Composite fermions: Experimental evidence for fractional charge and statistics

Apart, obviously, from the IQHE states, (ν = integer), the values of ν allowed for the FQHE by the Laughlin argument are at first sight only of the form $n_0 + 1/q$ where n_0 is a positive integer (including zero) and q is an odd integer. Actually we can generalize the argument a bit, by using "particle-hole symmetry": by starting with a filled Landau level and taking electrons out (i.e. generating holes) we can generate the "mirror image" of any state with $\nu < 1/2$, and it should have essentially the same properties. Hence we can generate FQHE states with $\nu = n_0 + (q-1)/q$ (e.g. from the well known $\nu = 1/3$ state we can generate a $\nu = 2/3$ one). Such states are indeed seen.

However, while a number of states corresponding to the above values of ν are indeed seen in experiment, we also see many others, e.g. $\nu = 2/5$ or $4/11$, which while "odd-denominator" are not of the form $\nu = n_0 + 1/q$ or $\nu = n_0 + (q-1)/q$. The original Laughlin argument does not obviously explain the existence of such states (if we try to generalize it naively, it produces values of q which are nonintegral, yielding a MBWF, which is nonanalytic in the arguments $z_i - z_j$). Incidentally, one point to note is that although the *integral* QHE is observed at least up to $\nu = 7$, no FQHE has ever been observed for $\nu > 4$.

There exist a number of apparently different, but essentially equivalent schemes for generating a "hierarchy" of FQHE states: see Yoshioka ch. 5. To my mind the simplest is the "composite-fermion" scheme of Jain,[1] and I now sketch that.

Let us suppose that we know the solution of the many-body problem for a particular filling factor ν^* (which need not be integral, cf. below); let this function be $\Phi_{\nu^*}(z_1 z_2 ... z_n)$ (and let the field $n_s\phi_0/\nu^*$ be denoted B^*). Now, while keeping n_s and hence N constant, let us change the field B so as to change the filling factor to a new value ν, and consider as a possible ansatz for the new situation the MBWF

$$\Psi_\nu(z_1, z_2, \dots z_N) = \Phi_{\nu^*}(z_1, z_2 \dots z_N) \times \prod_{j<k}^{N} (z_j - z_k)^{2p} \tag{1}$$

where p is a positive integer. Clearly if Φ_{ν^*} is a nonsingular, analytic, antisymmetric function of the z_i's (apart from the exponential factor) then so is Ψ_ν. Consider now a "large" orbit of say particle j in the states Φ_{ν^*} and Ψ_ν and, compare the extra phase acquired in the latter case relative to the former. If the orbit were to encircle just one particle k, this extra phase would be $2\pi \cdot 2p$, and thus if it encloses *exactly* n particles it will be $2\pi n \cdot 2p$. Now comes the delicate step: If the orbit area is A, then *on average* the number of electrons encircled will be $n_s A$ and therefore the phase picked up will be

[1] See Physics Today, Apr. 2000, p. 39, and references cited therein, or for much more detail J.K. Jain, Composite Fermions, CUP 2007.

$2\pi \cdot 2p \cdot n_s A$: note that for general values of A this will not be integral. But this is just the phase that would have been picked up in an extra magnetic field $B' \equiv 2pn_s\phi_0$. So we can argue that, at least at the mean-field level, the new problem (filling factor ν) is equivalent to the old one with an extra field B'. For this to work the new field B must be related to the old one B^* by $B = B^* + B'$, i.e.

$$B - 2pn_s\phi_0 = B^* \tag{2}$$

But by definition the old filling factor ν^* was $n_s\phi_0/B^*$, so this gives $B = (2p\nu^* + 1)B^*$, and hence the new filling factor $\nu \equiv n_s\phi_0/B$ is given by the formula

