Problem Sheet 3

1. Imagine a world in which ^{4}He has never been liquified, but theorists know that ^{4}He atoms are bosons (and know about BEC) and have calculated the approximate density of a possible liquid phase ($\approx 2.2 \times 10^{22}$ atoms/cm^3). By using Hohenberg's lemma, what can they say about the maximum condensate fraction (a) at 4 K (b) at 2.17 K?

2. (a) (very easy) By using standard Ginzburg-Landau (mean-field) theory and the definition of the superfluid density $\rho_s(T)$, show that in the limit $T \to T_c$ ρ_s has the form for both superfluid ^{4}He and superconductors (within mean-field theory)

$$\rho_s(T) = \text{const. } \rho(1 - T/T_c) \equiv \text{const. } \rho\, t$$

where ρ is the total density, and give an argument that for a translation-invariant system the constant must be of order unity.

   (b) Consider liquid ^{4}He in an annular pore of (circular) cross-section (50Å)^2 and circumference 1 cm. (This might be a primitive model of ^{4}He in Vycor). Make a rough estimate, using the result of part (a), of the value t_0 of t ($\equiv 1 - T/T_c$) at which the system loses long-range order.

   (c) The same as for (b), for a film of ^{4}He of thickness 10 Å and area 1 cm^2.

   (d) Are the results qualitatively affected by the replacement of the mean-field form of $\rho_s(T)$ in part (a) by what is believed to be the true behavior in the limit $T \to T_c$, namely $\rho_s(T) \sim \rho(1 - T/T_c)^\zeta$, $\zeta \approx 2/3$?

3. Consider the "Langer-Fisher" (LF) mechanism for decay of superflow, in which a (3D) vortex ring is nucleated and (may) expand to the boundaries of the system.

   (a) Using the formulae of lecture 10, determine the approximate dependence of the free energy barrier for this process on the superfluid velocity v_s.

(b) For the situation discussed in problem 2, part (b), at temperatures of the order of t_0, which process is more probable: the LF process, or a "bulk" phase slip in which a complete cross-section of the pore is turned normal?[1] Consider this question (i) for the lowest circulating state ($\kappa = 1$) (ii) for $v_s \sim 10\,\text{m/sec}$.

(c) Under the conditions of part (b), is the system "effectively" superfluid?

Solutions to be put in 598PTD homework box (2nd floor Loomis) by 9 a.m. on Mon. 12 Oct.

[1] You may assume that so long as $v_s \ll \hbar/m\xi(T)$ where $\xi(T)$ is the GL healing length, the superflow does not contribute appreciably to the free energy of the bulk phase-slip processes.