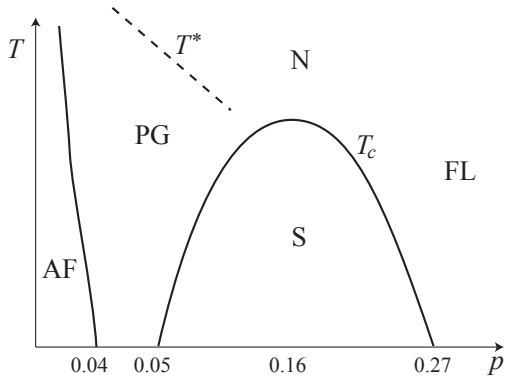
Aspects of two-dimensionality in the cuprates

Among the various bulk 3D materials that have a strongly layered structure, the cuprates stand out both as being among the most "2-dimensional" (see below) and as possessing the most interesting phase diagram. The general structure and composition of the cuprates was briefly reviewed in lecture 2: here we concentrate on the phase diagram, just recalling that the basic structure consists of n CuO_2 planes per unit cell, spaced relative to one another at $\sim 3.1\,$Å by "spacer" atoms such as Ca or Y and separated from the next multilayer (next unit cell) by a "charge reservoir" layer that may be up to $\sim 10\,$Å thick.



The variable whose effect has been most extensively examined, apart of course from the temperature T, is the stoichiometry; it is by now generally agreed that an appropriate way of parametrizing the latter is by converting it to a "number of holes per CuO_2 unit p", and this idea is reinforced by the fact that when this is done in a chemically plausible way, the phase diagram looks to a first approximation universal as regards its p-dependence. However the height of the superconducting "dome" varies by more than an order of magnitude between different cuprates (and in some is missing altogether). A very interesting observation is that in all homologous series (that is, series of cuprates with the same chemical composition except for having different numbers of CuO_2 planes per unit cell) in which the intercalant is Ca the maximum transition temperature T_{c0} depends in a systematic way on the layer multiplicity n, increasing up to $n = 3$ and thereafter probably[1] decreasing gradually. However, it should be emphasized that, almost without exception[2], when the bilayer homologue of a cuprate such as $La_{2-x}Sr_xCuO_4$ is spaced with an alkaline earth other than Ca (Ba or Sr), it is found not to be superconducting at all.

A couple of regions of the phase diagram show relatively familiar behavior. First the "parent" compound ($p = 0$, e.g., La_2CuO_4 or $YBa_2Cu_3O_6$) appears to be a standard antiferromagnetic Mott insulator, and the antiferromagnetism persists for weak doping (up to $p \cong 0.04$). On the other side of the diagram, in the region to the right of the superconducting dome, the cuprates appear to behave[3] roughly like textbook Fermi liquids, with an effective mass $m^* \sim 4m$ and an (in-plane) resistivity proportional to T^2 (as expected from electron-electron umklapp collisions). The superconducting (S) phase also behaves

[1] The caveat is that it is not certain that compounds with $n \geq 4$ are homogeneously doped, or even if they are, that optimal doping can be attained.

[2] the one exception: di Luccio et al., Phys. Rev. B **67**, 92504 (2003) (Sr-intercolated BSCCO).

[3] As regards the "in-plane" behavior.

qualitatively much like that of a conventional textbook superconductor, with however the exception of the behavior in phase-sensitive (Josephson-type) experiments, which is usually taken to indicate that the Cooper pairs are formed with the so-called $d_{x^2-y^2}$ symmetry. It should be noted that despite the fact that the OP is anisotropic, unlike the situation in e.g. the A phase of superfluid ^{3}He the anisotropy is "locked" to the crystal axes and so cannot fluctuate in the bulk material. For this reason, when considering effects such as a possible KT transition which depend on the effect of fluctuations, it is generally believed to be adequate to treat the broken symmetry as simply U(1) as in a classic BCS superconductor. For most of the purposes of this lecture it will be adequate to ignore the $d_{x^2-y^2}$ structure of the OP (except perhaps insofar as it means that the normal density at low temperatures is much greater than that of a standard s-wave superconductor).