$$\nu = \frac{\nu^*}{2p\nu^* + 1} \tag{3}$$

At this point we notice that there is actually nothing to stop us replacing the factor $(z_j - z_k)^{2p}$ by its complex conjugate, $(z_j^* - z_k^*)^{2p}$. (The resulting prefactor is then not in general analytic, but this does not matter—after all, the quantity $|z_j - z_k|^2$ is nothing but $|\mathbf{r}_j - \mathbf{r}_k|^2$, and there is no reason why the wave function should not be a function of this variable!) The effect is simply that the "fictitious" magnetic field B' is now $-2pn_s\phi_0$ rather than $2pn_s\phi_0$, and this leads to a $-$ sign in the denominator of the expression for ν (i.e. $1 - 2p\nu^*$, so B^* is opposite in sign to B). Thus the most general allowed form of ν for which our "mean-field" maneuver works is

$$\boxed{\nu = \frac{\nu^*}{2p\nu^* \pm 1}} \tag{4}$$

So, given any value ν^* of the FF for which we can construct a solution, we can do so for any ν that is related to ν^* by (4), with any (positive integer) value of p and either sign.

There is one delicate point that we need to notice here: Since the effective magnetic field has been increased from B^* to $B = (1 + 2p\nu^*)B$, the magnetic length changes correspondingly, so we need to adjust the exponential factor to be $\exp - \sum_i |z_i|^2/4l_M^2$ where l_M is the actual magnetic length $(\hbar/eB)^{1/2}$ rather than $l_M^* \equiv (\hbar/eB^*)^{1/2}$. Unfortunately, this point tends to be swept under the rug in the literature because of the unfortunate convention of using dimensionless units.

Now, one choice of ν^* for which we can certainly construct a solution is 1, the value corresponding to the *integral* QHE. The ν-values directly derived from this are of the form $\nu = 1/(2p + 1)$, i.e. precisely $1/q$ where q is an odd integer; so we recover the FQHE for such states. Indeed, it is easy to see that in this case the ansatz we have written down is precisely the Laughlin wave function.[2] So far, we have obtained nothing new. Moreover, it is easy to see that choosing ν^* to be a Laughlin value $(2m + 1)^{-1}$ simply gives another Laughlin state.

[2] Since as we saw earlier, the Slater-determinant GSWF of the IQHE can be written as the special case $q = 1$ of the Laughlin wave function.

However, there is nothing that tells us that the original filling factor ν^* has to be ≤ 1, and the next most obvious choice is some integer n greater than 1. For example, if we choose $n = 2$ and set $p = 1$ we generate values of ν equal to 2/3 and 2/5 – again values at which the FQHE is seen experimentally. And so on, in fact, most of the values seen experimentally can be generated in this first step of the possible iteration.

There is, however, one obvious objection to this technique: If we start with $\nu^* > 1$, then in general we would expect the wave function to contain single-particle "components" corresponding to higher LL's. On the other hand, in the limit $V_c \ll \hbar\omega_c$ we have already argued that for $\nu < 1$ the MBWF should be made up entirely from single-particle states in the LLL! If we apply both these statements to e.g. the case $\nu = 2/5$ ($\nu^* = 2$) we appear to get a contradiction.

Remarkably, as shown by Jain, this is not so: Once we have multiplied the higher LL functions by the factor $(z_j - z_k)^{2p}$, the resulting MBWF is to a good approximation made up from only LLL single-particle functions, and to the extent that it is not, any difficulty can be resolved by projecting the ansatz Ψ_ν on the subspace formed by products of the LLL states. Indeed, numerical calculations of the true groundstate for (e.g.) $\nu = 2/5$ show that it is excellently approximated by the projected wave function so obtained.

The most remarkable prediction that follows from the Jain approach (or from other, equivalent approaches based on bosonization or "anyonization") concerns the case $\nu = 1/2$ (Yoshioka, ch. 7). We would, of course, not expect this state to show a FQHE, since it is not "odd-denominator." However, if we reverse the above argument, we find it can be derived from a starting state with $\nu^* = \infty$! But such a state is nothing but the limit $B^* \to 0$, i.e. a simple noninteracting Fermi gas. Moreover, states with ν close to but not equal to 1/2 should correspond to values of ν^* large compared to 1, which translates to $\hbar\omega_c \ll \epsilon_F$. Thus one predicts that for ν close to 1/2, the QH system should behave very much like a standard (3D) metal in a *weak* magnetic field! We will return to this remarkable prediction in the next lecture.

