~~Note: lecture 3 is also handout II~~

Physics 598PTD, 2009
Physics in Two Dimensions

Introduce class, self: start on time.

Circulate sign-up sheet.

Motivation:

1) many specific 2D systems of interest, eg cuprates, graphene, helium films, Sr_2RuO_4, Si MOSFETs..., ultracold atomic gases...
2) (almost) unique theatre for realization of TQC.
3) forces us to rethink just about everything we took for granted in 3D, from scratch.

Draft plan of course (note degree of flexibility, at end) omissions (magnetism...)

Textbooks handouts.

* Assessment

Meaning of 2D:

in a many-body system, many characteristic lengths:

- atomic dimension (~ 3 Å) or, in e^- gas, k_F^{-1} (~ 1 Å) (but may be considerably larger eg in GaAs heterostructures)
- thermal dB wavelength, $\lambda_{DB} \sim \hbar/(mk_BT)^{1/2}$ (or $\sim \hbar c/k_BT$ for phonons)
- elastic mfp corrl^n length
- inelastic mfp (con h $\sim \mu$)

A given system may be "2D" in one context and "3D" in another.

(ex.: a metal film of thickness $d \sim 100$ Å at 1 K is thoroughly 2D from the point of view of the phonons ($\lambda_{DB} \sim 600$ Å) but still 3D from the point of view of the electrons.)

(note on correlation length)

A further complication: in many cases of interest, we have a 3D matrix containing many 2D planes (eg. organics, cuprates...). Then an important quantity is the degree of coupling between the planes, both due to tunnelling and due to (Coulomb) interaction (Note that even when a system is entirely 2D in all other respects (eg. graphene), the C.I. propagates in the (3D) vacuum and hence its (unscreened) 3D form is always prop. $|\underline{r}-\underline{r}'|^{-1}$)

Generally speaking, it is a necessary (and often sufficient) condition to regard a given exptl. system as "effectively 2D" ~~if~~ at temp. T that

(a) $k_B T \ll$ en. of all nontrivial excitns. ⊥'r plane (ie $d \ll \lambda_{dB}$)

(b) (in case of multilayers) $k_B T \gg$ interplane coupling.

↑: "interplane coupling" ambiguous! e.g. in case of cuprates, usually $k_B T_c \gg t_\perp$ (s.p. interplane hopping matrix element), ⇒ in formulating microscopic theory, can take to be "2D": on the other hand, for a macroscopic system, $k_B T \ll$ total interplane coupling ⇒ for calculating degree of LRO, etc, must take as 3D (more in l. 15). [also, $k_B T_c \gg \frac{e^2}{\epsilon r}$!

(2D magnetic systems)

Some peculiarities of 2D (vis-à-vis 3D)

(1) Bending energies in d dimensions: (a) consider KE in single-particle QM: If we try to contain the (normalized) Schrödinger wf over a distance L, then the KE is $\sim (\nabla\psi)^2 L^d \sim L^{d-2}|\psi|^2$ which since $L^d|\psi|^2 \sim 1$ is $\sim L^{-2}$. On the other hand, if the (attraction) potential is restricted to a region $\ll L$ then $\langle V\rangle \propto |\psi|^2 \propto L^{-d}$. Hence in 1D it is always advantageous to let $L \to \infty$, whereas in 3D it is not: 2D is "marginal" so it is not immediately clear what will happen. (see l. 8).

(b) similarly, if one has an order parameter with continuous symmetry which must be "bended", eg from a prescribed value (eg 0) at the origin to its bulk value at ∞, the bending energy is now $\propto (\overline{\nabla\phi})^2 L^d \sim L^{d-2}$ (note the OP is not normalized!) On the other hand any bulk energy assoc. with breaking of the continuous ·

symmetry (e.g. ~~[illegible]~~ in a Bloch wall in a ferromag. insulator, the Xtalline anisotropy) $\propto L^d$. Hence in 3D advantageous to shrink the domain wall to a point, whereas in 1D it has finite width. Again, 2D is marginal.....

