implies that

$$\begin{aligned} C_j' &= +C_j \text{ for } V \text{ or } A \text{ coupling} \\ C_j' &= -C_j \text{ for } S,\ P,\ T \text{ couplings} \end{aligned} \tag{14}$$

In order to distinguish the two possible scenarios in Equation 14, it was necessary to determine the ν_e helicity. This was accomplished in an ingenious experiment by Goldhaber, Grodzins, and Sunyar who used the following reactions:

$$\underset{(0-)}{e^- + {}^{152}Eu} \rightarrow \nu_e + {}^{152}Sm^*\left(1^-\right) \tag{15}$$

$$\hookrightarrow {}^{152}Sm\ (0^+) + \gamma$$

From the measurement of the circular polarization of γ, it was concluded that ν_e has a negative helicity: $h\ (\nu_e) = -1$. This result showed that *S*, *P*, *T* coupling sin Equation 14 are small if not zero. Therefore, only C_A and C_V are mainly responsible for β-decays.

From the rates of the $^{10}C \rightarrow {}^{10}B$, $^{14}O \rightarrow {}^{14}N$, $0^+ \rightarrow 0^+$ transition, where only C_V can contribute, one determined that

$$C_V = 1 \tag{16}$$

From neutron lifetime measurement, where $\tau_n \alpha C_V^2 + 3C_A^2$, one obtained

$$\left| C_A / C_V \right| = 1.25 \tag{17}$$

The relative phase between C_A and C_V was determined through angular correlation experiment in polarized neutron decay:

$$C_A = +1.25\ C_V \tag{18}$$

Collecting Equations 13, 14, 16, and 18, one finds

$$\mathscr{L}_\beta = -\frac{G_F}{\sqrt{2}} \overline{\psi}_p \gamma_\mu \left(1 - 1.25\gamma_5\right) \psi_n \overline{\psi}_e \gamma^\mu \left(1 - \gamma_5\right) \psi_{\nu e} \tag{19}$$

In 1958 Feynman-Gellmann and Sudarshan-Marshak proposed the universal *V-A* form for charged weak current:

$$\mathscr{L}_\beta = -\frac{G_F}{\sqrt{2}} \left(J_\mu J^{\mu +} + J_\mu^+ J^\mu \right) \tag{20}$$

where J_μ is the 'charge-lowering' and J_μ^+ the 'charge-raising' weak current. J_μ consists of both a lepton part and a hadron part

$$J_\mu = J_\mu^{hadron} + J_\mu^{lepton} \tag{21}$$

$$J_\mu^{lepton} = \overline{\psi}_e \gamma_\mu \left(1 - \gamma_5\right) \psi_{\nu e} + \overline{\psi}_\mu \gamma_\mu \left(1 - \gamma_5\right) \psi_{\nu \mu} + \ldots \tag{22}$$

$$J_\mu^{hadron} = \overline{D} \gamma_\mu \left(1 - \gamma_5\right) U \tag{23}$$

Note that the deviation of C_A/C_V from 1 (Equation 18) was attributed to strong interaction effect in the nucleon.

Equation 23 only explained weak decays with $\Delta S = 0$. However, $\Delta S = 1$ weak decays, such as

$$K^+ \to \mu^+\nu,\ \Lambda \to P\pi^-,\ \ldots$$

had also been observed with reduced strength. In 1963 Cabbibo combined the $\Delta S = 0$, $\Delta S = 1$ transitions by suggesting the following form for the hadronic weak current:

$$J_\mu^{hadron} = \bar{D}_C \gamma_\mu \left(1-\gamma_5\right) U \tag{24}$$

where

$$D_C = \cos\theta_C D + \sin\theta_C S \tag{25}$$

The weak interaction eigenstate is therefore a mixture of the strange and down quarks. The mixing angle θ_C is determined from the relative strength of the $\Delta S = 0$ versus $\Delta S = 1$ transitions.

$$\theta_C \sim 13^o \tag{26}$$

The Feynman-Gellmann-Cabbibo's scheme is a generalization of Fermi's theory. It suffers the same difficulties of the Fermi's theory, as recognized by Heisenberg in 1936. Namely, the 'contact' interaction violates unitarity at sufficiently high energies. This problem can be illustrated by considering the $\nu_e + e^- \to e^- + \nu_e$ reaction. As will be shown later, the cross-section for this reaction is

$$\sigma = \frac{G_F^2 S}{\pi} \tag{27}$$

However, the cross-section can also be expressed as

$$\frac{d\sigma}{d\Omega} = \left|f\left(\theta\right)\right|^2 \tag{28}$$

where

$$f\left(\theta\right) = \frac{1}{2E}\sum_{\ell=0}^{\infty}\left(2\ell+1\right)M_L P_L\left(\cos\theta\right)$$