Given that we have identified a particular FQHE plateau close to a filling fraction ν given by

$$\nu = \frac{\nu^*}{2p\nu^* \pm 1} \tag{5}$$

what do we expect to be the charge e^* on the quasiparticle excitations of this state? This is not entirely obvious; we know that for $\nu^* = 1$ the answer is $e^* = e\nu$, but this would be consistent (for example) with either $\nu^*/(2p\nu^* + 1)$ or $1/(2p\nu^* + 1)$. A simple, if crude, argument to resolve the dilemma is that it must be possible to add or subtract *one* (and only one!) electron to the system by creating an appropriate number of quasiparticles, and thus the numerator cannot be other than 1. It ought to be possible to construct a more convincing argument[3] by writing down the explicit form of (e.g.) the hole wave function,

[3] I find the argument given by Yoshioka in paragraph 3 of p. 113 impossible to follow.

for which we might guess

$$\Psi_{\text{qh}} = \prod_{i=1}^{N} (z_i - z_0)^{2p+1/\nu^*} \Psi_\nu \tag{6}$$

and integrating over z_0 as in the Laughlin case, but this would require a knowledge of the IQHE and GSWF Ψ_{ν^*} for $\nu^* > 1$, which we have not explored. At any rate, the accepted result is that for general ν^* we have

$$e^* = \pm e/(2p\nu^* + 1) \tag{7}$$

so that for example in the $\nu = 2/5$ FQHE the quasiparticles have charge $\pm e/5$ (not $2e/5$).

Let's now turn to the question of the experimental observation of fractional charge. This is actually a slightly slippery question, as so often when what one is really testing is a whole complex of ideas [cf. situation with photoelectric effect]. In particular, it is sometimes difficult to sort out what are "genuine" effects of fractional charge as such, and what merely effects of the "fractional" conductance of the Laughlin state. (Yoshioka: "the actual system is composed of electrons, and the quasiparticles are only a convenient way to describe the excitations. Therefore, if one is too critical, one may not see [the existing] experiments as clear evidence for fractional charge.")

Indirect evidence for the existence of fractionally charged quasiparticles in the *bulk* of a FQH system may be obtained from the fact that the longitudinal resistance R_{xx} has an activated form as a function of temperature:

$$R_{xx} \sim R_0 \exp -\Delta/T \tag{8}$$

with a "gap" Δ which is considerably smaller than $\hbar\omega_c$. If fractionally charged quasiparticles and quasiholes exist in the system, with excitation energies ϵ_{qp} and ϵ_{qh} respectively, then we should expect a temperature-dependence of the form (8), with $\Delta = (\epsilon_{\text{qp}} + \epsilon_{\text{qh}})/2$ (the factor of 2 comes, just as in the standard theory of semiconductors, from entropy considerations.[4] On dimensional grounds we should expect that Δ is of order $e^2/4\pi\epsilon\epsilon_0 l_M$ and hence is proportional to $B^{1/2}$, and this appears crudely consistent with the experimental data for the case $\nu = 1/3$ (see Yoshioka fig. 4.10); the experimental coefficient is somewhat smaller than the value calculated on the simplest model, but the discrepancy is believed to be qualitatively understood (ibid., p. 87). A second piece of indirect evidence comes from the observation in light-scattering experiments, of an excitation with an energy $\sim e^2/4\pi\epsilon\epsilon_0 l_M$, which may (or may not!) be identified with a pair of "magneto-rotons" (essentially, exciton-like quasiparticle-quasihole bound states).

Most of the evidence for fractional charge and possibly fractional statistics comes, however, from experiments that probe the edges rather than the bulk of the sample. Apart

[4] One might worry that the situation is not comparable to that in a semiconductor because the statistics obeyed by the quasiparticles is not fermionic but anyonic. However, in the dilute limit this should not matter as in that limit both reduce to Maxwell-Boltzmann.

