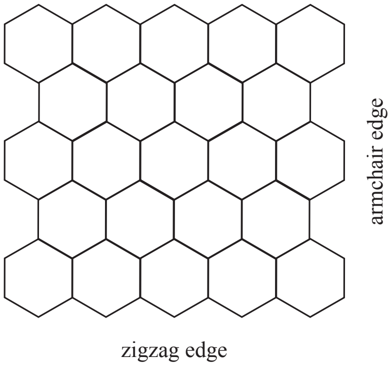
Graphene III: Disorder, transport, interactions

The question of possible sources of disorder in graphene are rather different for the epitaxial case and that of a freely suspended sheet. Since it seems likely that the purest (highest-mobility) samples will be achievable in the latter case, so that it will increasingly be used in experiments, I will concentrate on that. To remind you, these sheets are typically suspended from a metallic "scaffold" and, at least to date, are of area $\sim (1\,\mu)^2$.

Is the suspended sheet likely to be ideally flat? One rather trivial consideration is that if it is horizontal, gravity will distort it; however, a simple calculation shows that the resulting vertical displacement of the center is of order gL^2/c^2 where L is the linear dimension and c the speed of (longitudinal) sound ($\sim$ 10 km/sec). For $L \sim 1\,\mu$ this is $\sim 10^{-19}$ m, so completely negligible. A rather more tricky question is the effect of *transverse* thermal fluctuations of the film: this is discussed in CN section III. The problem is that for an infinite graphene sheet under zero tension the only restoring force for such vibrations is the bending energy, which is proportional to $\left(\nabla^2 h(\mathbf{r})\right)^2$ ($h(\mathbf{r})$ = height above equilibrium plane at $\mathbf{r}$).



Thus the frequency is proportional to k^2, with a coefficient that turns out to be $\sim 4\times10^{-6}$ in SI units. For a sheet of dimension $\sim 1\,\mu$ the lowest excitation energy is therefore of order 1 mK, so that at room temperature many such transverse vibrations are excited. This does not in itself imply (contrary to what CN seem to suggest) that the film is not effectively flat, but a proper calculation of the *rms* transverse displacement certainly indicates that it is indeed very large compared to the lattice constant. Taking account of the tension due to gravitational distortion (if any) does not appear to remedy this problem, so prima facie one would expect the thermal fluctuations around the planar configuration to be very large. It is, therefore, quite surprising that electron diffraction experiments[1] on freely suspended films, while indicating the presence of large "crumpling" distortions, seem to indicate that those are *static*; there are in fact substantial deviations (typically $\sim 5°$ of the surface normal from its "reference" value), which appear to extend over distances of the order of 250 Å. Both the origin of these apparently static distortions (which are considerable larger than would be given by thermal fluctuations) and why the effect of thermal fluctuations seems to be invisible in the electron diffraction experiments, is apparently at present not entirely clear.

Next, let us investigate the edges of the sample. As is well known from earlier work on C nanotubes, the edge of a planar sheet of graphene can be of two types, depending on the orientation of the edge relative to the honeycomb lattice.

Interestingly, it turns out from a solution of the TB Hamiltonian with the appropriate

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[1] Meyer et al., Nature **446**, 60–63 (2007).

boundary conditions (CN section II.H) that zigzag edges can sustain electronic states localized near the boundary but armchair edges cannot. Of course, the question presumably becomes more complicated for a "jagged" edge, say one oriented on average at an angle of 45° in the plane of the above diagram; to the best of my knowledge one has no guarantee that such edges may not occur, at least in places, in realistic suspended samples of graphene.

In addition to the boundaries, other inhomogeneities may in principle be introduced in the bulk by various kinds of dislocations etc. in the bulk lattice; however, these should be energetically costly and as far as I know there is no positive evidence for them in existing experiments.

One important consequence of the existence of extra electron states at edges and possibly bulk dislocations is the possibility of "self-doping": see CN section III.E for details. While for a dislocation-free bulk sample the magnitude of this doping vanishes in the thermodynamic limits, they estimate that for a $1\,\mu$ sample it is of order 10^{-5} electrons per unit cell, so roughly equivalent to a bias voltage of the order of $100\,\mathrm{V}$ (in the original epitaxial geometry of Novoselov et al.). Thus, in practice one often identifies the offset voltage corresponding to zero doping empirically, e.g. from the minimum of the dc conductivity.

