Problem Sheet 4

1. Using general notions relating a quantum critical point (QCP) in d dimensions to the corresponding classical one in $d+1$ dimensions, show that for a thin film close to the QCP the resistivity should depend on electric field $\mathcal{E}$ as

$$R(\mathcal{E}, \delta) = R_c\, f(\delta\; \mathcal{E}^{-1/\nu(z+1)})$$

where ν is the exponent of the (spatial) correlation length in the classical problem, z is the "dynamical scaling exponent", δ is a control parameter such as film thickness, with the "critical" value subtracted, and R_c is the value at the QCP. What is the behavior as a function of $\mathcal{E}$ and T at $\delta = 0$?

[Hint: Relate the electric field to some "characteristic" energy.]

2. Dependence of $T_c(n)$ in cuprates on number of CuO_2 planes per unit cell:

   (a) Show that a model based on the KT mechanism in individual multilayers produces, with "reasonable" approximations, the relation

$$T_c(n) = T_c(1) + \Delta T_c(1 - 1/n) \tag{1}$$

   (b) For the KT mechanism of (a), find as quantitatively as possible the difference between the T_c's for $n = 2$ and $n = 3$ for the Hg series.

   [You will need to look up some experimental numbers.]

   (c) (easy) Show that the "full-blooded" Anderson inter-layer tunneling (ILT) model (i.e. that in which *all* the superconducting condensation energy is due to a "pseudo-Josephson" interplane coupling of the form of eqn. (7) from lecture 15) also leads naturally to a formula of the form (1).

   (d) What would this model predict for the c-axis London penetration depth of Tl-2201?

3. Consider the experimental quantum Hall system described by Champagne et al., Phys. Rev. B **78**, 205310 (2008), in a magnetic field $\mathbf{B}$ of 5 T oriented at 60° to the normal $\hat{\mathbf{n}}$ to the 2DEG planes and at 100 mK. For the purposes of the problem, consider the two planes to behave independently. Using the data provided and standard data on GaAs, etc., calculate

   (a) the elastic mean free path

   (b) the cyclotron and Zeeman energies

   (c) the "characteristic" intra-plane Coulomb energy E_c in plane 1 at filling $\nu_1 = 1/3$ (take E_c to be $e^2/4\pi\epsilon\epsilon_0 l_{\mathrm{M}}$)

   (d) the maximum inter-plane Coulomb energy

   (e) a rough order of magnitude of the (theoretically expected) $R_{xx}^{(1)}$ at $\nu_1 = 1/3$.

   (f) a rough order of magnitude of the inter-plane tunneling matrix element t.

   (g) the periodicity of t. In which direction is t periodic?

   Is the approximation of treating the planes as independent likely to be valid under the given conditions (at $\nu_1 = 1/3$)?

Solutions due by 9 a.m. on Mon. 28 Oct.