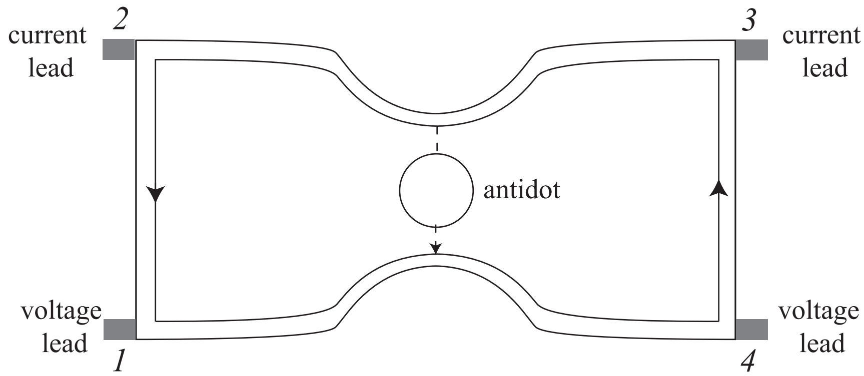


Fig. 1:

from experiments on the $I - V$ characteristics of the edge states (which in some sense test the combination of the Laughlin theory and Tomonaga-Luttinger theory of 1D systems, and will not be discussed here) the experiments that currently provide the best evidence for fractional charge in the FQHE are of two types: the conductance through a quantum "antidot" and shot noise. Both certainly *can* be interpreted in terms of fractional charge: the argument is about whether such an interpretation is *necessary*.

The earliest experiments on transmission through a quantum "antidot" were reported by two groups: Goldman et al.,[5], who interpret them as direct evidence for fractional charge, and Franklin et al.,[6] who do not. The experimental setup is similar in the two cases (see Fig. 1).

GaAs heterostructures with an areal electron density $\sim 10^{11}\,\mathrm{cm}^{-2}$ and mobilities $\sim 1 \times 10^6\,\mathrm{cm}^2/\mathrm{V\,sec}$ were patterned to provide the structure shown (in both cases the antidot radius is ~ 300 nm), and the resistance between "leads" 2-3 and 1-4 measured by a four-terminal technique in the presence of a variable magnetic field (0-15 T) perpendicular to the plane of the sample. In the QH regime (integral or fractional) one expects a non zero conductance[7] between the leads only because of tunneling between the edge states, which in this case should take place via intermediate states on the quantum dot and should be sensitive to the behavior of these states as a function of B *and of the potential on the antidot* (controlled by the gate voltage V_s, which is negative with respect to the bulk). What one actually sees is an oscillatory component in the resistance (i.e. the current) as a function both of B and of V_g and the question is what this tells us about the nature of the many-body states involved. Note that the center of the antidot is devoid of electrons.

[5]Science **267**, 1010 (1995).

[6]Surface Science **361**, 17 (1996).

[7]In the actual experiments the situation is a bit more complicated because two different QH fillings occur in different regions of the sample. The authors subtract this effect out.

Goldman and Su argue as follows: For a negative potential V_g applied to the antidot, the effect should be to create a region of positive charge surrounding it, i.e. a region of quasiholes in the (electron) Laughlin state. We do not know the exact area, S_m, of this region (though if we estimate its order of magnitude from the size of the antidot, the number of flux quanta it contains should be several hundred). The tunneling current should be controlled by the properties of the last occupied (electron) state, i.e. that closest to the Fermi level. It is clear that the properties of this state should be periodic in both B and V_g. However, the periodicity ΔB as a function of B should be controlled simply by the periodicity of the electron states, so that $\Delta B = \phi_0/S_m$; on the other hand, the periodicity ΔV_g in V_g should be controlled by the *total charge* on the antidot (plus surrounding hole), i.e. $\Delta V_g = e^*/CS_m$ where e^* is the unit in which the charge can change and C is the capacitance per unit area between the antidot and the gate, which can be reliably calculated from the geometry. Thus the unit of charge e^* is given by

$$e^* = C\phi_0 \Delta V_g/\Delta B. \tag{9}$$

Using the calculated value of C Goldman et al. obtain, in the IQHE regime, a value of $e^* = (0.97 \pm 0.04)e$, and in the $\nu = 1/3$ FQHE regime, a value $e^* = (0.325 \pm 0.01)e$, evidently consistent with $e* = \nu e$.