In addition to the above kinds of "mesoscopic" disorder, there may of course exist sources of microscopic disorder, the most obvious being chemical substitutional impurities (such as B or N). The effect of such impurities on the electron states is rather different from that in a traditional textbook 3D solid. Let us for the moment consider a single isolated impurity of charge Z. The potential generated by it has the standard Coulomb form (recall that the electromagnetic field is always fully 3-dimensional!)

$$V_{\text{imp}}(r) = Ze^2/4\pi\epsilon\epsilon_0 r \tag{1}$$

where $\epsilon \cong 1$ for a freely suspended graphene sheet (the polarizability of the C cores is very small), while for epitaxial graphene ϵ is (of order of) the dielectric constant of the substrate, which can easily be $\sim 10 - 15$. If the electron energy spectrum were parabolic near the minimum as in an ordinary semiconductor such as Si or GaAs (or as in bilayer graphene), then the Coulomb potential would of course produce the familiar hydrogenic levels. However, the linear character of the spectrum near the Dirac points can change things qualitatively. Imagine that we try to localize an electron within some distance r_0 of a positively-charge impurity. We thereby gain a Coulomb energy $-|Z|e^2/4\pi\epsilon\epsilon_0 r_0$ but we have to pay a price in kinetic energy. To estimate the latter we take the spread Δk, in wave vector k to be the minimum compatible with the uncertainty principle, namely $1/(2r_0)$; then the KE is (of order) $\hbar v_{\mathrm{F}} \Delta k \sim \hbar v_{\mathrm{F}}/2r_0$. We see that if the dimensionless effective coupling g_Z defined by

$$g_Z \equiv \frac{Ze^2}{4\pi\epsilon\epsilon_0 \hbar v_{\mathrm{F}}} \tag{2}$$

exceeds 1/2, then it will always be energetically advantageous to decrease r_0 without limit.[2] Now, with $v_{\mathrm{F}} \cong 10^6$ m/sec the quantity $e^2/4\pi\epsilon_0\hbar v_{\mathrm{F}}$ is about 2, so in suspended graphene we need to take this instability seriously even for a singly-charged positive impurity. Of course, the argument fails when r_0 becomes comparable to the lattice constant a, since then the corresponding values of $\mathbf{k}$ are large enough that the linear Dirac spectrum is no longer correct, but it does indicate that we should expect some electronic bound states that are localized in a spatial region of the order of the unit cell.

The above considerations assume that the impurity is unscreened. Actually, as we shall see, the screening behavior of the electrons in graphene turn out to be rather different from that in a textbook metal or semiconductor, but we can anticipate the result that the screening length is of order $n_s^{-1/2}$, so for weak doping it is (as we might intuitively expect) completely ineffective on the scale of a few lattice spacings.

Let's now turn to the question of electronic transport in graphene. We start by discussing the semiclassical dynamics of electrons near a Dirac point. The form of the equation of motion is identical to that in a standard textbook crystal:

$$\hbar\frac{d\mathbf{k}}{dt} = \boldsymbol{F} = e(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) + (\mathrm{coll}) \tag{3}$$

where "(coll)" indicates the effect of so far unspecified collisions with static impurities, ripples of the surface, phonons or other electrons, and $\mathbf{v} \equiv (1/\hbar)\partial\epsilon/\partial\mathbf{k}$ is the semiclassical velocity. The electronic current carried by the electron is just $e\mathbf{v}$. Now, in view of the Dirac spectrum in zero magnetic field, namely,

$$\epsilon(\mathbf{k}) = \hbar v_{\mathrm{F}}|\mathbf{k}| \tag{4}$$

we have ($\hat{\boldsymbol{n}} \equiv \mathbf{k}/|k|$)

$$\mathbf{v} = v_{\mathrm{F}}\hat{\boldsymbol{n}} \equiv \hbar\mathbf{k}/m^*(k) \tag{5}$$

where the k-dependent "effective mass" $m^*(k)$ is given by $\hbar k/v_{\mathrm{F}}$. In the absence of an electric field eqns. (3) and (5) lead to circular motion around the Dirac point in $\mathbf{k}$-space with angular ("cyclotron") frequency

$$\omega_c \equiv eB/m^*(k) \tag{6}$$

(note that $k \equiv |\mathbf{k}|$ is constant along the orbit). In the context of a realistic cyclotron resonance experiment we are normally interested in a situation where $k_{\mathrm{B}}T \ll \hbar v_{\mathrm{F}} n_s^{1/2}$, so that the Fermi sea of electrons (or holes) is degenerate; in that case the relevant value of

