Problem Sheet 1

1. (very easy). Consider an attractive square well potential of depth V_0 and dimension a, in one, two or three dimensions (i.e. $V(\mathbf{r}) \equiv V(r) = -V_0\theta(a-r)$ where in the 1D case $r \equiv |x|$).

   (a) In the 1D case, find the condition for at least one *odd-parity* bound state to exist.

   (b) Hence, or otherwise, find the condition for the existence of at least one bound state in 3D.

   (c) Now consider the 2D case with a thin solenoid along the cylindrical axis supplying flux Φ. Find a *sufficient* condition for a bound state to exist whatever the value of Φ.

2. Consider an AlGaAs–GaAs heterostructure at $T = 0$, with an electric field $\mathcal{E}$ perpendicular to the interface (in the z-direction).

   (a) Use the virial theorem and a semiclassical argument (or dimensional considerations) to obtain the dependence on $\mathcal{E}$ of (i) the extent of the groundstate wave function in the z-direction (ii) the energy splitting between the ground state and first excited state in this direction.

   (b) If we are given that the extension of the groundstate wave function is ~ 200 Å, at roughly what areal density does the first excited state become occupied?*

   (c) Consider an areal density of 10^{11} cm^{-2}. The measured low-temperature mobility is 10^6 cm^2/V s. Estimate (i) the total mean free path (ii) the "typical" Coulomb interaction between electrons (iii) the filling fraction (number of electrons per flux quantum) if a field of 10T is applied.

   (d) *Very roughly,* below what temperature do you expect the total mean free path to be approximately temperature-independent? (The electron mobility in GaAs at room temperature is ~8000 cm^2/V$_s$)

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* The accuracy will be improved if you keep the factors of 2π, etc., in the semiclassical argument (this was not done in the very rough estimate made in lecture 3).

3. For this problem, refer to sections 3.1-2 of the paper of Hadzibabic *et al.*, NJP **10**, 045006 (2008).

Suppose you are trying to achieve a similar situation with the fermion isotope ^{6}Li.

(a) Find suitable values of parameters V_o and ω_z to achieve a single 2D layer.

(b) Estimate the gap against excitation of the first excited state in the z-direction

(c) If the parameters ω_x and ω_y are as in the Hadzibabic experiment, estimate the Fermi energy for a total of 10^6 ^{6}Li atoms in a single hyperfine state.

Solutions to be submitted by 9am on Monday 12 September
(Please see e-mail message for possible methods of submission.)