(2) (a somewhat related point): In the theory of 2nd order ϕT, one would like to introduce a "symmetry-breaking field" $\mathcal{H}$, and then take the limit $V \to \infty$ then $\mathcal{H} \to 0$. What exactly does this mean? We want to take the limit in such a way that the energy associated with the SB field, $M\mathcal{H}$ where $\overset{\equiv N\mu}{M}$ is the total magnetization, is $\gg k_B T$ but the single-spin energy $\mu\mathcal{H} \ll \epsilon_0$, the minimum collective excitation energy of the system (if this is not so, ~~a~~ the external field is in some sense no longer a "small" perturbation). Since for a continuously broken symmetry ~~the~~ ϵ_0 typically $\propto k_{min}^2 \propto L^{-2}$, ~~these requirements imply~~ while $N \propto L^d$, these requirements imply

$$o(L^{-2}) \gg \mathcal{H} \gg o(L^{-d})$$

This condition ~~may be~~ is automatically satisfied in 3D by taking (e.g.) ~~because and the~~ ~~$\mathcal{H}$~~ (say) $\mathcal{H} \propto L^{-5/2}$, $L \to \infty$. It clearly cannot be satisfied in the same way in 2D (or $d<2$). This point is related to theorems about the absence of LRO in $d \leq 2$, see l.4. More generally, fluctuations tend to be much more important in 2D than in 3D.

(3) Random walks: we will explore this topic in detail in l.4, but note for now that in 2D (or of course in 1D) a RW returns infinitely many times to the origin, whereas in 3D the probability of return $\to 0$ as $t \to \infty$. This has profound implications for the theory of localization.

(4) The Coulomb interaction: as noted above, the (virtual) EM field is always 3D, so in real space the CI $\propto r^{-1}$ irrespective of d. However, the F.T. is $\propto q^{1-d}$ (for $d>1$) As a result, the energy of a charge oscillation (plasmon), which is $\propto (q^2V(q))^{1/2}$, → const. for $q \to 0$ in 3D, but in 2D $\propto q^{1/2}$ and thus → 0. A related point is that there is no analog of the (uniform) dielectric constant ϵ for a 2D system: in fact, the screening → 0 ~~with~~ as $q \to 0$ (see L.15).

(5) Scattering processes for degenerate fermions: this is quite a tricky point. Consider a pair of ~~short~~ fermions scattering at $T=0$ in the presence of the Fermi sea; suppose they have (small) total momentum $\underline{P}$ and (small) total energy E relative to $2\epsilon_F$. In ~~S~~ both 2 and 3D, conservation of energy means that the available phase space (outside the F. sea) $\propto E$; when combined with the DOS for initially exciting the 2^{nd} particle, this gives the standard result that the total scattering probability of a single electron of energy ϵ relative to the F. surface $\propto \epsilon^2$. What about conservation of momentum? In 3D this does not introduce any other critical factor, because ~~the~~ there is a "cone" of mom^m-conserving final states available, obtained by rotating the initial states around the total mom^m. vector. In 2D things are quite different: as $\underset{\sim}{P} \to 0$ the scattering becomes restricted to the forward direction (see Problem)

(6) The "strong-coupling" condition: in many MB systems we find that ∃ a correlation ("healing") length ξ which is proportional to $n^{-1/2}$ where n is the (d-dim^l.) density. Generally speaking, the o-of-m criterion for some kind of MF approach to be valid is that the no. of particles in a vol. whose side is ξ is $\gg 1$, i.e. $n\xi^d \gg 1$. Thus for 3D the condition is $c.n^{-1/2} \gg 1$, i.e. the low-density limit, while in 1D it is $c'n^{1/2} \gg 1$, i.e. the high-density limit. For $d=2$ the criterion to be in the MF limit is independent of density at least to log'c accuracy.

(7) However, perhaps the most important difference of all between 2D and 3D (or 1D) ~~is~~ lies in the effects of <u>topology</u>: this is really what makes 2D unique.

Imagine a particle moving in a d-dimensional space, with the origin excluded. In 1D the situation is trivial; if the particle ~~stays~~ starts on the R of the origin it stays there for ever, and vice versa. In ~~3D~~, d ≥ 3, imagine an arbitrary closed trajectory which avoids the origin; such a trajectory can explore arbitrary regions of the space before returning to its starting point. However, any such trajectory can be continuously contracted to a point, and so any transformation induced by it ~~is~~ must be "homotopically equivalent" to the identity. (Thus, for example, the orbital Schr. w.f. must be single-valued)

An important application is to the interchange of two identical particles: We know that this cannot affect the prob. density, so we immediately have that $\psi(\underline{r}_1;\underline{r}_2) = \exp i\alpha\, \psi(\underline{r}_2, \underline{r}_1)$, α real. However, <u>two</u> interchanges with the same "sense" are equiv^t (up to an irrelevant translation of the COM) to taking one particle completely around the other, and by the above argument, in ~~3D~~ d ≥ 3 dimensions the result of this must be the identity. Hence it immediately follows that in 3 (or more) dimensions $\alpha = 0$ or π (mod. 2π), giving the standard Bose or Fermi statistics. (The association of these with integral and half-integral spin respectively requires further (nontrivial!) argument).