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[2] Although the argument given is somewhat "hand-waving", the critical value 1/2 of g_Z follows from a more rigorous calculation. The result (2) is the analog of one well known in quantum electrodynamics, where however the corresponding limit on the fine structure constant is very large compared to its physical value.

m^* is $m^*(k)$ for $k = k_{\mathrm{F}} = \pi\sqrt{n_s}$. From now on, therefore, we shall drop the argument on m^*, defining

$$m^* \equiv m^*(k)_{k=k_{\mathrm{F}}} \tag{7}$$

We see then that the cyclotron frequency depends on the areal coverage n_s:

$$\omega_c = \frac{eBv_{\mathrm{F}}}{\hbar k_{\mathrm{F}}} = \frac{1}{2}\nu^{1/2}\omega_0 \tag{8}$$

where ν is the filling factor and $\omega_0 \equiv (e\hbar v_{\mathrm{F}}B)^{1/2}$ is the quantity defined in the context of the QHE.[3] Since the Landau levels are given by $E_n = n^{1/2}\omega_0$, eqn. (8) ensures the correct behavior in the correspondence limit.

Let's now consider the conductivity in the absence of a magnetic field. For this purpose we crudely model the effect of collisions by a phenomenological relaxation time τ:[4]

$$\left.\frac{d\mathbf{k}}{dt}\right|_{\mathrm{coll}} = -\mathbf{k}/\tau \tag{9}$$

We then find that in a steady state $\mathbf{k} = -e\boldsymbol{E}\tau$, so the current carried by a single electron is $e^2\boldsymbol{E}\tau/m^*$ and the conductivity σ (sheet conductance) is

$$\sigma = n_s e^2\tau/m^* \equiv n_s e\mu \tag{10}$$

where we define the mobility $\mu \equiv e\tau/m^*$. Thus, if the scattering mechanism is independent of carrier concentration, we find the anomalous n_s-dependence

$$\mu \propto n_s^{-1/2}, \qquad \sigma \propto n_s^{1/2} \tag{11}$$

An alternative and suggestive form for σ, introducing the mean for path $l \equiv v_{\mathrm{F}}\tau$, is

$$\sigma = \frac{2e^2}{h}(k_{\mathrm{F}}l) \tag{12}$$

Note that apart from the numerical factor this is identical in form to the expression for the Drude conductivity of an ordinary 2D metallic film, and, as this suggests that interesting effects might occur when l becomes $\lesssim k_F^{-1}$.

Generally speaking, experimental measurements of the cyclotron frequency in moderately pure graphene sheets confirm the prediction $\omega_c \propto n_s^{-1/2}$. However, the electrical conductivity does not always scale as $n_s^{1/2}$; indeed, in the early measurements of the Manchester group[5] it is accurately linear in V_g, i.e. in $|n_s|$, except at the very lowest dopings

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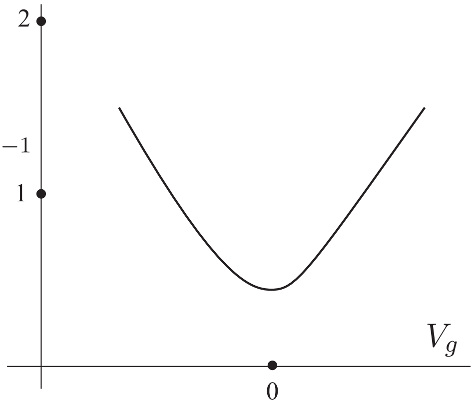
[3] It should be noted that the subscript "F" has a rather different significance in k_{F} and v_{F}. In the latter, it indicates $\hbar^{-1}\partial\epsilon(k)/\partial k$ at the Fermi energy (i.e. close to the Dirac point), which is approximately independent of the filling; in the former, it is related to the actual filling.