By contrast with these more or less familiar patterns of behavior, that in the normal (N) and even more in the pseudogap (PG) regime is very anomolous. The most striking feature of the normal phase is that the in-plane resistivity appears to depend on temperature as T^α, where the exponent α depends on p, varying from approximately 2 for $p > 0.35$ (the "Fermi-liquid" regime) to considerably less than 1 for $p \leq 0.1$. For optimal[4] doping ($p \cong 0.16$) the in-plane resistivity appears to be very close to exactly linear in T for all the cuprates, over a range from $\sim 1000\,\mathrm{K}$ right down to the superconducting T_c (which may be as low as $10\,\mathrm{K}$). It seems very unlikely that this behavior has much to do with the standard linear-in-T resistivity of textbook metals for $T \geq \theta_D$, since in the cuprates the Debye temperature θ_D is of order of room temperature (and, additionally, the exponent in the textbook case should be 1 independently of the details of the electronic band structure, i.e. the degree of doping).

The "pseudogap" (PG) or "underdoped" regime is even more unusual. In the first place, while very few direct measurements have been carried out at the lowest temperatures in the region of doping p between 0.04 and 0.05 which separates, at $T = 0$, the AF and S phases, everything appears consistent with the hypothesis that the (in-plane) resistivity $\rho_{ab}(T)$ is tending to ∞ with T, indicating that at $T = 0$ the system is a perfect insulator. Secondly, many features of the experimental behavior in the PG region are reminiscent of superconductivity: the electronic specific heat and spin susceptibility are much reduced, both ARPES and tunnelling see an energy gap which is comparable to that of the S state and whose energy is virtually independent of temperature, the high-frequency electromagnetic response is suggestive of a "residual" superconducting effect at nonzero frequencies, and a huge Nernst effect is seen, suggesting the presence of "virtual" superconducting vortices.

A feature of the phase diagram whose interpretation is very controversial is the so-called "T^* line" that, crudely speaking, separates the pseudogap regime from the "normal metal" that exists at and around optimal doping. Experimentally, this line manifests itself as a continuous but fairly sharp drop in quantities such as the electronic specific heat and spin susceptibility as T is decreased at constant p. An important question is whether the

[4] "Optimal" doping conventionally means that giving the highest superconducting T_c.

$T^*(p)$ line joins on smoothly to the critical temperature $T_c(p)$ at $p \cong 0.16$, or whether it continues into the S phase, reaching the $T = 0$ axis at $p \cong 0.19$. My personal belief is that the arguments of Tallon and Loram[5], together with the rather few experiments in which superconductivity is suppressed by doping or a strong magnetic field, favor the latter view.

Before raising some specific questions that may be related to the 2D nature of the cuprates, let's briefly address the question: while their structure is clearly strongly two-dimensional, how far is this reflected in their experimental properties? As regards static properties such as uniaxial compressibility, there is some anisotropy, particularly in the bi- and trilayer cuprates (e.g. Bi-2212 is more compressible along the c-axis than in the ab-plane by a factor ~ 3), but it is not dramatic. With regard to the electrical conductivity the effects of anisotropy are much more spectacular. In fact, this quantity is not even qualitatively universal; e.g. at optimal doping, while all components of $\rho(T)$ behave as a power law T^α, as we have seen α is invariably exactly 1 for the in-plane component, while for the c-axis component it varies in the range between -1 and $+1$ depending on the material. Moreover, the actual numerical ratio $\rho_c(T)/\rho_{ab}(T)$ at room temperature is always $\gg 1$, varying from ~ 30 for optimally doped YBCO (probably the "most 3-dimensional" of all the cuprates) to $\sim 10^5$ for Bi-2212. A similar situation is found as regards the penetration depth in the superconducting state: at $T = 0$ this again shows very strong anisotropy, with $\lambda_{ab}(0)$ typically $\sim 1000 - 2000$ Å but $\lambda_c(T)$ varying from $\sim 1\mu$ for optimally doped YBCO to the enormous value of 100μ (almost visible with the naked eye!) for Bi-2212. (Note that since most simple models give $\lambda^2(0) \sim \rho_n$, the ratios of the normal and superconducting-state parameters are reasonably consistent.) The temperature-dependence of $\lambda_c(T)$ is also noticeably different from that of $\lambda_{ab}(T)$.

The above pattern of behavior raises a number of obvious questions to which the two-dimensionality may (or may not!) be relevant, as follows:

1. Is the thermodynamic and transport behavior best regarded as 2- or 3-dimensional (a) in the N phase (b) in the S phase?