In 2D the above argument <u>does not work</u>, because it is impossible (while continuing to exclude the origin) to shrink the loop trajectory to a point. Hence in principle ~~too~~ (if we know nothing about the nature of the "particles" in question) the "exchange phase" α can be any real number. Of course, if the "particles" are real electrons or atoms we know that they "really" live in 3D space + hence must obey B/F statistics ~~or~~ even when physically confined (eg. by low T) to a plane; but there may be more exotic composite objects which can only be defined within the plane, and these can have, a priori, any value of α. Such objects were originally envisaged by Leinaas + Myrheim in 1977, and were subsequently christened "anyons" by Wilczek.

Generally speaking, we will find that the nontrivial effects of topology in 2D are strongly enhanced, in the case of charged particles such as electrons, by a magnetic field ⊥'r to the plane of motion. The reason is that, in any d, in a nonzero magnetic field (and zero $\underline{E}$) and in the absence of complicating band-structure effects*, the projection of the semiclassical orbit of an electron (etc) on a plane ⊥'r to the field is always closed. Since in the 2D case this is the only plane available for the motion, the orbit itself is automatically closed, which is just the state of affairs which is liable to activate "anyonic" effects.

[Advice to brush up elementary chemistry, particularly of C compounds]

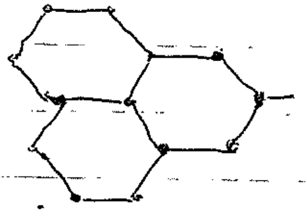
* which can produce "open" orbits.

Lecture 2. Some important (quasi-) 2D systems.

A. Naturally occurring

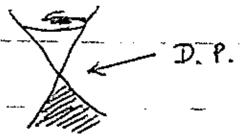
1. Graphene

A single sheet of C atoms — most "perfect" 2D system known to occur naturally. Although most experiments use graphene on (or laid on) some insulating substrate (eg SiC) it is perfectly possible to produce (at least μ-sized) free-floating (suspended) sheets[*].
(graphene can be grown by heating SiC to >1300 K)



Structure is honeycomb, (so 2 inequivalent sublattices), with C–C separation ~ 1·42 Å.
~~Most~~ "Hardest" system known in nature. (breaking strength ~100 times that of steel)
(+ most expensive: \$100 M/cm^2) — by 2013, down to ~\$60/in^2
Chemistry: C electronic str. is $(1s^2)2s^2 2p^2$

of the 4 valence electrons, 3 form the 3 sp^2 bonds necessary to maintain the 120° bond config. at each atom, leaving 1 per atom over. Thus, ~~prima facie~~ pure graphene should be metal with half-filled conduction band. Actually, more interesting: band str. is peculiar, with a "Dirac point" exactly halfway up the band. Hence pure graphene is insulator, or more strictly a semimetal.



Easy to dope with either donor or acceptor impurities, i.e. provide extra electrons/holes. Mobility is enormous (~ 2·5 × 10^5 cm^2/V sec at RT.[†] (why?)

From the point of view of basic physics, special interest of graphene is the nature of the electron states close to the D. P.:

$$\hat{H}_{eff} = \text{const.}\ \underset{\sim}{\sigma}\cdot\underset{\sim}{p} \quad (\text{like Majorana neutrino}) \qquad (v_F \sim 10^6\ \text{m/sec})$$

⟹ many analogies with relativistic physics. Also v. interesting from applied point of view. Thus, enormous surge of interest (> 2,500 papers since 2004)

[†] For comparison, μ of most semiconductors at RT ~ a few × 10^3 (Kittel, 55ϕ, p. 308)

[*] See e.g. Meyer et al., Nature 446, 60 (2007)