For a contact interaction of zero range, only the *S*-wave contributes, and Equation 28 becomes

$$\frac{d\sigma}{d\Omega} = \frac{1}{4E^2}\left|M_0\right|^2 = \frac{\left|M_0\right|^2}{S} \qquad \sigma = \frac{4\pi}{S}\left|M_0\right|^2 \tag{29}$$

unitarity implies

$$\left|M_0\right| \le 1 \tag{30}$$

Hence

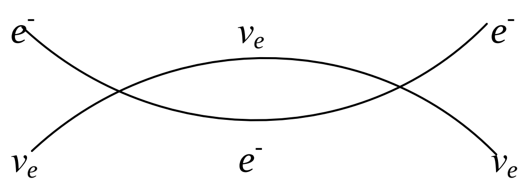
$$\sigma \le \frac{4\pi}{S} \tag{31}$$

From Equations 27 and 31, one concludes that unitarity limit is violated at

$$\frac{4\pi}{S} = \frac{G_F^2 S}{\pi} \tag{32}$$

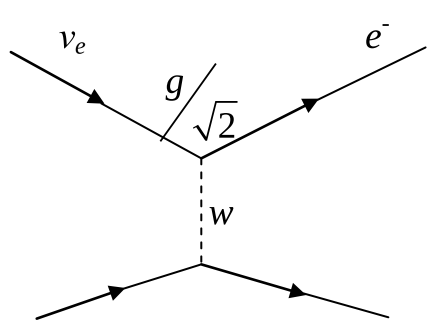
which occurs at $\sqrt{2} \sim 600\ GeV$

Can second-order diagram cancel the leading-order and make the result finite? It turns out that the second-order diagram for point-interaction such as



actually diverges. In QED, one also encounters divergences. However, they can be removed to all orders by mass and charge renormalization. This is not possible for the Fermi theory.

Another way out is to introduce a vector boson mediating the weak interaction. The $\nu_e\, e^- \rightarrow e^-\, \nu_e$ interaction is therefore



e^- ν_e

where

$$M = \frac{-ig^2}{2}\left[\bar{u}_e \gamma_\mu \frac{1-\gamma^5}{2} u_{\nu e}\right]\left[\frac{-g^{\mu\nu} + q^\mu q^\nu / \mu_w^2}{q^2 - M_w^2}\right]\left[\bar{u}_{\nu e}\gamma_\nu \frac{1-\gamma^5}{2} u_{e^-}\right] \tag{33}$$

At low energies, the Fermi theory should emerge. Therefore

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_w^2} \tag{34}$$

The expression of the cross-section resulting from the lowest order diagram is

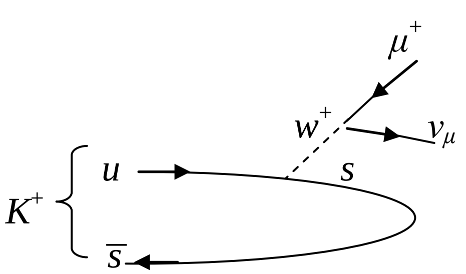
$$\frac{d\sigma}{dy}\left(\nu_e e^- \to e^- \nu_e\right) = \frac{g^4}{32\pi}\frac{S}{\left(q^2 - M_w^2\right)^2} \tag{35}$$

which does not have the unitarity problem. However, higher order diagrams still diverge. Furthermore, for processes such as

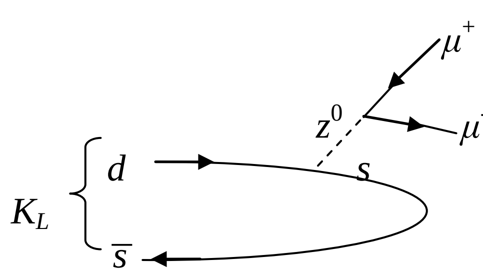
$$\nu_e + \bar{\nu}_e \to w^+ + w^-$$

it can be shown that the cross-section diverges even in the Born term. The origin of this divergence is the $q^\mu q^\nu$ term in the *w*-propagator, signifying a longitudinally polarized massive *w* (This term is absent for QED).

Despite the problem encountered by the Intermediate vector Boson model, it was proposed in the 1960s that there should be neutral weak interaction mediated by neutral vector bosons. However, no neutral weak interaction was observed. In particular, it was very puzzling why $K^+ \to \mu^+ \nu_\mu$ has a B.R. of 63%, while $K_L \to \mu^+\mu^-$ has a B.R. of only 9 x 10^{-9}, which can be accounted for by radiative effect.



$K^+ \to \mu^+ \nu_\mu$



$K_L \to \mu^+ \mu^-$

Should neutral weak current only occur for $u \to u$, $d \to d$, $s \to s$ and not for the flavor-changing case $d \to s$, $s \to d$, $u \to c$, etc.?