2. (a related question): Does the superconducting transition have anything to do with the Hohenberg-Mermin-Wagner theorem, or with the KT transition?

3. Is the "pseudogap" regime a regime in which Cooper pairs form locally but lack long-range phase coherence? ("pre-formed pairs")

4. Does the $T^*(p)$ line reflect the presence of a quantum phase transition at $T = 0$, $p \cong 0 \cdot 19$? If so, what is the symmetry that is broken?

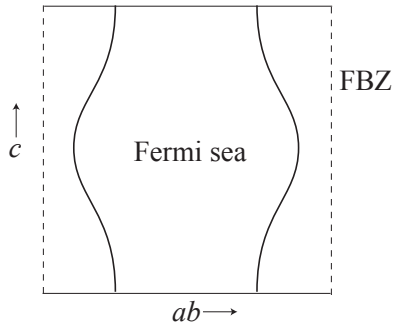
5. Is the apparent upturn of $\rho_{ab}(T)$ in the pseudogap regime a "weak-localization" effect?

6. What is the origin of the systematic dependence of $T_{c0}(n)$ on the layer multiplicity n?

[5] Physica C **349**, 53 (2001).

I shall attempt to address these questions in turn. In doing so, I shall usually make the default assumption that where a particular kind of experimental behavior has been observed in one or a few cuprates it is typical of the class of cuprates as a whole, unless of course its absence has been positively observed in others.

(1a). Is the normal phase 2- or 3-dimensional?



The most obvious question is: Should we think of the N phase in terms of a 3D Fermi sea as in a textbook metal, or rather regard the electrons in each CuO_2 plane as independently described by their own 2D Fermi sea (or indeed perhaps as not described by a Fermi sea at all)? The traditional way of investigating the shape of the Fermi surface (in a 3D textbook metal) is by dHvA and related techniques. Unfortunately, even if some or all of the cuprates really do have a 3D Fermi surface, it is almost certain to be so flat in the c-direction (see below) that it seems very unlikely that one would get any closed orbits in the ac- or bc-planes; and in fact, to the best of my knowledge no dHvA-type experiments have even been tried with the magnetic field oriented perpendicular to these planes. In the last few years a number of experiments have been done[6] with the field oriented along the c-axis: while the results are very interesting, they really only probe the (cross-section of the) Fermi surface that lies in the ab-plane, so cannot descriminate between the 2D and 3D scenarios. Similarly, the strongly layered structure of the cuprates makes ARPES almost automatically a probe only of the in-plane band structure.

That leaves one with transport measurements, and in particular[7] with the c-axis dc electrical conductivity[8]. Crudely speaking, there are two obvious models:

(1) Coherent transport: it could be that the Fermi surface really is 3-dimensional, but then, unless the scattering mechanisms are themselves very strongly anisotropic, the observed values of ρ_c/ρ_{ab} ($\gtrsim 30$) would seem to imply a similar anisotropy of the effective mass (since $\rho \sim m^*/ne^2$). With such a large mass anisotropy the Fermi surface cannot be closed, but must have roughly the shape shown in the figure. This is not ridiculous a priori, but counter-arguments include (a) that band-structure calculations (for what they are worth, which may not be very much in a strongly correlated system such as the cuprates) predict a very much smaller degree of anisotropy, and

[6] See Jaudet et al., Physica B **404**, 354 (2009), and earlier references cited therein.
[7] the c-axis thermal conductivity is almost certainly phonon-dominated.
[8] A good discussion is given by Cooper and Gray in Ginsberg IV, section II.

(b) that in that case one would prima facie expect the temperature-dependence of $\rho_c(T)$ to be similar to that of $\rho_{ab}(T)$. This is true for most cuprates in the "FL" regime to the right of the superconducting dome, and approximately true for optimally doped YBCO; however, it is not at all true, even at optimal doping, for the more anisotropic cuprates such as BSCCO, which often have $\rho_c(T) \propto T^\alpha$ with $\alpha < 0$ ("semiconducting" behavior).