[4] This is of course really only appropriate to a Drude (classical) model, but just as in the case of a textbook metal should work for a degenerate Fermi system provided τ is suitably chosen.

[5] Nature **438** 197 (2005).

where it reaches a nonzero minimum value $\approx 0.15\,(\text{k}\Omega)^{-1}$ (the significance of this number will be discussed below). What is at first sight equally surprising is that, at least in epitaxial samples, σ appears to be approximately independent of temperature right up to room temperature; since typically this is still $\ll T_\text{F}$, neither k_F nor m^* can be changed appreciably, so this must indicate a *temperature-independent* value of the collision time τ (or mean free path l). Let's first discuss this result.



The obvious source of temperature - dependence of the mean free path is collisions with phonons. Now the speed of the *slowest* acoustic phonon (sound) mode in graphene is about 8 km/sec; thus, the shortest-wavelength phonon which is appreciably excited at room temperature has $k \sim 0.14\,\mu^{-1}$, which is about a factor of 100 smaller than the Fermi wave vector of the carriers at a doping of $10^{12}\,\text{cm}^{-2}$, and $\lesssim 1/6$ of k_F even at $2\times 10^9\,\text{cm}^{-2}$. Thus, we are in the Bloch-Grüneisen regime, and we expect that the effective collision rate τ^{-1}, which enters the expression for the electrical conductivity to the "main" collision rate, which we can estimate as of order [6] $(v_\text{F}/a)\cdot(T/\theta_\text{D})^2$ (so that the corresponding mean free path is only a few lattice constants) multiplied by a factor of order $(k/k_\text{F})^2$, thus making it comparable to the sample dimension. Of course, we should really put in the correct numerical factors (which can be quite large!), but this argument makes it at least plausible that ordinary phonons do not contribute much to the resistivity even at room temperature.

However, while this argument may be satisfactory for epitaxial graphene, it is incomplete for the case of a freely suspended sheet, since in that system we need to worry also about the effect of thermally excited flexural phonons. If as above we take those phonons to have a spectrum $\omega \sim Ak^2$ with $A \sim 4\times 10^{-7}$ in SI units, then the k-value of the shortest-wavelength flexural mode excited at room temperature is of the inverse lattice constant,[7] so we are not at all in the Bloch-Grüneisen limit and at first sight one would expect these phonons to contribute very substantially to the resistivity. Indeed, recent measurements[8] of the conductivity of suspended graphene sheets do show a strong temperature-dependence, which the authors speculate is due to phonons. What is at first sight quite surprising is the relatively weak strength of this effect: even at $T \sim 1-2K$, the temperature of the two recent FQHE experiments, the maximum k should be down only by a factor ~ 10, yet means free paths $\sim 0.3\,\mu$ see to be obtained! At the time of writing, this appears to be

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[6] a = lattice constant, θ_D - Debye temperature. The factor $(T/\theta_\text{D})^2$ is, to within a factor of order units, the number of thermally excited phonons per lattice site in 2D.

[7] Of course by this point the assumptions used in deriving the dispersion relation have long broken down.

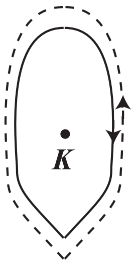
[8] Bolotin et al. PRL **101**, 096802 (2009)

something of a mystery.

Let's return to the epitaxial case and consider the possible sources of the temperature-independent scattering that seems to be observed. The most obvious sources are charged chemical impurities in either the graphene itself or (more likely) the substrate. Naively, one would think that such impurities would give rise to a scattering rate that is independent of n_s, and thus predict $\sigma \propto n_s^{1/2}$ as in eqn. (11). However, it is important (just as in semiconductors) to take into account the screening of the impurity potential by the electrons themselves. We will consider this in detail below, but anticipate the result that the relevant screening length l_{TF} is proportional to $n_s^{-1/2}$. Naively, one would expect that in a 2D system the scattering "cross section" would be proportional to l_{TF}, so that the mean free path varies as l_{TF}^{-1}, and thus to $n_s^{-1/2}$. If so, then we expect the conductivity σ to be proportional to n_s, as is often seen. Note that this argument implicitly assumes that n_s and the number of charged impurities in the substrate are independent variables, i.e. that we are not in the "self-doping" regime.

