Problem Sheet 7

1. Decoherence by classical noise:

   Consider a particle of spin 1/2 in a constant magnetic field H_0 along the z-axis. Imagine that at time zero the spin is pointing along the x-axis.

   (a) (easy): Write down the form of the density matrix in the standard (σ_z) basis as a function of time, and evaluate the quantity $\langle \sigma_x + i\sigma_y \rangle(t) \equiv P(t)$.

   (b) Now suppose we have an extra random Gaussian field acting along the z-axis, with correlations specified by

   $$\langle H(t_0)H(t_0+t)\rangle = H_r^2 f(t)$$

   where $f(t)$ falls off at least as fast as t^{-2} for $t \to \infty$. Find the form of the (ensemble-averaged) density matrix and of $P(t)$. Does it matter whether H_r is large or small compared to H_0?

   (Hint:Transform to the rotating frame and use the result,valid for any Gaussian-distributed quantity A, $\langle \exp(iA)\rangle = \exp -\langle A^2\rangle/2$. You may find it easier to evaluate $P(t)$ first).

2. Using the results obtained in lectures 10 and 11, or otherwise, find approximate expressions for the density of (a) fermionic excitations and (b) vortex-antivortex pairs in a 2D single-spin Fermi superfluid just below its KT transition, as a fraction of the areal particle density, and compare them. (Note: in this problem the numerical factors are important – do not set $2\ \pi \sim 1$!) [This problem is important in the context of the $\nu = 5/2$ QHE, see lecture 27.]

3. Consider a triplet superconductor (Sr_2RuO_4?) in which to a first approximation we can regard the up and down Cooper pairs as forming independently. Suppose we try to create a “half-quantum” vortex line, that is a configuration such that at distances

$\ll$ the London penetration depth λ_{L} the down-spin order parameter is constant in space but there is a simple vortex line in the up spins. Show that just as in the case of a simple Abrikosov vortex in an s-wave superconductor, the Ampere current-current interaction leads to screening out of the circulating electric current over a distance $\sim \lambda_{\mathrm{L}}$, and that the total trapped flux is $h/4e$. If the core lies at the center of a cylinder of radius R ($\gg \lambda_{\mathrm{L}}$), what is the approximate energy per unit length of such a configuration?

Please submit solutions by 9.00 am on Monday 9 December.