Problem Sheet 1

1. (very easy). Consider an attractive square well potential of depth V_0 and dimension a, in one, two or three dimensions (i.e. $V(\mathbf{r}) \equiv V(r) = -V_0\theta(a-r)$ where in the 1D case $r \equiv |x|$).

   (a) In the 1D case, find the condition for at least one *odd-parity* bound state to exist.

   (b) Hence, or otherwise, find the condition for the existence of at least one bound state in 3D.

   (c) Now consider the 2D case with a thin solenoid along the cylindrical axis supplying flux Φ. Find a *sufficient* condition for a bound state to exist whatever the value of Φ.

2. Consider an AlGaAs–GaAs heterostructure at $T = 0$, with an electric field $\mathcal{E}$ perpendicular to the interface (in the z-direction).

   (a) Use the virial theorem and a semiclassical argument to obtain the dependence on $\mathcal{E}$ of (i) the extent of the groundstate wave function in the z-direction (ii) the energy splitting between the ground state and first excited state in this direction.

   (b) If we are given that the extension of the groundstate wave function is ~ 200 Å, at roughly what area density does the first excited state become occupied?*

   (c) Consider an area density of 10^{11} cm^{-2}. The measured mobility is 10^6 cm^2/V s. Estimate (i) the elastic mean free path (ii) the Coulomb interaction between electrons (iii) the filling fraction (number of electrons per flux quantum) if a field of 10T is applied.

* The accuracy will be important if you keep the factors of 2π, etc., in the semiclassical argument (this was not done in the very rough estimate made in lecture 3).

(d) To see quantum-Hall-effect-related phenomena clearly, a minimum requirement is that the total mean free path against both elastic and inelastic collisions be $\gg$ the "magnetic length" (essentially, the radius of the flux quantum). Can this condition ever be fulfilled for the sample described in part (c)? If so, estimate *very roughly* the temperature to which we need to cool.

3. For this problem, refer to the paper of Hadzibabic *et al.*, Nature **441**, 1118 (2006) [see also NJP **10**, 045006 (2008)].

   Suppose you are trying to achieve a similar situation with the fermion isotope ^{6}Li.

   (a) Find suitable values of the maximum laser intensity I_m, detuning Δ and magnetic field dependence $B(z)$ in the z-direction to achieve a single 2D layer.

   (b) Estimate the gap against excitation of the first excited state in the z-direction

   (c) If the x- and y-dependencies of the magnetic field are as in the Hadzibabic experiment, estimate the Fermi energy for a total of 10^6 ^{6}Li atoms.

   [Note: This problem is fairly open-ended and you will need to do some literature searching on the relevant properties of ^{6}Li. Some of the material in AJL, RMP **73**, 307 (2001), section IIB, may also be helpful.]

Solutions to be put in 598PTD homework box (2nd floor Loomis) by 9 a.m. on Mon. 14 Sept.