Franklin et al. obtain raw experimental results that are consistent with those of Goldman et al. However, they point out that the interpretation of ΔV_g as reflecting a fundamental "unit of charge" e^* is questionable; one could equally well say simply that as soon as the *average* charge in the region of the antidot has changed by $1/3\,e$, the MBWF relaxes back to its original form. At this point it may become a matter of "theology"...

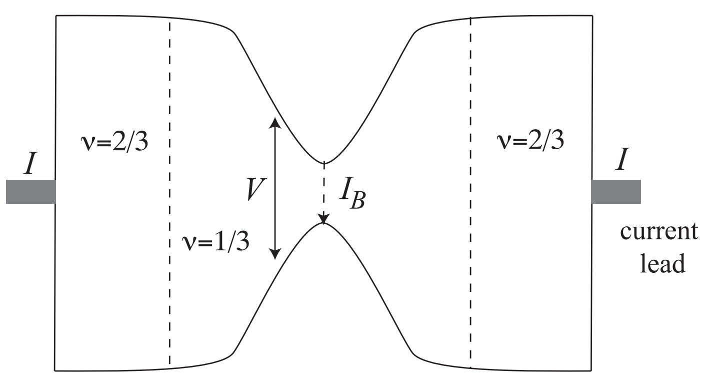


Fig. 2:

The second group of experiments is on shot noise and has, again, been carried out by two groups, de Picciotto et al.[8] and Saminadayar et al.[9] Again, the experimental setups were very similar: GaAs heterostructures with $n \sim 10^{11}\,\mathrm{cm}^{-2}$, $\mu \sim 10^{6}\,\mathrm{cm}^{2}/\mathrm{V\,sec}$ patterned to make a "point contact" whose effective width is controllable by the gate voltage (cf. lecture 3); see Fig. 2. Fields from 0 to 15 T were used. Again the d.c. current I_B that results from tunneling between the edge states is measured, and moreover so is the *noise spectrum* of this current. In the simplest theory of shot noise, if the elementary unit of charge is e^*, we have for the noise per unit frequency bandwidth S_I at zero temperature and for $I_B \ll I$ the standard result

$$S_I = 2e^* I_B \tag{10}$$

However, because of the difficulty of absolute measurements of noise one would prefer not to rely too heavily on this. A more detailed theory gives (for $I_B \ll I$, i.e. low transmission of contact)

$$S_I = 2e^* I_B \coth(e^* V / 2k_{\mathrm{B}}T) \tag{11}$$

This has the advantage that one can read off e^* also from the *shape* of the S_I-V characteristic at given T. Since the "transmitted" current $I - I_B$ is related to the voltage V by the standard relation $I - I_B = (\nu e/h)V$, one can plot S_I directly against I_B and deduce a value of e^*. Both groups find that in the region of FQHE plateau corresponding to $\nu = 1/3$, both the $eV \ll k_{\mathrm{B}}T$ value of S_I and the shape of the $S_I(I_B)$ curves are consistent with the value $e^* = e/3$, and clearly inconsistent with $e^* = e$. Again, one might quibble about whether this experiment really tells us that fractional charge "exists", or merely about the properties of the Laughlin wave function; and again, this controversy may or may not be regarded as "theological."

There has been a further set of experiments by the Goldman group over the last few years that while still using an (anti)dot scheme have varied the experimental geometry:

(1) Camino et al., PRL **95**, 246802 (2005): see Fig. 3. This experiment is more complicated than the previous generation, since while the overall magnetic field is constant the number density on the right and left sides (controlled by the voltage applied to the substrates) is different, and thus the value of ν is different ($\equiv \nu_B = 1/3$ on the left, $\equiv \nu_C = 2/5$ on the right). What is measured in this experiment is the 4-terminal resistance V_{2-3}/I_{1-4}, which in view of the geometry may be called a "longitudinal" resistance R_{xx}. Clearly, for $\nu_B = \nu_C$ (i.e. a uniform value of ν) we should expect $R_{xx} = 0$; for unequal values the prediction (not derived here) is, in the absence of the antidot

$$R_{xx} = (h/e^2)(1/\nu_C - 1/\nu_B) = (-)h/2e^2 \tag{12}$$

The point of the experiment is that R_{xx} is measured as a function of both the magnetic flux Φ applied to the antidot and the back-gate voltage applied to it (hence, if we know the capacitance, of the charge Q accumulated on it). The result:

[8] Nature **389**, 162 (1997).
[9] PRL **79**, 2526 (1997).