2. Graphite

Basically the stuff of "lead" pencils: pure C sheets (i.e. graphene), sp^2-bonded, honeycomb lattice, (hexagonal) lattice const. ~1.42 Å, layer spacing (unintercalated) ~ 3.35 Å. ~~[illegible]~~ Interplane bonding is principally vdW, hence v. friable (although π-orbitals stick out of plane, they are antibonding (?) hence do not contribute much to interplane cohesion). Ratio of conductivities, $\sigma_\perp/\sigma_\parallel$, ~ 3×10^{-4}. ($\sigma_\parallel \sim 10^4-10^5\ \Omega^{-1}\,\mathrm{cm}^{-1}$ at RT) Can be intercalated* with e.g. alkalis or SbF_6; in latter case $\sigma_\perp/\sigma_\parallel \sim 10^{-6}$ Apart from intrinsic interest, useful substrate for e.g. He films. Sometimes supercond. but only at a fraction of a K. [but rises with intercalation] max. app. ~0.55K

[note: for graphite host, free carrier concⁿ only ~10^{-4}/atom at RT]

3. TM dichalcogenides†

These are of the formula MX_2, where M is a group-V transition metal (usu. Ta or Nb) and X is a chalcogen (usu. S or Se): e.g. TaS_2, $NbSe_2$. (†: do not confuse with the TM trichalcogenides, such as TaS_3 and $NbSe_3$, which are essentially 1D systems). Composed of sheets of metal atoms sandwiched by the chalcogen.

Recall: electronic str. of Ta is $5d^3 6s^2$ and of Nb $4d^4 5s$: so 4 of the 5 valence electrons form (double) ionic bonds with the 2 chalcogens (S is $3s^2 3p^4$, Se $4s^2 4p^4$, so 2 holes/atom), leaving 1 "free" e^- per formula unit ⇒ half-filled band. This e^- is essentially in a d-state and lies mainly in the plane of the metal atoms ⇒ expect system to be essentially 2D, with transport ∥ plane easy and that along c-axis (⊥) difficult. In practice, $\sigma_\perp/\sigma_\parallel \sim 0.03$. (Absolute value of $\sigma_\parallel \sim 10^4\ \Omega^{-1}\,\mathrm{cm}^{-1}$ at RT, rising to ~$10^6-10^7\ \Omega^{-1}\,\mathrm{cm}^{-1}$ at low T, see FJ fig. 13)

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† Review: Friedel + Jerome, J. Phys. C 12, 1441 (1979), section 3.4 ("FJ")

* Review of intercalated graphite: Dresselhaus et al., Adv Phys 30 (1980), reprinted in Adv. Phys 51, 1 (2002). Note σ of Cu at RT ~ $6\times10^5\ \Omega^{-1}\,\mathrm{cm}^{-1}$

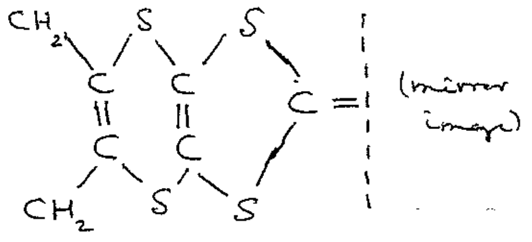
Calculated F.S. v. flat along c-axis, as expected. However, also has parts which are v. flat in plane, so quite close to nesting (FJ fig 12). So not surprising that many such materials form CDW's at low T. 2H polytype also superconducting at LT ($\lesssim 1$ K).

Easy to intercalate the TM chalcogenides, eg with flat organic mols, and then $\overset{\text{superconducting}}{\wedge}$ T_c rises dramatically (to ~ 6 K for TaS_2)

4. 2D organics

Most Xtals made up of aromatic hydrocarbons likely to be strongly 2D. Polyacenes (benzene, naphthalene, anthracene, tetracene, pentacene, ...) insulating in natural state (HOMO-LUMO gap ~ 2.5-4 eV)).

However, most studied 2D organic Xtals are based on bis(ethylene-dithio)-tetrathiafulvalene (BEDT-TTF ⇒ ET) and are of general formula



$(ET)_2 X$ Note ET v. nearly flat except at ends

where X = (eg) I_3^-, IBr_2^-, AuI_2^-, or more complex groups like Cu-$(NCS)_2$. Many types of str (eg α, β, κ ...) even for same compound, but generally speaking Xtals tend to form layers of $(ET)_2$ separated by layers of X.