(2) Incoherent hopping: in this scenario the whole idea of Bloch waves propagating along the c-axis is thrown out, and one considers instead that the c-axis conductivity is due to incoherent hopping between cells (multilayers) with some phenomonological hopping time τ. To estimate τ, we use the fact that the diffusion rate is a^2/τ where a is the interlayer spacing, and the (quantum) Einstein relation $\sigma = e^2 D\chi$ where χ is the relevant "susceptibility" (compressibility); it seems reasonable to take the latter to be equal to the 2D DOS $m^*/\pi\hbar^2$ times the mean density of planes along the c-axis, which is of order[9] a^{-1}. Thus we get up to a constant of order unity

$$\tau \sim (R_c/R_Q)\tau_0 \tag{1}$$

where we have defined

$$R_c \equiv \rho_c/a, \qquad \tau_0 \equiv m^* a^2/\hbar \qquad (\text{and } R_Q \equiv h/e^2) \tag{2}$$

For a typical cuprate with $a \sim 10\,\text{Å}$, and (from specific heat data) $m^* \sim 4m$ the quantity τ_0 is $\sim 4 \times 10^{-14}$ secs., which coincidentally is about equal to $\hbar/k_B T$ at the highest onset temperature for superconductivity ($\sim 150K$). Now, putting in the numbers, we find that the ratio R_c/R_Q is always $\gg 1$ except possibly for optimally doped or overdoped YBCO. We therefore draw the conclusion that for almost all cuprates at the onset temperature T_c for superconductivity we have

$$\tau(T_c) \gg \hbar/k_B T_c \tag{3}$$

and thus that, prima facie, we should be able to consider the process of formation of Cooper pairs in terms of individual 2D planes.

What is the mechanism that decoheres the inter-plane (or more accurately inter-cell) hopping process? A very tempting idea is to think of the problem as analogous to the spin-boson problem already mentioned in lecture 14. We saw there that if the dissipation is "ohmic" it is characterized by a single dimensionless parameter α, with a zero-temperature QPT occurring at $\alpha = 1$. A more detailed consideration[10] shows that at high enough temperatures the transitions between the two "wells" in the SB problem are by incoherent hopping, with a rate $T^{2\alpha-1}$, so that the rate decreases with increasing T for $\alpha < 1/2$

[9] but not in general equal to it, except in single-plane materials.

[10] see e.g. AJL et al., RMP **59**, 1987.

(because the attempts at coherent tunnelling are detuned by the environment) while for $\alpha > 1/2$ it increases with T (because the incoherent hopping is "activated" by the environment). Thus, if the value of the dimensionless parameter α differs for different cuprates, this could explain why the dependence of $\rho_c(T)$ on T is almost always a power law, but the exponent varies, roughly between -1 and 1, for different materials. However, to make this idea quantitatively plausible one evidently needs a microscopic theory of the detuning process.

(1b) Is the superconducting phase 2- or 3-dimensional?

Above we argued that the *process of formation of the Cooper pairs* in the cuprates should be regarded as "2-dimensional." However, it is a different question whether the fluctuations of the superconducting OP have a 2- or 3D character. Actually, according to the arguments of lecture 8, we should expect that the appropriate version of a GL-like theory for the cuprates would be the Lawrence-Doniach (LD) model as implemented by eqn. (8). Such a model will behave at least for the purposes of its critical behavior, like a 2D or 3D model according as the c-axis healing length $\xi_c(T)$ is less or greater than the inter(multi)layer spacing a. Now, we recall that $\xi_c(T)$ is proportional to $(1 - T/T_c)^{-1/2}$, with a coefficient that is proportional to $\rho_{so}^{-1/2}(0)$. Thus, we should expect the system to behave as 2D far from T_c but to switch over to 3D behavior as T approaches T_c, the 3D region being larger the greater the degree of anisotropy. Actually, so long as we are in the "mean-field" regime far from T_c there is no qualitative distinction between 2D and 3D behavior; however, in the "critical" region close to T_c the exponents are predicted to be different.

The most accurately measured property of the superconducting state is the (in-plane) London penetration depth[11]; for the critical regime of the XY model in 3D, this is predicted to behave as $(T_c - T)^{-1/3}$, and the experimentally observed temperature dependence indeed seems to fit this dependence rather well (see fig. 9 of Bonn and Hardy, ref. cit.).

