Problem Sheet 5

1. Consider the experimental quantum Hall system described by Champagne et al., Phys. Rev. B **78**, 205310 (2008), in a magnetic field $\mathbf{B}$ of 5 T oriented at 60° to the normal $\hat{\mathbf{n}}$ to the 2DEG planes and at 100 mK. For the purposes of the problem, consider the two planes to behave independently. Using the data provided and standard data on GaAs, etc., calculate

   (a) the elastic mean free path

   (b) the cyclotron and Zeeman energies

   (c) the "characteristic" intra-plane Coulomb energy E_c in plane 1 at filling $\nu_1 = 1/3$ (take E_c to be $e^2/4\pi\epsilon\epsilon_0 l_\mathrm{M}$)

   (d) the maximum inter-plane Coulomb energy

   (e) a rough order of magnitude of the (theoretically expected) $R_{xx}^{(1)}$ at $\nu_1 = 1/3$.

   (f) a rough order of magnitude of the inter-plane tunneling matrix element t.

   (g) the periodicity of t. In which direction is t periodic?

   Is the approximation of treating the planes as independent likely to be valid under the given conditions (at $\nu_1 = 1/3$)?

2. (a) For a phonon of frequency $\omega = 10\,\mathrm{GHz}$ propagating in a 2DEG of overall density $10^{11}\mathrm{cm}^{-2}$ in GaAs, which of the following is a good approximation? (q = wave vector of phonon, v_F = Fermi velocity of electrons, $l \equiv v_\mathrm{F}\tau$ = electron elastic mean free path)

      i. $\omega_a \tau \gg 1$

      ii. $ql \gg 1$

      iii. $qv_\mathrm{F} \gg \omega$

   (b) Show, by solving the collisionless Boltzmann question in the approximation $\omega = 0$, or otherwise, that for when the conditions (ii)−(iii) are fulfilled the transverse conductivity of a free degenerate 2D Fermi gas is given by

$$\sigma(q,\omega) = \text{const.}\ (e^2/\hbar)(k_\mathrm{F}/q)$$

What is the constant?

(c) Under the conditions of part (a), is the result of part (b) still approximately true in a field of 0.1 T?

3. Consider the "poor man's version" of the Hohenberg-Mermin-Wagner argument as applied to crystalline long-range order (LRO) in which one considers the thermal fluctuations in the relative positions of atoms i and j as the distance between them tends to infinity. If we apply this argument to a graphene sheet of area $(100\mu)^2$ at (a) room temperature (b) 4 K, what, if anything, can we conclude about the degree of LRO? (For the purposes of the problem you may, if you wish, replace "real" graphene by a square lattice of C atoms with the same order of magnitude of the elastic constants).

Solutions to be put in 598PTD homework box (2nd floor Loomis) by 9 a.m. on Mon. Nov 9.