Band str. as calculated by TB method fairly close to cylindrical, with in-plane hopping matrix element $\overset{t_{\parallel}}{\wedge}$ ~ 0.05 eV

Many of the 2D organics supercond., with T_c up to ~ 14 K. In view of small $t_{\parallel}$, this may be regarded as "high-temperature" superconductivity!*

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* cf. P.A. Lee, Reps Prog Phys. 71, 012501 (2008), p. 3

4DS1313
PTD 2.4

Cuprates[*]

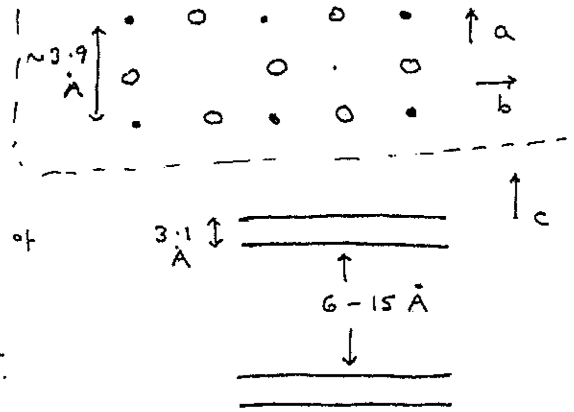
The cuprates are a group of fairly strongly 2D materials whose main claim to fame is that many of them (~300 different compounds by now) show superconductivity at anomalously high T (max. ~ 160 K (Hg-1223) : dozens in range 110-120 K). Note that not all cuprates show HTS, and those that do generally show it only in a restricted range of doping (cf. below).

Composition: traditional chemical notation, eg $YBa_2Cu_3O_{7-\delta}$, not v. informative. More sensible to write generic formula for a cuprate in form

$$(CuO_2)_n A_{n-1} X \qquad (n = 0, 1 \ldots)$$

where A = alkaline earth, rare earth, Y, La ... , X = (almost) anything (not nec. stoichiometric), but frequently a heavy-metal oxide.[ξ]

CuO_2 units occur in planes, with str. If more than 1 CuO_2 plane per (physical)[†] unit cell, then they occur together (in "multilayers") spaced (intercalated) by the element A. The remainder of the formula, X, is the "charge reservoir" and lies between the "multilayers". In the literature one sometimes partitions the charge reservoir material into a "spacing" layer, near to the CuO_2 planes, typically an oxide, and one or more "insulating" layers (~~often~~ which can actually be weakly metallic). E.g. in YBCO, where the reservoir is $BaCuO_{3-\delta}$, the ~~spa~~ spacing layers (near to the CuO planes) are BaO, and the (not very insulating!) "insulating" layer is the $CuO_{1-\delta}$ chains. In some compounds, eg $La_2CuO_{4-\delta}$, the insulating layer is absent.



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[*] Ref.: Shaked et al., ANL "pocketbook"

[ξ] Thus, the compound usually called $YBa_2Cu_3O_{7-\delta}$ is in this notation $(CuO_2)_2 Y (BaCuO_{3-}$

KPD 3.4

PTD 2.5

Electronic str.: It is often possible to construct, for a given choice of elements in the charge reservoir X, a stoichiometric compound s.t. the valences balance, eg. La_2CuO_4 or $YBa_2Cu_3O_6$ (note not O_7!). When one balances the valences in this way, the Cu's (which start off in the $3d^{10}\,4s^1$ configuration) are left in the Cu^{++}, $3d^9$ config^n, thus with 1 hole per ~~unit cell~~ CuO_2 unit in the d-shell. So band theory would predict that at least the single-layer compounds of this class, such as La_2CuO_4, would be metals (since they have one odd electron per unit cell, hence a half-filled band). In fact, both La_2CuO_4 and $YBa_2Cu_3O_6$ are insulators, and all the ev. is that they are Mott ins^n (one localized e^-/unit cell) and moreover AF.

If now we deviate from exact stoichiometry, eg by ~~changing~~ reducing the no. of O's in the chains from 1 (YBCO) or by substituting say Sr for La (LSCO), the effect is to inject extra holes (or sometimes e^-'s); these tend to sit in the planes, predominantly on the Cu sites, and hence are predominantly in one of the Cu 3d states: all the evidence is that they are specifically in the $d_{x^2-y^2}$ states (the $d_{z^2-r^2}$ ~~and~~, d_{xy}, d_{xz} and d_{yz} states being already filled) and hybridize somewhat with the O $2p_x$ and $2p_y$ states.

