Problem Sheet 2

All these problems have to do with weak localization and/or interaction in 2D systems such as metallic films. Unless otherwise stated you should assume that the system is "locally" 3D (i.e. thickness $d \gg k_{\mathrm{F}}^{-1}, l$). In problems 1 and 2, neglect effects of interactions, except in so far as they may be phase-breaking.

1. Consider scattering of the conduction electrons by a set of localized spins $\mathbf{S}_i$, where the interaction is

$$H_{\mathrm{ss}} = -\sum_i J_i \boldsymbol{\sigma} \cdot \mathbf{S}_i f(|\mathbf{r} - \mathbf{R}_{0i}|), \quad f(r) = \theta(a - r)$$

where the positions $\mathbf{R}_{0i}$ and directions of the $\mathbf{S}_i$ are random unless otherwise stated, and the value of $J|\mathbf{S}|$ is Gaussian-distributed with mean square J_0^2.

(a) If the localized spins are polarized in a strong Zeeman field, is there any effect on weak localization? If so, which sign does it have? (You should convince yourself (and me!) that the Zeeman polarization of the *conduction* electrons is irrelevant to leading order in $\mu_{\mathrm{B}} H/\epsilon_{\mathrm{F}}$. Ignore the orbital effects of the magnetic field.)

(b) Find an expression for the single rotation which is effected by the sequence $R_1 R_2 R_1^{-1} R_2^{-1}$, where $R_j(\hat{\boldsymbol{\omega}}_j, \theta_j)$ is a rotation through angle θ_j around axis $\hat{\boldsymbol{\omega}}_j$.

(c) (easy) Show that for random directions of the $\mathbf{S}_i$ the mean free time τ_{sf} of a conduction electron against spin flip is of order $\tau_0 \equiv 1/n_s a^2 v_{\mathrm{F}}$ if $J_0 a/\hbar v_{\mathrm{F}} \gg 1$ and of order $(\hbar v_{\mathrm{F}})^2/J_0 n_s v_{\mathrm{F}}$ if $J_0 a/\hbar v_{\mathrm{F}} \ll 1$. ($n_s$ = number of impurities per unit volume.)

(d) Using the results of (b) and (c), show that in the context of weak localization the effect of spin-spin scattering is phase-breaking, and find (the order of magnitude of the) equivalent dephasing time τ_ϕ in terms of τ_0 and τ_{sf}.

(e) Assuming that the elastic scattering time τ is $\ll \tau_0$, what is the relation of the corresponding dephasing lengths?

2. A given metallic film has thickness $d = 1000\,\text{Å}$, elastic mean free path $l = 10\,\text{Å}$, and spin-orbit length $L_{\text{SO}} = 10\,\mu$ and typical values of the electron-electron and electron-phonon interactions. It contains no magnetic impurities. Discuss the *qualitative* behavior of the magnetoresistance in a perpendicular magnetic field up to $10\,\text{T}$ at

   (a) room temperature

   (b) $5\,\text{K}$

   (c) $0.1\,\text{K}$,

   giving rough estimates of any characteristic "crossover" fields introduced. (Assume that any "classical" contributions to the magnetoresistance are negligible on the scale of $10\,\text{T}$).

3. This question relates to the density-density response function $\chi_0(\mathbf{q},\omega)$: For the definition and some basic properties in 3D see, e.g., Pines + Nozières, *Quantum Liquids*, ch. 2

   (a) Derive the form of $\operatorname{Im}\chi_0(\mathbf{q},\omega)$ for a $2D$ Fermi gas at $T = 0$, $q \ll k_{\text{F}}$, $\omega \ll \epsilon_{\text{F}}$. Does it have any singularities? If so, where and of what type?

   (b) If one inserts this expression into the formulae of lecture 7, what is the correction to the single-particle density of states from interactions in the limit $\epsilon \to 0$?

   (c) Suppose we substitute for $\chi_0(\mathbf{q},\omega)$ the "full" density-density response function as calculated from Landau Fermi-liquid theory, with, for simplicity, all Landau parameters $F_l^{s,a}$ set equal to zero except for F_0^s. How are the results affected?

   [Note: In part (c) you do *not* need to determine the real part of $\chi(\mathbf{q},\omega)$ for arbitrary $s \equiv \omega/qv_{\text{F}}$, only its approximate behavior close to the singularity.]

Solutions to be put in 598PTD homework box (2nd floor Loomis) by 9 a.m. on Mon. 28 Sept.