We should not expect to find many charged impurities in a suspended graphene sheet, and indeed the zero-B conductivity reported by the Rutgers group is proportional to $n_s^{1/2}$, which is consistent with a mean free path that is of the order of the sample dimensions. In fact these samples appear to be in or close to the true "ballistic" limit, at least at temperatures 1-2 K.

One obvious question is: why is single-layer graphene metallic at all, since the considerations of lectures 4–7 seem to imply that metals in 2D should not exist, and those that are experimentally found to do so (in Si MOSFETs, etc., see lecture 14) are much less "2D" than a single sheet of C atoms? There has been quite a bit of discussion in the theoretical literature of the question of weak localization in ideal (infinite) graphene sheets, with some interesting conclusions: in particular, it turns out that if an electron is elastically scattered only with a single "valley," then the time-reversed paths that interfere to give (in the normal case) a weak-localization effect actually acquire an extra relative phase of π $\big($see the diagram: the solid (dotted) trajectory in $\mathbf{k}$-space acquires an extra phase of $\pi/2(-\pi/2)\big)$, so impurities actually give rise to an *antilocalization* effect (cf. the case of (3D) spin-orbit scattering in the normal case). There is a little experimental evidence concerning such effects in fairly dirty epitaxial samples, see CN section IV.H. However, if one asks the question "why do current suspended samples violate the "theorem" of no metallicity in 2D", the answer is actually rather trivial: recall that the 2D localization length is of order of magnitude



$\psi \sim \exp \pm i\phi_q/2$

$$L_{\mathrm{loc}} \sim l \exp k_{\mathrm{F}} l \tag{13}$$

For all but the very lowest carrier densities, $k_{\mathrm{F}} l \gg 1$, so the RHS is much larger than the sample dimension!

This leads naturally to the question: What happens when n_s and hence k_{F} really tend to zero, so that $k_{\mathrm{F}}l$ need not be large compared to 1? Recall that the semiclassical (Boltzmann) expression for the conductivity of a 2D metal can be written in the form

$$\sigma_{2D} = \left(\frac{e^2}{h}\right)(k_{\mathrm{F}}l) \tag{14}$$

Mott's original argument, applied (originally in 3D) in the context of dirty metals, was that it is unphysical for the mean free path to be much less than k_{F}^{-1} while the system remains metallic, so there should be a "minimum metallic conductivity" that in 2D is of order e^2/h. It is usually believed[9] that this result should be stable against the effects of interactions. It is thus very significant that all measurements of the conductivity of graphene, whether epitaxial or suspended, do seem to show that at the Dirac point (i.e. at zero doping) it does seem to level off at a value that is precisely of order e^2/h, although the numerical constant differs somewhat from experiment to experiment. Of course, at the very lowest dopings in ultrapure samples ($l \sim$ sample dimensions), the condition $k_{\mathrm{F}}l \sim 1$ corresponds to only a dozen or less electrons in the whole sample, so no doubt the theory (which is usually done for the infinite case) may have to be revised, and in particular it would not be surprising if the numerical constant in the prediction $\sigma = \text{const.}(e^2/h)$ were to depend on the sample geometry.

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So far, apart from taking into account the screening of a charged impurity, we have totally neglected the effect of electron-electron interactions, and have got in most cases relatively decent agreement between theory and experiment. This is actually quite puzzling: as we have observed, the "graphene fine structure constant" α_g, which is the analog of the familiar fine structure constant $\alpha \equiv e^2/4\pi\epsilon_0\hbar c$ of quantum electrodynamics, is given by the expression

$$\alpha_g = e^2/4\pi\epsilon\epsilon_0\hbar v_{\mathrm{F}} \tag{15}$$

Since v_{F} is about 1/300 of c, α_g is of order 0.2 for epitaxial graphene ($\epsilon \sim 10 - 12$) and for a freely suspended sheet is ~ 2! So, prima facie, the electrons in graphene are by no stretch of the imagination a weakly interacting system.