Both because of the strongly anisotropic Xtal str., and because the "free" electrons are primarily in the $d_{x^2-y^2}$ state, the electronic properties of the cuprates are expected to be highly anisotropic, and indeed the exptl. value of $\sigma_{\parallel}/\sigma_{\perp}$ ranges from ~ 30 (in optimally doped YBCO) to $\sim 10^5$ (Bi 2212) (and even higher in some intercalated cuprates).* Typically $\sigma_{\parallel} \sim 10^3$ S/cm at RT (and "optimal" doping) ($\sim 10^{-3}$ that of Cu); interestingly, this corr. to a $R_\square$ per square of CuO_2 plane which is of the order of the "quantum unit of resistance" h/e^2 (l.5).

[In addition to sup^y, weak localiz^n may be realizable in cuprates]

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* All the evidence is that the electronic transport in the c ($\perp$) dir^n is by incoherent hopping, so that the concept of a BS is inapplicable.

6. Strontium ~~[illegible]~~ ruthenate* (Sr_2RuO_4)

Original motivation for study: similarity in both structure + chemistry to cuprates. Structure is layered, with RuO_2 planes separated by SrO_2 spacers (so similar to eg La_2CuO_4), similar plane spacing (~ Å).

Chemistry: $Cu(Z=29)$ is $[Ar] + 3d^{10}4s^1 \longrightarrow 3d^9$

$Ru(Z=44)$ is $[Kr] + 4d^7 5s^1 \longrightarrow 4d^4$

In normal state, $\sigma_\perp / \sigma_\parallel \sim 10^{-3}$ (similar to cuprates). (but T-dependence in each case nothing like cuprates). Residual resistance ~ 1 μΩ cm (comparable to Cu at RT).

Becomes superconducting at $T_c \sim 1K$ (cuprates: ~100 K!)

Most interesting feature: strong ev. that Cooper pairs form in spin triplet state, and moderately strong evidence (inc. Kidwingira et al.) that they form in the so-called ($p + ip$) orbital state.

Hence of great interest in context of TQC.! ↖ like ^{3}He-A,

but 2-dimensional so no nodes in gap

7. Films of liquid He ~~[illegible]~~

Neither ^{3}He nor ^{4}He crystallizes under own vapor pressure → can (inter alia) form liquid film on certain kinds of substrate (eg graphite, metallic Cs). General belief is that first 2 layers are "solid", not "liquid" (but conceptual problem regarding the meaning of solid/liquid in this kind of situation). On some substrates however ev. for superfluidity of ^{4}He (hence, a fortiori "liquidity") down to a fraction of a monolayer.

A related 2D system is electrons on the surface of liquid He (typically v. low density, ~ cm^{-2})

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* Comprehensive review: Mackenzie + Maeno, RMP 75, 1 (2003)

8. Monolayers (of rare gases etc)*

Although in bulk all elements other than He (including the other rare gases, Ar, Ne, Xe ...) form solids, it is often possible to put down monolayer or sub-monolayer amounts on some appropriate surface (eg exfoliated graphite) and measure properties such as the sp ht or NMR. ~~A~~ Many of these systems show interesting phase transitions between solid/liquid or commensurate/incommensurate phases

9. Ferropnictides

[use l.13 of SAM, 2008]

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* Cf. Thouless, op.cit.

Lecture 3 Some important (quasi-) 2D systems, cont.

B. Engineered

(note influence of development of MBE on 2D physics)

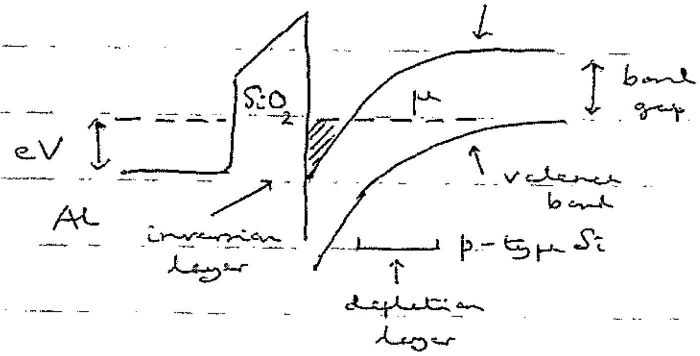
1. Semiconductor interfaces

The great advantage of semiconductors over metals is that because of the low density + hence relatively poor screening properties, it is easy to influence the carrier density close to a surface by applying modest potentials (~ a few V), and thereby bending the bands over an energy quite comparable to the ~~bo~~ intrinsic band gaps.

nb. band gap of Si = 1.14 eV
binding en of B acceptor = 0.045 eV

Si MOSFETs

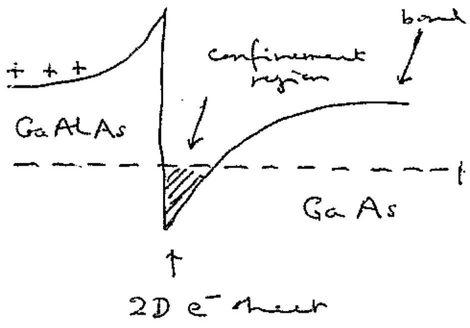
This is probably the conceptually simplest case.