In analogy with the charged weak current, one should have, for the weak neutral current, a term like

$$\bar{d}_c d_c \quad \text{where} \quad d_c = \cos\theta_c d + \sin\theta_c s \tag{36}$$

now,

$$\begin{aligned} \bar{d}_c d_c &= \left(\cos\theta_c \bar{d} + \sin\theta_c \bar{s}\right)\left(\cos\theta_c d + \sin\theta_c s\right) \\ &= \cos^2\theta_c \bar{d}d + \sin^2\theta_c \bar{s}s + \left(\sin\theta_c \cos\theta_c\right)\left(\bar{s}d + \bar{d}s\right) \end{aligned} \tag{37}$$

Equation 37 shows that the flavor-changing neutral current terms $\bar{s}d + \bar{d}s$ should exist, which is in disagreement with the tiny B.R. for $K_L \to \mu^+\mu^-$ decay.

The solution to this puzzle was the suggestion that a new quark flavor, called charm, exists. We now have two quark doublets

$$\begin{pmatrix} u \\ d \end{pmatrix} \text{ and } \begin{pmatrix} c \\ s \end{pmatrix}$$

It is natural, in an extension of the Cabbibo mixing, to expect that the weak eigenstate for S (i.e. S_c) can be written as

$$S_c = \cos\theta_c S - \sin\theta_c d \tag{38}$$

Therefore, the combined neutral current from d_c and S_c is

$$\bar{d}_c d_c + \bar{S}_c S_c = \bar{d}d + \bar{S}S \tag{39}$$

and there is no flavor-changing $\bar{d}S$, $\bar{S}d$ terms.

The presence of the charm quark also implies that the hadronic weak charged current becomes

$$J_\mu = \bar{D}_c \gamma_\mu \left(1-\gamma_5\right) U + \bar{S}_c \gamma_\mu \left(1-\gamma_5\right) C \tag{40}$$

One expects that second-order charge-current processes can contribute to $K_L^0 \to \mu^+\mu^-$:

Note that these two diagrams involve an up and a charm quark exchange, respectively. If $m_u = m_c$, these two diagrams would cancel and would not contribute the $K_L^0 \to \mu^+\mu^-$ decay.

Based on the experimentally observed B.R., one can determine that the mass of the charm quark is $m_c \sim 1-3\ GeV$.

In 1973 Kobayashi and Maskawa extended the Cabbibo theory to 3 generations. In this case, there are four parameters in the 3 x 3 matrix; three of them are the mixing angles θ_1, θ_2, θ_3 plus one phase δ ($e^{i\delta}$). A non-zero value of S implies CP-violation in charged-current weak interactions.

The electroweak theory of Glashow, Weinberg and Salam in the 1960s, which unified the electromagnetic and the weak interactions, had a unique prediction on the existence of neutral current:

However, it is difficult to identify effects of neutral current in the quark sector, since the NC effect is overshadowed by the electromagnetic process. To overcome this problem, one can use neutrino beam, which is not subjected to EM interaction.

In 1973, neutral current was discovered in CERN using a Freon bubble chamber (Gargamelle). The NC reaction

$$\nu_\mu\left(\bar{\nu}_\mu\right) + N \to \nu_\mu\left(\bar{\nu}_\mu\right) + \text{hadrons}$$
$$(\text{no } \mu^- \text{ or } \mu^+)$$

was observed. The CC events from

$$\nu_\mu\left(\bar{\nu}_\mu\right) + N \to \mu^-\left(\mu^+\right) + \text{hadron}$$

were also observed. The ratio of the neutral current yield versus charged current yield was found to be

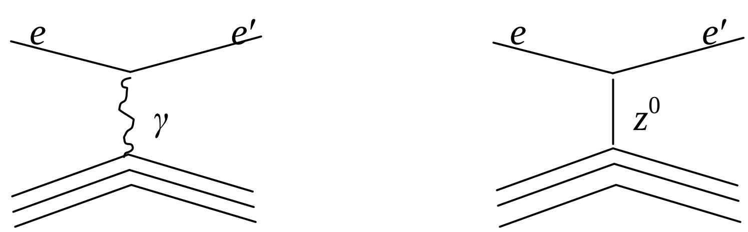
$$\left(NC/CC\right)\nu = 0.21 \pm 0.03$$
$$\left(NC/CC\right)\bar{\nu} = 0.45 \pm 0.09$$

The standard model prediction is

$$\begin{aligned} R_\nu &= \left(\frac{NC}{CC}\right)\nu = \frac{1}{2} - \sin^2\theta_w + \frac{20}{27}\sin^4\theta_w \\ R_{\bar{\nu}} &= \left(\frac{NC}{CC}\right)\bar{\nu} = \frac{1}{2} - \sin^2\theta_w + \frac{20}{9}\sin^4\theta_w \end{aligned} \qquad (41)$$

Hence one deduces $0.3 < \sin^2\theta_w < 0.4$ from the Gargamelle data.