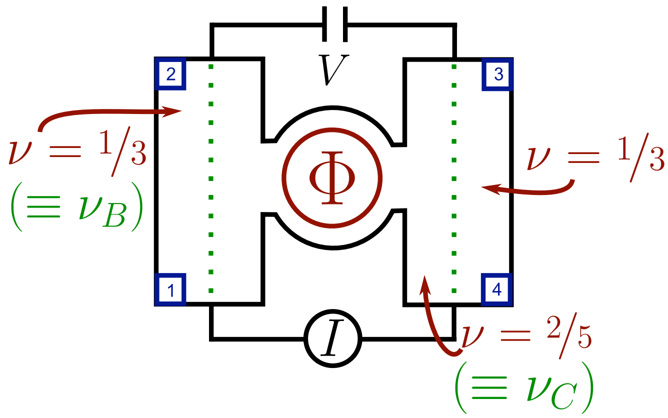


Fig. 3:

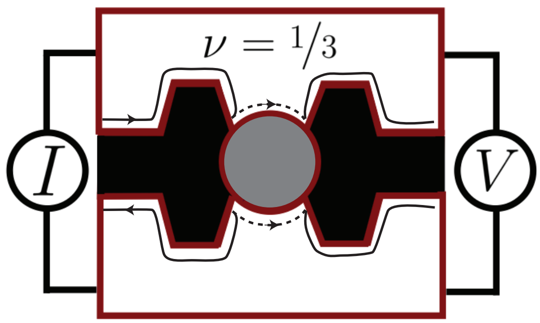


Fig. 4:

$$R_{xx} \text{ is periodic in } \Phi, \text{ with period } \Delta\Phi = 5\phi_0$$
$$R_{xx} \text{ is periodic in } Q, \text{ with period } \Delta Q = 2e$$

Thus $\Delta Q/\Delta\Phi = 2/5 = \nu_C$.

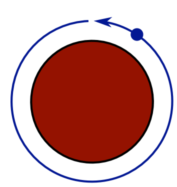


Fig. 5:

A second experiment used the geometry shown in Fig. 4. This is similar in spirit to the original 1995 experiment, but with the difference that the area enclosed by the two paths which are presumably showing the interference now contains electrons rather than void. When the 4-terminal resistance R_{xx} is plotted as a function of the flux Φ and back-voltage V_{BG} (i.e. charge Q) applied to the dot, one finds as previously that R_{xx} is periodic in Φ with period Φ_0 and in Q with period $e/3$.

In a further theoretical paper[10] Goldman et al. argue that their experiments are evidence not only for fractional charge but also for fractional statistics. Their argument goes roughly as follows: Consider for definiteness the case of a $\nu = 1/3$ with a central dot (not antidot) as in Fig. 5. Experimentally, the flux periodicity is ϕ_0 and the charge periodicity is $e/3$. Thus, a change by ϕ_0 in flux corresponds to a change $\equiv e/3$ in (average) charge. Suppose we require that the wave function of a charge $-1/3$ quasiparticle encircling the dot returns to its original value every ϕ_0, without any extra phase factor. That is

$$\Delta\varphi = 2n\pi \tag{13}$$

Now the total extra phase factor is the sum of an AB term $\Delta\varphi_{\text{AB}} = e^*\Delta\Phi/\hbar = e\phi_0/3\hbar = 2\pi/3$, and a Berry term $\Delta\varphi_{\text{Berry}}$ coming from the encirclement of the extra quasiparticles induced on the dot by the flux change by ϕ_0. Thus, from (13),

$$\Delta\varphi_{\text{Berry}} = 2n\pi - \Delta\varphi_{\text{AB}} = -2\pi/3\,(\text{mod } 2\pi) \tag{14}$$

so the exchange phase, which is half[11] of $\Delta\varphi_{\text{Berry}}$, is $\pi/3$ as expected theoretically.