~~The~~ A positive potential applied to the metal bends the bands, as shown. The p-type Si ~~has~~ contains acceptor impurities which are ionised, so there is a nonzero concentration of holes in the bulk, but close enough to the surface the potential for holes drives them away, giving a "depletion layer" which is insulating. Meanwhile, if the applied potential exceeds $E_c - \mu$, there will be a region (shaded) close to the surface where the electron energy is $< \mu$, thus it will be advantageous for electrons from the metal to migrate there. ("inversion layer")

The width of the inversion layer is typically ~ 25 Å, and the electrons have to a first approx^n their bulk-Si values of m^* (~0·2 m) and also ϵ (~12). Hence the energy of the first excited transverse state is ~ 200 K. Mobilities can be high, ~ ~~[illegible]~~ 10^6 cm²/V sec., corr. to a mfp ~ ~~1000 Å~~ 5μ.

GaAs – GaAlAs ($Al_xGa_{1-x}As$) heterostructures.



Note that although "quantum wells" are often mentioned in this context, all that is really needed is an interface between two semiconductors with appreciably different band gaps. We dope the AlGaAs ~~p~~ with donor impurities, which become ionized and thus generate an (approximately quadratic) potential ($\operatorname{div} E = \rho/\epsilon_0 \Rightarrow E \sim \rho(x-x_0)$ $\Rightarrow \phi \sim \rho(x-x_0)^2$). The donated electrons migrate to the point of lowest potential, on the GaAs side of the interface. The confinement region typically has width ~ 200 Å, and the eff. mass of an e^- in bulk GaAs is $m^* \sim 0{\cdot}07\,m$ (and $\epsilon \sim 11$), so the en. of the first excited state is ~ 10 K. Densities (controllable by gate voltage) are typically ~ $10^{10} - 10^{12}$ ~~Ao~~ e^-/cm^2: note that this means that in a ⊥'r magn. field of a few T, the filling factor is ~ 1.

An advantage of this geometry is that the electrons filling the surface states are ~~phys~~ spatially separated from their donors. As a result, mobilities can be v. high indeed (~ 3×10^7 cm²/V sec in recent expts), higher even than attained in the best Si MOSFETs. Another advantage of this system vis-à-vis Si is that in the latter there are 2 different "valleys", with different geom^l. props, while in GaAs there is only one valley centered at the Γ point of the FBZ. For these reasons GaAs/AlGaAs heterostructures have (in recent years) become the system of choice for sophisticated QHE expts.

Engineered systems, cont.

Trapped ultracold atomic gases

Ultracold atoms may be trapped by a spatially varying magnetic field and/or by counterpropagating laser beams. In the case of magnetic trapping the relevant potential is just $-\mu\cdot B(r)$ where μ is the atomic magnetic moment. Since it is impossible to produce a maximum of the (time-ind.) magnetic field in free space (Earnshaw's theorem) but perfectly possible to produce a minimum, one usually tries to trap ~~those~~ atoms in the HF states in which μ is oppositely oriented to the field (the "low-field seekers"); contrary to what one might perhaps think, these HF states ~~are~~ are often stable over timescales ~ minutes (comparable to the decay time of the gas). In the case of optical trapping, the basic principle is that if the detuning $\Delta \equiv \omega_{las} - \omega_{tr}$ of the laser field from the relevant atomic transition is ≫ the spontaneous linewidth Γ ~~sp~~ of the transition, then ~~to~~ after averaging over timescales ≫ ω_{laser}^{-1} one gets an effective potential which is proportional to I_0/Δ where I_0 is the ~~and Δ is the detuning $\omega_{las}-\omega$~~ laser intensity at the point in question. Note (a) the potential is repulsive (attractive) for blue-shifted (red-shifted) laser frequency (b) the intensity is prop'l to the square of the [(optically averaged)] total electric field E; thus by using ~~two~~ counterpropagating beams one can get a strong spatial dependence of the trapping potential. If the laser wavelength is λ_L and the two beams are propagating at relative ~~angle~~ angle θ, then one gets a periodic potential with "periodicity $\lambda_L/2\sin\theta$. This potential has a depth (height) V_0 which is normally expressed in terms of the "recoil" energy $E_R \equiv \hbar^2 k_L^2/2M$ where k_R ~~is~~ $\equiv 2\pi/\lambda_L$ and M is the mass of the atom in question: a typical value of E_R is ~ ~~100 kHz~~ 5 kHz (200 nK), and ~~E_R~~ typical values of V_0/E_R range from 0 to ~40. The potential is sinusoidal, and the frequency of small oscillations around the minimum is $2\sqrt{V_0 E_R}/\hbar$. ~~[illegible]~~

