## Problem Sheet 1

1. (very easy). Consider an attractive square well potential of depth $V_0$ and dimension $a$, in one, two or three dimensions (i.e. $V(\mathbf{r}) \equiv V(r) = -V_0\theta(a-r)$ where in the 1D case $r \equiv |x|$).

   (a) In the 1D case, find the condition for at least one *odd-parity* bound state to exist.

   (b) Hence, or otherwise, find the condition for the existence of at least one bound state in 3D.

   (c) Now consider the 2D case with a thin solenoid along the cylindrical axis supplying flux $\Phi$. Find a *sufficient* condition for a bound state to exist whatever the value of $\Phi$.

2. Consider an AlGaAs–GaAs heterostructure at $T = 0$, with an electric field $\mathcal{E}$ perpendicular to the interface (in the $z$-direction).

   (a) Use the virial theorem and a semiclassical argument (or dimensional considerations) to obtain the dependence on $\mathcal{E}$ of (i) the extent of the groundstate wave function in the $z$-direction (ii) the energy splitting between the ground state and first excited state in this direction.

   (b) If we are given that the extension of the groundstate wave function is $\sim 200$ Å, at roughly what area ℓ density does the first excited state become occupied?*

   (c) Consider an area ℓ density of $10^{11}$ cm$^{-2}$. The measured low-temperature mobility is $10^6$ cm$^2$/V s. Estimate (i) the total mean free path (ii) the "typical" Coulomb interaction between electrons (iii) the filling fraction (number of electrons per flux quantum) if a field of 10T is applied.

   (d) *Very roughly,* below what temperature do you expect the total mean free path to be approximately temperature-independent? (The electron mobility in GaAs at room temperature is ~8000 cm$^2$/$V_{sec}$)

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* The accuracy will be improved if you keep the factors of $2\pi$, etc., in the semiclassical argument (this was not done in the very rough estimate made in lecture 3).

3. For this problem, refer to sections 3.1-2 of the paper of Hadzibabic *et al.*, NJP **10**, 045006 (2008).

   Suppose you are trying to achieve a similar situation with the fermion isotope $^6$Li.

   (a) Find suitable values of parameters $V_o$ and $\omega_z$ to achieve a single 2D layer.

   (b) Estimate the gap against excitation of the first excited state in the $z$-direction

   (c) If the parameters $\omega_x$ and $\omega_y$ are as in the Hadzibabic experiment, estimate the Fermi energy for a total of $10^6$ $^6$Li atoms in a single hyperfine state.

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Solutions to be submitted by 9am on Monday 16 September

(Please see e-mail message for possible methods of submission.)