Thus far, everything seems consistent with the "standard wisdom", including the hypothesis that for $\nu = p/q, p \neq 1$, the quasiparticle charge e^* is e/q rather than ep/q. However there are at least two experiments[12] which cast doubt on this conclusion. These are shot-noise experiments conducted on the plateaux corresponding respectively to $\nu = 2/5$ and to $\nu = 2/3$, and in both cases the data indicate that while at high temperature the effective charge of the carriers is indeed e/q, at low temperatures it is ep/q (with a transition regime when it appears to vary smoothly from one to the other as a function of temperature). This is as far as I know not currently understood: for a recent discussion see Snizhko, Low Temp. Phys. **42**, 60 (2016). Note that while in bulk the $\nu = 2/3$ state should be the "mirror image" of the $\nu = 1/3$ one, this is not so near the sample edge, so if the shot noise is dominated by the edge states it is not so surprising that the behavior of the quasiparticles in the two states is qualitatively different.

Let's now stand back and try to assess how far the experiments cited are evidence for fractional charge and/or fractional statistics. The basic problem is that, at least at first sight, most of the experiments seem to need only the FQHE itself plus the standard AB effect. Let's consider e.g. the question of the periodicity of the properties of a simple ring

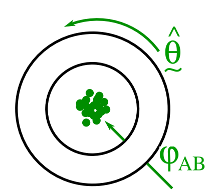


Fig. 6:

[10] PRB **71**, 153303 (2005).
[11] Recall that 2 exchanges = 1 encirclement.
[12] Chung et al., PRL **91**, 216804 (2003): Bid et al., PRL **103**, 286802 (2009).

(annulus) with an AB flux through it (Fig. 6). Suppose the electrons in the ring are in the FQH state corresponding to some fractional charge e^*, e.g. $e/3$. Then prima facie one might expect the basic periodicity of the properties of the ring to be $h/e^* = 3\phi_0$, so that they would *not* in general be periodic in ϕ_0. However, this conclusion would contradict a rigorous theorem (the so-called Byers-Yang theorem) to the effect that all properties of such a ring must indeed be periodic with period ϕ_0. I now give a brief demonstration of this result:

With the AB geometry, the flux φ_{AB} is completely taken into account by the replacement in the Hamiltonian of the canonical momentum $\mathbf{p}$ by $\mathbf{p} - e\mathbf{A}$, i.e.

$$\hat{H} = \sum_i (\mathbf{p}_i - eA(\mathbf{r}_i)^2)/2m + \sum_i U(\mathbf{r}_i) + \frac{1}{2}\sum_{ij} V(\mathbf{r}_i - \mathbf{r}_j), \quad \mathbf{A}(\mathbf{r}_i) = (\varphi_{\mathrm{AB}}/2\pi)\hat{\boldsymbol{\theta}} \tag{15}$$

which must be solved subject to the "single-valuedness boundary condition" (SVBC)

$$\psi\big(\theta_i, \{R_j\}\big) = \psi\big(\theta_i + 2\pi, \{R_j\}\big) \qquad (j \neq i) \tag{16}$$

(in words: if we take a single electron around the ring, leaving all others untouched, we must recover the original wave function). If now we perform a gauge transformation,

$$\psi \to \psi' \equiv \psi \, \exp -ie \sum_i \int_0^{\mathbf{r}_i} \mathbf{A}(\mathbf{r}_i) \cdot d\boldsymbol{l}/\hbar \tag{17}$$

then the new Hamiltonian is

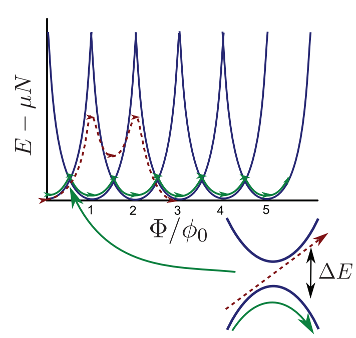
$$\hat{H}' = \sum_i (\mathbf{p}_i^2/2m) + \sum_i U(\mathbf{r}_i) + \frac{1}{2}\sum_{ij} V(\mathbf{r}_i - \mathbf{r}_j) \tag{18}$$