In order to get an effectively 2D system, one combines magnetic trapping in all 3 dimensions with laser trapping in ~~the~~ one direction (i.e. ⊥ to the desired 2D plane). Thus, in the experiment of Hadzibabic et al*, a cloud of ^{87}Rb atoms was subjected to a total potential

$$V(r) = V_{mag}(r) + V_{laser}(r)$$

where

$$V_{mag}(r) \cong \tfrac{1}{2} M(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2)$$

$$V_{laser}(\underset{\sim}{r}) \cong V_0 \exp(-2x^2/W_x^2) \sin^2 k(z - z_0)$$

where W_x is the laser "waist" along the x-dir^n and z_0 is the offset of the laser potential (which does not necessarily coincide with the min. of the magnetic pot^l). The parameters of the expt. were

$$V_0/\hbar = 50 \text{ kHz}, \quad \omega_x/2\pi = 11 \text{ Hz}, \quad \omega_y = 130 \text{ Hz},$$

lattice period π/k ≈ 3μ

$\omega_z = 3.6$ kHz, (so $\omega_z \gg \omega_x, \omega_y$, i.e. ~~[illegible]~~ magnetic z-trapping is much stronger than the in-plane trapping). ~~[Bearing in mind that 20 kHz corresponds to 1 μK, we see that for T ~ 80 nK (the typical temp. used in the expt) the population of the]~~ ~~It is interesting that if the $z_0 = 0$, the laser trapping in the z direction seems to be a small pert^n on the magnetic trapping~~. (In the actual expt., they often set a maximum of the pot^l at z_0, so as to obtain 2 degenerate minima). ~~If the~~ Bearing in mind that 20 kHz corr. to 1 μK, we see that at 80 nK (a typical temp. in the expt) the second harmonic oscillation in the z-direction is only v. weakly thermally excited, so indeed the system can be regarded as "2D".

A major difference from solid-state systems is that the "system" can be rapidly taken apart by turning off the pot^{ls}; this is extremely useful in diagnosing its behavior, e.g. by looking for interference effects.

* Nature 441, 1118 (2006)
cf also NJP 10, 045006 (2008)

Diagnostics of (quasi-) 2D systems.

The question of the experimental methods available for examination of the structure + dynamics of (quasi-) 2D systems is liable to depend heavily on whether we are dealing with a single system (eg. graphene, He films, QHE systems) or with a macroscopically large number of similar systems embedded in a 3D matrix (eg. organics, cuprates). In the latter case most of the standard techniques of CM physics are available: eg. X-ray/neutron scattering can be used to probe the structure, and ~~at~~ at least if we can reliably subtract the contribution of the matrix, thermod^c quantities such as the sp. ht. C_v and spin susceptibility χ can be measured.

For genuinely isolated single 2D systems most of these methods are impractical, simply because the no. of atoms/electrons involved is too small. However, this difficulty should not be overestimated: remarkably, it is possible to examine the structure of even a single suspended graphene sheet by TEM*, and while conventional NMR in (say) QHE systems is unfeasible because of the weakness of the signal, resistively detected NMR has been used† to measure the spin susceptibility of some of the more robust QH states.

The most commonly used diagnostic for (quasi-) 2D systems is the transport properties. In the case of a film / monolayer on a substrate, the thermal conductivity is likely to be shorted out by the substrate, but for free-standing films it should be measurable. Even in the case of a substrate provided it is insulating, the electrical conductivity of the 2D system is relatively easy to measure, and so is the Hall conductivity. In the case of the QHE, in fact, until recently these have been just about the only things that can be measured. In principle, optical abs^n + photocond^n should be measurable

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* J.C. Meyer et al., Nature 446, 60 (2007)

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