2. Is the HMW theorem and/or the KT mechanism relevant?

Since the system we are dealing with is at bottom 3-dimensional, it is fairly obvious that we shall not get complete suppression of the LRO by the HMW argument. However, in view of the very large anisotropy of some of the cuprates, it is worth asking whether by using Hohenberg's lemma we can set any interesting upper limit on the condensate fraction n_0/n. Unfortunately, without a specific model of the motion of electrons along the c-axis, it appears impossible to exploit Hohenberg's lemma. Just for interest, let's ask what would result if we were to assume (probably unrealistically, see (1a) above) that the c-axis motion is coherent with an effective mass m_c^* (and that in the ab-plane coherent with effective mass m_{ab}^*). The problem can then be mapped on to an isotropic 3D problem with the replacement $m \to \left(m_c^*(m_{ab}^*)^2\right)^{1/3}$. Now for an isotropic 3D superconductor Hohenberg's lemma may

[11] See Bonn and Hardy, in Ginsberg V.

be shown to lead to an upper limit on n_o/n which is of order $(T_{\rm F}/T)^{3/2}$, where the Fermi temperature $T_{\rm F}$ varies as m^{-1}. Hence for our problem, assuming $m^*_{ab} \sim m$, the upper limit will be of order $(T_{\rm F}/T)^{3/2}(m/m^*_c)^{1/2}$, which for BSCCO would be $\sim 10^{-3}(T_{\rm F}/T)^{3/2}$. Assuming $T_{\rm F} \sim 1\,{\rm eV}$ $(10^4\,{\rm K})$ and $T_c \sim 100\,{\rm K}$, this just fails to give a nonvacuous (i.e. < 1) upper limit on n_0/n at any $T < T_c$. (However, it might be interesting to put in the numerical factors carefully.)

What about the relevance of the KT transition? Let us first estimate $T_{\rm KT}$ for a *single isolated* CuO_2 multilayer. For most of the higher-T_c cuprates the naive estimate which equates the $T = 0$ GL superfluid density per multilayer $\rho_{s0}(0)$ to n (the layer multiplicity) times $n_a(m^2/m^*)$ where n_a is the number of conduction electrons (actually holes) per unit area is reasonably good, so if we take $\rho_{s0}(T)$ for $T \to T_c$ to have the temperature-dependence $\rho_{s0}(T) \sim \rho_{s0}(0)(1 - T/T_c)^\zeta$ (cf. above), then using $n_a\hbar^2/m^* \sim E_{\rm F}$ we find

$$(1 - T_{\rm KT}/T_c)^\zeta \sim \frac{1}{n}\left(\frac{T_c}{T_{\rm F}}\right) \tag{4}$$

so whatever the (reasonable) value of ζ, $T_{\rm KT} - T_c$ will be $\ll T_c$, in fact $< 10^{-2}T_c$. The problem now is that by the time we reach this point we are already in the regime where the fluctuations are effectively 3-dimensional (see (1b) above). Hence it looks very unlikely that KT physics has much to do with the superconducting transition in the cuprates.

One might possibly object to the above argument on the grounds that the fact that the *long-wavelength* fluctuations are essentially 3D need not necessarily imply that the behavior of the vortex-antivortex pairs which are responsible for the KT transition is equally 3D. Indeed, there is substantial evidence in the more anisotropic cuprates such as BSCCO for the existence, in an external magnetic field, of so-called "pancake" vortices, that is vortices confined to a single plane. Unfortunately, a proper discussion of this question would require us to examine in detail the various possible mechanisms of coupling between vortices in neighboring multilayers, including not only Josephson and Ampere couplings but the second-order effect of the Coulomb interaction[12]—a topic which would be several lectures' worth in itself. However, irrespective of the details, it seems implausible that the main mechanism of T_c is a single-multilayer KT one, for a reason given below under (6).

3. "Pre-formed pairs" in the pseudogap regime?

A very attractive hypothesis, which has been advanced in particular by Emery and Kivelson[13], is that the reason why so much of the experimental behavior of the cuprates in the pseudogap regime of the phase diagram is so reminiscent of that in the superconducting phase is that in the PG regime Cooper pairs are indeed formed "locally," but there is no long-range phase coherence (LRO). If this is true, one might regard the T^* line as a continuation of the T_c line for doping below optimal, and as indicating the order of magnitude of