which is independent of $\mathbf{A}$ (hence of φ_{AB}), while the SVBC becomes

$$\psi(\theta_i + 2\pi, \{R_j\}) = \exp 2\pi i(\varphi_{\mathrm{AB}}/\phi_0)\, \psi(\theta_i, \{R_j\}) \tag{19}$$

where the exponential factor is of course periodic with period ϕ_0. Hence all energy levels and wave functions must be rigorously periodic with period ϕ_0, from which immediately follows the Byers-Yang theorem.

However, there is a subtlety:[13] By bending the 2 edges of a Corbino disk and joining them, we can form a torus (see Fig. 7). Now there is an exact result of Wen and Niu:[14] for the FQHE with $\nu = 1/q$ on a torus, there are



[13] Thouless and Gefen, PRL **66**, 806 (1991).
[14] Phys. Rev. **41**, 9377 (1990).

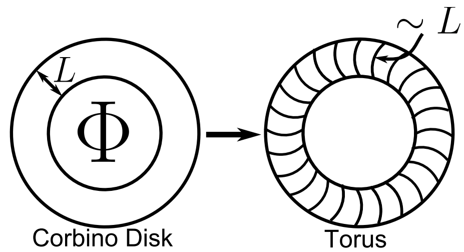


Fig. 7:

q groundstates that are degenerate (and mutually inaccessible) to $\mathcal{O}(e^{-L/\xi})$ where $\xi \sim$ magnetic length l_M. How does this result translate to the Corbino-disk geometry?

A picture consistent with Byers-Yang theorem is the following: For all but the slowest sweeps, system follows - - - giving a periodicity of $3\phi_0$. For the *very* slowest sweeps, system follows —; so at *very* low voltages, FQHE should $\Rightarrow$ IQHE! The probability of following – rather than - - - is of order $\tau\Delta E$ where $\tau \sim \phi_0/V$ is the period of the sweep and $\Delta E \sim \exp -L/\xi$ is the level splitting. Hence, the part of the Hall resistance which is periodic with period ϕ_0 is of order $\exp -L/\xi$, so not practically visitle for a "thick" ring.

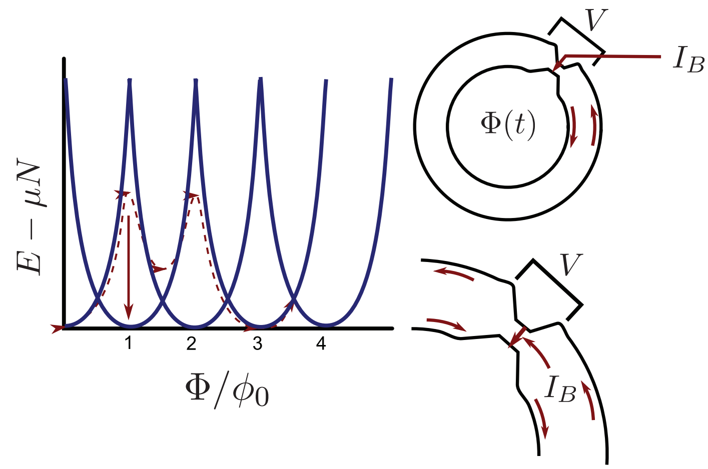


Fig. 8:

But, now consider a Corbino disk with a constriction (Fig. 8: this is close to geometry of the real shot-noise experiments). Even if the system (tries to) follow the dotted curve, interaction with its "environment" (e.g. phonons) may cause it to drop back to the lowest state for that Φ. This process is irreversible, so generates nonzero V and nonzero "backscattered" current I_B. Both V and I_B will be characterized by shot noise, and inspection of fig. 8 shows that this is periodic with period $3\phi_0$ (not ϕ_0). The \$64K question is: Is the quasi-periodicity alone sufficient to generate the quantitative formula (confirmed by experiment) for shot noise, with $e^* = 1/3$?

If yes, then experiments don't tell us anything about charge/statistics of quasiparticles, even edge quasiparticles!