Problem Sheet 7

1. Decoherence by classical noise:

   Consider a particle of spin 1/2 in a constant magnetic field H_0 along the z-axis. Imagine that at time zero the spin is pointing along the x-axis.

   (a) (easy): Write down the form of the density matrix in the standard (σ_z) basis as a function of time, and evaluate the quantity $\langle \sigma_x + i\sigma_y \rangle(t) \equiv P(t)$.

   (b) Now suppose we have an extra random Gaussian field acting along the z-axis, with correlations specified by

   $$\langle H(t_0)H(t_0+t)\rangle = H_r^2 f(t)$$

   where $f(t)$ falls off at least as fast as t^{-2} for $t \to \infty$. Find the form of the (ensemble-averaged) density matrix and of $P(t)$. Does it matter whether H_r is large or small compared to H_0?

   (Hint:Transform to the rotating frame and use the result,valid for any Gaussian-distributed quantity A, $\langle \exp(iA)\rangle = \exp -\langle A^2\rangle/2$. You may find it easier to evaluate $P(t)$ first).

2. Stability of superfluid ^{4}He in an annulus:

   For the purposes of the problem, let's model ^{4}He as a gas of bosons with a repulsive contact interaction of strength V_0.

   (a) (easy) Consider two atoms in plane-wave states $\psi_{\mathbf{k}}$, $\psi_{\mathbf{k}'}$. Show that the expectation value of the interaction for $\mathbf{k} \neq \mathbf{k}'$ is twice that for $\mathbf{k} = \mathbf{k}'$.

   (b) Suppose the helium is contained in an annular (toroidal) geometry and that initially all the N atoms are in a state with nonzero angular momentum l. In the groundstate all atoms have angular momentum zero (all of this is of course in the

Hartree-Fock approximation). If we consider a trajectory interpolating between these two states in which at time t $N_0(t)$ atoms are in state 0 and the rest in state l, what is the maximum energy barrier which must be surmounted?

(c) Same as (b), but now with the interpolating trajectory having all N atoms occupying a *linear superposition* of ψ_0 and ψ_l.

(d) For what value of l does the energy barrier tend to zero? Express the corresponding *velocity* in terms of the speed of sound (as calculated from the compressibility derived in the Hartree-Fock approximation).

3. Consider a triplet superconductor (Sr_2RuO_4?) in which to a first approximation we can regard the up and down Cooper pairs as forming independently. Suppose we try to create a "half-quantum" vortex line, that is a configuration such that at distances $\ll$ the London penetration depth λ_L the down-spin order parameter is constant in space but there is a simple vortex line in the up spins. Show that just as in the case of a simple Abrikosov vortex in an s-wave superconductor, the Ampere current-current interaction leads to screening out of the circulating electric current over a distance $\sim \lambda_L$, and that the total trapped flux is $h/4e$. If the core lies at the center of a cylinder of radius R ($\gg \lambda_L$), what is the approximate energy per unit length of such a configuration?

Solutions to be put in 598PTD homework box (2nd floor Loomis) by 5 p.m. on Mon. 14 Dec.