[12] See AG Rojo and AJL, PRL **67**, 3614 (1991).
[13] Nature **374**, 434 (1995)

the temperature at which the "local" Cooper pairs begin to dissociate into their constituent electrons. If one were to take the prima facie "2-dimensionality" of the cuprates seriously, it would bring us back to the KT scenario. If on the contrary one believes the system to be essentially 3D in nature, then it seems more natural to regard it as analogous to the "BEC-BCS crossover" which is now a major topic of study in the ultracold-gas field, and a lot of theoretical work has been done to exploit this analogy. In an extreme version of this scenario, propagated in particular by Uemura, one actually regards the local "pairs" in the pseudogap regime as tightly bound bosons analogous to diatomic molecules, and views T_c as simply the temperature for onset of BEC. Because of the strongly anisotropic nature of the single-boson spectrum, one predicts that T_c should scale as $\rho_s(0)$, a prediction that is partially though not totally in agreement with experiment.[14]

4. Is the T^* line a result of a zero-temperature QPT?

If one does not believe that the line of anomalies denoted T^* joins are smoothly to the T_c line for $p > p_{opt}(\cong 0.16)$, and thus does not have directly to do with Cooper pair formation, but rather dives into the superconducting phase and (once superconductivity is suppressed intersects the p-axis at $p \cong 0.19$, then the obvious question is: is the point $T = 0$, $p = 0.19$ a QCP? And if it is, what is the nature of the "ordered" phase(s) involved?

One feature of the experimental data that is strongly suggestive of a QCP is the behavior of the dc ab-plane resistivity, which appears to behave as a power-law T^α for all doping p close to p_c, but with α varying from (roughly) -1 to 2 as p is varied. This is strongly suggestive of a scaling form for $\rho_{ab}(T, p)$ such as

$$\rho_{ab}(T, p) \sim T^n f\left(T(p - p_c)^\nu\right) \tag{5}$$

though it does not appear that easy to find a form of the function $f(x)$ that fits both the temperature—and the doping-dependence observed experimentally. However, if this scenario is correct, then presumably there must be some symmetry that is spontaneously broken for $p < p_c$ but unbroken for $p > p_c$. Moreover, the T^* line must mark a genuine second-order phase transition; since there is very little direct evidence (e.g. in discontinuities in the specific heat, etc.) to data for such a transition, advocates of this point of view have to argue that simple inhomogeneities, etc., have obscured it. As to the nature of the symmetry that is supposedly broken in the ordered phase, then are numerous suggestions in the theoretical literature, but none has so far been unambiguously established experimentally. A more conservative view, expressed in particular by Tallon and Loram, is that while the $T^*(p)$ line is not a continuation of $T_c(p)$ and indeed intersects the p-axis at $p = p_c \cong 0.19$ as above, it corresponds not to a second-order phase transition but rather to a smooth "crossover" (as in the Emery-Kivelson-Uemura scenario, but presumably now connected not with pair formation but with something else of unknown nature).

[14] It works reasonably well for a single material as a function of doping, less well for the comparison between different materials.

Over the last few years a number of experiments, in particular on spin-dependent neutron scattering and optical rotation, have made plausible the idea that some kind of exotic order develops below the T^* line, and very recently (June 2013) there has appeared an ultrasound propagation experiment which has been widely hailed as definitive proof of a second-order phase transition at T^*. This raw data consistent of measurements of the ultrasound velocity is a function of temperature, which appear to be consistent with a slope discontinuity at T^*, and a peak in the absorption there; however, a determined sceptic might still be able to argue that these are merely signatures of a smooth crossover.

5. Is the upturn of $\rho_{ab}(T)$ in the PG regime a symptom of weak localization?

Although this is an intriguing question, it is difficult to say much definitive because of the sparseness of the experimental data. Actually, a logarithmic dependence of $\rho_{ab}(T)$ is seen not just in the PG regime of some superconducting cuprates, but in some nonsuperconducting ones and, sometimes, in the (underdoped) N phase of superconductors with low T_c. However, to my knowledge the only systematic studies have been those of Jing et al.[15] on Bi-2201, which deduced from magnetoresistance measurements that the phase-breaking rate τ_φ^{-1} had the unusual dependence $\tau_\varphi^{-1} \propto T^{1/3}$, and the more recent work of Rullier-Albenque et al[16], which concluded that while the behavior of *overdoped* Tl was consistent with a weak-localization picture, that of underdoped YBCO was better explained by a Kondo scenario.

6. Does the dependence of T_c on n in homologous series have to do with 2-dimensionality?

As remarked above, one of the most striking systematic features of the superconducting transition temperature T_c in the cuprates is the dependence on the layer multiplicity n; in all of the homologous Ca-spaced series, T_c rises with n up to $n = 3$, then (probably) slowly decreases. This has been a puzzle from the earliest days; various theories have tried to fit a supposed dependence of the form

$$T_c(n) = T_c(1) + \Delta T_c(1 - 1/n) \tag{6}$$

but the basis for this in the data seems rather thin.