In the familiar case of the 3D electron gas, the minimal approximation that gives even qualitative agreement with experiment is the random-phase approximation (RPA) of Bohm and Pines. The principal effect of the RPA is that the charge density response function $\chi(\mathbf{q},\omega)$ is strongly normalized from the form $\chi_0(\mathbf{q},\omega)$ calculated for the free electron gas (in 3D the Lindhardt function):

$$\chi_0(\mathbf{q},\omega) \to \chi_{\mathrm{RPA}}(\mathbf{q},\omega) \equiv \frac{\chi_0(\mathbf{q},\omega)}{1 + V_{\mathbf{q}}\chi_0(\mathbf{q},\omega)} \tag{16}$$

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[9] The original argument (for a "standard" metal) was given by Fradkin, PRB **33**, 3257 (1986).

where $V_{\mathbf{q}}$ is the matrix element of the Coulomb interaction ($\propto q^{-2}$ in 3D, q^{-1} in 2D). For a 3D degenerate Fermi gas with the usual quadratic spectrum, this gives inter alia a plasmon resonance at a frequency which is approximately q independent at small q, and for realistic metallic parameters is usually appreciably larger than ϵ_{F}, thus the plasmon is very well separated from particle-like excitations. In the 2D quadratic-spectrum case, because of the fact that $V_{\mathbf{q}} \propto q^{-1}$, the plasmon frequency is proportional to $q^{1/2}$, but is still well separated at small q from the particle-like continuum whose width scales as q.

The obvious question, therefore, is: what is the behavior of $\chi_0(\mathbf{q}, \omega)$ in graphene, with its peculiar "Dirac" spectrum $\epsilon(\mathbf{q}) \propto q$ for small q? Let us first consider the situation at zero doping. We use the general formula (valid for any spectrum and in any number of dimensions)

$$\chi_0(\mathbf{q}, \omega) = \sum_{nm'} \frac{(\epsilon_{m'} - \epsilon_m)|\langle m|\rho_{\mathbf{q}}|m'\rangle|^2}{(\epsilon_{m'} - \epsilon_m)^2 - \hbar^2\omega^2}\, n_m(1 - n_{m'}) \tag{17}$$

where $\rho_{\mathbf{q}}$ is the Fourier transform of the electronic density operator. In the standard case of a filled Fermi sea $\rho_{\mathbf{q}}$ is just $\sum_{\mathbf{k}} a^{+}_{\mathbf{k}+\mathbf{q}/2} a_{\mathbf{k}-\mathbf{q}/2}$, so it simply constrains the (quasi)momentum of the states m and m' to differ by $\mathbf{q}$. In graphene, however, the coordinate-space form of ρ is a sum of contributions from the A and B sublattices: since the π band is completely filled at zero doping, the state m must lie in this band and the state m' in the π^* band, and their momenta must differ by $\mathbf{q}$. But the states in the π and the π^* bands are respectively symmetric and antisymmetric with respect to the $A \rightleftarrows B$ interchange, so for $\mathbf{q} = 0$ it is clear that the matrix element is zero.[10] For small nonzero $\mathbf{q}$ the matrix element is proportional to q (actually $\mathbf{q} \cdot \boldsymbol{\delta}_1$). Thus the expression on the RHS of eqn. (17) becomes

$$\chi_0(\mathbf{q}, \omega) = \sum_{\mathbf{k}} \frac{F(\mathbf{k}, \mathbf{q})(\mathbf{q} \cdot \boldsymbol{\delta}_1)^2}{F^2(\mathbf{k}, \mathbf{q}) - \hbar^2\omega^2} \tag{18}$$

where since n is in the π band and n' in the π^* band

$$F(\mathbf{k}, \mathbf{q}) \equiv \hbar v_{\mathrm{F}} \left\{ |\mathbf{k} + \mathbf{q}/2| + |\mathbf{k} - \mathbf{q}/2| \right\} \tag{19}$$

Because of the two-dimensional integral (sum) over $\mathbf{k}$ the regions near the Dirac point do not give a particularly significant contribution (to the real part). The expression (18) can actually be evaluated analytically[11] using the exact (general-$\mathbf{k}$) expression for $F(\mathbf{k}, \mathbf{q})$, and the result is

$$\chi_0(\mathbf{q}, \omega) = \frac{\mathbf{q}^2}{4\hbar(v_F^2\mathbf{q}^2 - \omega^2)^{1/2}} \tag{20}$$

The important point to note in this expression is that $\chi_0(\mathbf{q}, \omega) \to 0$ for $q \to 0$ independently of ω, and in particular in the limit $\omega \to 0$ where the imaginary part of (20), which is nonzero

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[10] This is, of course, just a special case of a theorem well known in the context of (interband) optical absorption in semiconductors.