In 1978, evidence for neutral current was found also at SLAC in a deep-inelastic scattering experiment using polarized electron beam. In this $\vec{e} + d \to \vec{e}' + x$ scattering, the following two diagrams can contribute:



The single-spin asymmetry, A, defined as

$$A = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \tag{42}$$

could have a non-zero value as a result of the interference of the γ and z^0 diagrams. This asymmetry is due to a term proportional to $\vec{\sigma}_e \cdot \vec{P}_e$, where $\vec{\sigma}_e$ is the spin of the electron (R, L refers to the right-handed and left-handed electron beam, respectively). $\vec{P}_e$ is the momentum vector of the scattered electron. It is clear that the $\vec{\sigma}_e \cdot \vec{P}_e$ term violates parity.

The predicted asymmetry is related to the Weinberg angle θ_w:

$$A = -\frac{G_F Q^2}{2\sqrt{2}\pi\alpha}\frac{9}{10}\left\{1 - \frac{20}{9}\sin^2\theta_w + \left(1 - 4\sin^2\theta_w\right)\left[\frac{1-(1-y)^2}{1+(1-y)^2}\right]\right\} \tag{43}$$

The experiment obtained

$$\frac{A}{Q^2} = (-9.5 \pm 1.6) \times 10^{-5}$$

which implies

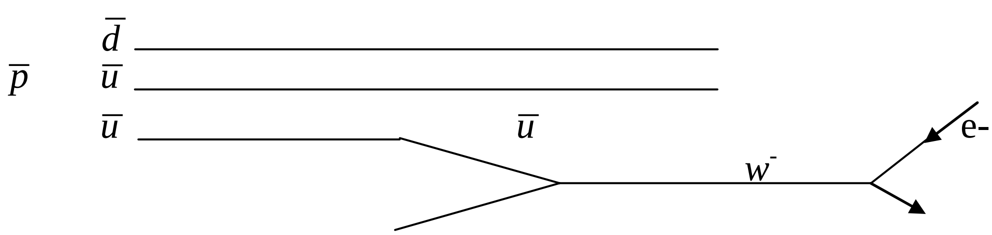
$$\sin^2\theta_w = 0.20 \pm 0.03$$

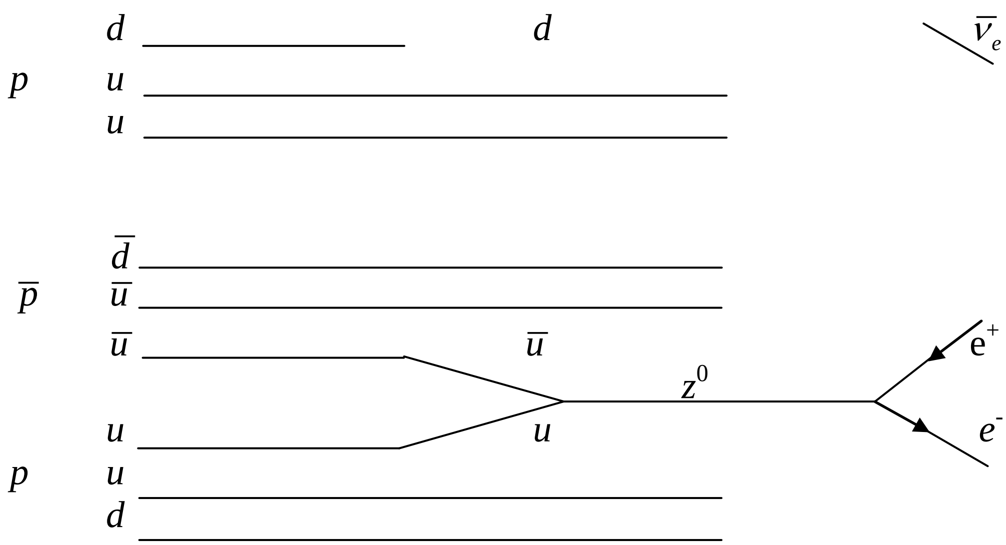
In the electro-weak theory, the masses of w and z can be determined once $\sin^2\theta_w$ is known:

$$\begin{aligned} M_w^2 &= \frac{\pi\alpha}{\sqrt{2}G_F \sin^2\theta_w} \\ M_z^2 &= M_w^2 / \cos^2\theta_w \end{aligned} \tag{44}$$

with $\sin^2\theta_w = 0.23$, Equation 44 predicts M_w = 80 GeV and M_z = 92 GeV.

The w and z bosons were observed in $p\bar{p}$ collision at CERN in 1982:





The observation of w/z^0 and their production and decay characteristics dramatically confirmed the electroweak theory.

Another important ingredient in the standard model, namely the Higgs particle, was found at LHC in 2012.