Crudely speaking, explanations of this behavior may be divided into two classes: In the first, "extrinsic," class it is attributed to the fact that multilayering improves in some respect or other the properties of one or more of the CuO_2 planes individually (usually

[15] PRL **67**, 761 (1991).
[16] PRL **87**, 157001 (2001).

the central one, for $n = 3$). In my view the only really plausible version[17] of this idea is one in which the "improvement" consists of reducing the polarizability of the immediate off-plane environment (for example, in the Hg series by replacing the highly polarizable BaO complexes in the 1-plane material by Ca^{++}—pretty much equivalent to vacuum—in the 3-plane compound). Should this explanation turn out to be correct, then it would seem to follow, perhaps counterintuitively, that a strong Coulomb repulsion actually favors high-temperature superconductivity.

The second, "intrinsic" class of explanations attributes the behavior of $T_c(n)$ to some kind of interaction between the different planes, and thus in some sense involves the 2-dimensionality of the structure in a more essential way. One variant of this class can be dismissed right away, namely that which attributes the effect to Josephson tunnelling between neighboring planes within the context of a standard ("Fermi-liquid") model of the in-plane behavior. The reason this will not work is that such a model gives rise to a Josephson coupling not of the form

$$\Delta F = -\text{const.} \sum_n (\Psi_n^* \Psi_{n+1} + \text{c.c.}) \tag{7}$$

(which would indeed give rise to an extra energy advantage for the S phase relative to the N phase) but rather one of the form

$$\Delta F = -\text{const.} \sum_n |\Psi_{n+1}^* - \Psi_n|^2 \tag{8}$$

so that the "best" superconducting state ($\Psi_{n+1} = \Psi_n$) is only at parity with the N state. For some years a prominent role was played by a ingenious variant of this idea, the Anderson "interlayer tunnelling model," in which (because the in-plane behavior is assumed not to be Fermi-liquid-like) the coupling is indeed of the form (7); however, it is now generally agreed (including, apparently, by its author) not to be by itself a viable explanation of HTS as originally advertised (though it is still possible that it may be a viable explanation of the n-dependence of T_c).

A second line of explanation is based on the idea of considering individual multilayers and applying the KT scenario. We have already indicated some difficulties with this idea; if however we sweep these for the moment under the rug, and moreover let ζ in the relation $\rho_{s0}(T) = (1 - T/T_c)^\zeta$ have its mean-field value of 1, we see that the prediction has the form

$$T_c(n) = T_{c0} \left(1 - \frac{1}{n} \left(\frac{T_c}{T_F} \right) \right) \tag{9}$$

where T_{c0} is the 3D mean-field transition temperature. This has the structure of eqn. (5), so perhaps at first sight seems promising. However, there are at least two major problems:

[17] One should however mention the work of O. K. Andersen et al., (PRL **87**, 047003 (2001)) which attributes the effect to a strengthening of the next-nearest-neighbor tunnelling matrix element because of the increased distance from the apical oxygen.

(1) since T_c/T_F is only ~ 0.01, the predicted dependence on n is much too small (2) even if we "adjust" the effective value of T_c/T_F (by putting in some fudge factor) to fit the data, the value of T_{c0} for (e.g.) the Tl series would be $\sim 180\,K$, and we should expect "local" quantities such as the specific heat to show some kind of anomaly around this temperature; while in Tl-2201 there is indeed a "precursor" specific heat anomaly, it extends only $\sim 10\,K$ above the actual T_c ($\sim 95\,K$).

A final model in the "intrinsic" class attributes the rise of T_c with n to the effect of the Coulomb interaction between neighboring layers of a multilayer. Since the matrix element for such interplane interaction, regarded as a function of the *in-plane* wave vector q falls off exponentially for $q \gtrsim$ the inverse interplane spacing d^{-1}, it follows that if this explanation is correct then much of the superconducting condensation energy must be associated with q-values $\lesssim d^{-1}$ –and that this must apply also that the *intra-plane* Coulomb interaction, contrary to what most microscopic theories of HTS implicitly assume.

[If time allows I will say more about this model.]