[11] Shung, Phys. Rev. B **34**, 979 (1986).

for $\omega > v_{\mathrm{F}} q$, vanishes). Thus the renormalization of the effective Coulomb potential $V_{\mathrm{eff}}(\mathbf{q})$ given by

$$V_{\mathbf{q}} \to V_{\mathrm{eff}}(\mathbf{q}) \equiv \frac{V_{\mathbf{q}}}{1 + V_{\mathbf{q}} \chi_0(\mathbf{q}, \omega)} \tag{21}$$

is equivalent simply to multiplication of $V_{\mathbf{q}} = \text{const.} q^{-1}$ by a factor $(1 + \frac{\alpha_g}{4})^{-1}$.

The situation is modified at finite doping n_s. Indeed it is rather obvious that where $q \ll k_{\mathrm{F}} = (\pi n_s)^{1/2}$ the situation must be exactly[12] equivalent to that in the usual (quadratic-spectrum) 2D Fermi gas, since it is only the nature of the spectrum close to the Fermi surface which now matters, and this is the same in the two cases. Thus we find the standard 2D result

$$\chi_0(\mathbf{q}, \omega) = \frac{dn}{d\epsilon} \frac{v_{\mathrm{F}} q}{\sqrt{v_F^2 q^2 - \omega^2}} \tag{22}$$

which for $\omega = 0$ is just the constant $dn/d\epsilon$. Unlike the usual 2D case (where it is independent of n_s) for the Dirac case $dn/d\epsilon$ is proportional to k_{F}, i.e. to $n_s^{1/2}$. Thus the Coulomb potential is nontrivially renormalized:

$$V_{\mathrm{eff}}(\mathbf{q}) = \frac{V(\mathbf{q})}{1 + \text{const.}(k_{\mathrm{F}}/q)} \to \text{const. as } q \to 0 \tag{23}$$

The real-space screened Coulomb interaction is the Fourier transform of $V_{\mathrm{eff}}(\mathbf{q})$; while it does not fall off exponentially at large distances, it does cross over, at a range $\sim k_F^{-1}$, from the unscreened r^{-1} form to r^{-2}, thereby giving rise to a finite cross-section proportional to k_F^{-1}, i.e. to $n_s^{-1/2}$. Thus the result we anticipated earlier for scattering by screened charged impurities is justified.

Finally, let's briefly return to the behavior of the transport coefficients in a high magnetic field (nowadays one can apply fields up to $\sim 45\,\mathrm{T}$). Of course, the original question of interest was the existence and location of integral and fractional quantum Hall plateaux. However, the two recent experiments that reported evidence for a FQHE at $\nu = 1/3$ also reported some very interesting behavior around $\nu = 0$ (similar behavior had been previously observed in *dirty* epitaxial samples). Both groups observed that at $T \sim 1 - 2\,\mathrm{K}$, $B > 5\,\mathrm{T}$, the state close to $\nu = 0$ was *insulating*, with $R_\square > 1\,\mathrm{G}\Omega$. They also observed that as the temperature was raised the resistivity shows an activated behavior

$$R_\square \sim \exp(T_0/T)^\alpha \tag{24}$$

However, the values of α and T_0 obtained by the two groups are rather different: the Columbia group finds $\alpha \approx 1$ (i.e. standard "Arrhenius" activation) with $T_0 \sim 120\,\mathrm{K}$ at $14\,\mathrm{T}$ and rapidly decreasing with decreasing B, while the Rutgers group finds a "best"

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[12] That is, the situation as regards intraband ($\pi^* \to \pi^*$) transitions. Of course we still get the original interband ($\pi \to \pi^*$) contribution, which is only slightly modified.

value of α close to 1/2 (reminiscent of Mott variable-range hopping) and $T_0 \propto B^2$. These results are very new and not at all well understood; it seems unlikely in particular that they can be understood on any noninteracting-electron picture.