# Superconducting-state properties (static + transport)

To avoid simply making a "laundry-list", we'll compare as far as possible with what is expected for a simple weak-coupling (s-wave) BCS picture. Data are for optimal or near-optimal doping unless otherwise stated.

## 1. Structural and elastic properties: essentially no change at $T_c$

In fact, even the in-plane electron density distribution, as measured by x-ray scattering, appears insensitive to the onset of superconductivity within the accuracy of the experiment.[1]

## 2. Macroscopic electromagnetic properties

The direct determination of meaningful values of $H_{c1}$ and $H_{c2}$ is complicated by the extreme type-II nature of the cuprates + hence the extreme importance of fluctuation effects. (Cf. the graphs of $R(T)$ as a function of field, Malozemoff in G I, Fig. 7.) However, a reasonable value of $\partial H_{c2}/\partial T$ YBCO, BSCCO and TBCCO seems to be $\sim$ 0.5T/K for $\mathbf{H} \parallel \mathbf{c}$ and $\sim 4-5$T/K for $\mathbf{H} \perp \mathbf{c}$, giving values $\sim$ 50T and 400T respectively at $T = 0$ (not currently reachable, except in pulsed experiments). This would correspond to a $\xi_{ab}(0)$ of 15−30 Å and $\xi_c(0) \sim 2-3$ Å (smaller than the intermultilayer spacing). For comparison, a BCS-type estimate $\xi_{ab}(0) \sim 0.18\hbar v_{\rm F}/k_{\rm B}T_c$, with $m^*/m$ taken $\sim 4$, would give a $\xi_{ab}(0) \sim 30$Å. $H_{c1}(0) \parallel \mathbf{c}$ is also problematic but is probably in the range of 900G, giving an (in-plane) $\lambda_{ab} \sim 1000$Å. As we shall see (lecture 8) direct measurements of $\lambda_{ab}$ are consistent with this estimate. The ab-plane $H_{c1}(0)$ is smaller, ∼200G. Generally, one can say that the HTS are (a) strongly type-II and (b) strongly anisotropic, with $\xi_c \ll \xi_{ab}$ and $\lambda_c \gg \lambda_{ab}$ (a natural result, in view of the strongly inhibited transport in the **c**-direction).

The EM properties will be discussed in more detail in lecture 9.

## 3. Specific heat and condensation energy[2]

As in the normal phase, the electronic specific heat has to be extracted from the raw data by subtraction of the data for a reference sample. When this is done, for optimal doping we find a jump at $T_c$ which is somewhat larger than the BCS value ($\Delta c_{n-s}/c_n \cong 2$ for YBCO and Tl-2201, ∼1.6 for LSCO). The jump appears to be abrupt for YBCO, but for Tl and to a lesser extent LSCO has a small precursor on the high-temperature side ($(T_{\rm onset} - T_{\rm max})/T_{\rm max} \sim 15\%$). For either underdoping or overdoping, (and also for doping with Zn) the peak not only shifts with $T_c$ but gets considerably more rounded.

Below $T_c$ the specific heat falls off sharply, and for $T \gtrsim 0.5\ T_c$ does not look qualitatively very different from that of a standard BCS superconductor. However, at low $T$ it

[1] Hu et al., in SNS 1997. The difference in the raw data at 300 K and 11 K is attributed by the authors to the effect of ionic thermal vibrations.

[2] Refs.: Loram et al., Physica C **235–240**, 134 (1994); Tallon & Loram, Physica C **349**, 53 (2001).

certainly does not go exponentially to zero. Extraction of the "true" electronic specific heat for $T \to 0$ is difficult because of the large effect of any Schottky anomalies due to "2-state" impurities etc.[3] but the behavior is probably consistent with a linear behavior in this limit.

A very interesting quantity is the *condensation energy* $U(0)$, which can be measured by integrating the specific heat (or equivalently entropy) data from 0 to $T_c$. This has been done for $Y_{0.8}Ca_{0.2}Ba_2Cu_3O_{7-\delta}$ (which can be overdoped as well as underdoped) as a function of hole concentration $p$ by Tallon & Loram and is shown in their Fig. 4; it is striking that it peaks sharply at $p = 0.19$ (somewhat larger than the value 0.16 corresponding to maximum $T_c$), at a value of $\sim 33$J/mole, which corresponds to a condensation energy of 2K per $CuO_2$ unit. A closely similar result has recently been obtained for Bi-22l2.

## 4. NMR properties[4]

As mentioned in lecture 5, the $^{63}$Cu(2) and $^{17}$O Knight shift in YBCO is independent of temperature in the (optimally doped) normal state, and the $^{89}$Y shift nearly so. The interpretation of the raw data in the superconducting state is complicated by uncertainty about how much of the "raw" shift is orbital in origin. In fact, for a field along the c-axis the Knight shift is temperature-independent below $T_c$, which at first sight would argue for spin triplet pairing. However, P. & S. argue convincingly that this is simply a consequence of the fact that the spin contribution to $K_{cc}$ is, by a coincidence, zero. The in-plane components of $K_s(T)$ show an approximately BCS-like behavior for $T \gtrsim 0.5T_c$ (as do all components of the Cu(1) $K_s$).

The spin-lattice relaxation rate $T_1^{-1}$ drops precipitously below $T_c$ for both $^{63}$Cu(2) and $^{17}$O(2,3) (it also drops for Cu(1) and O(1), but in these cases there was already a steep temperature-dependence in the normal state so the effect is less dramatic). There is no evidence of any HS peak.

The low-temperature behavior of $K_s$ and $T_1^{-1}$ of great interest in the context of assignation of a pairing state. See Annett et al., in G II.: $T_1^{-1} \propto T^3$.

## 5. Penetration depth[5]

In the literature one can find numerous claims to have measured both the absolute value and the temperature-dependence of the penetration depth for many different cuprates: the results are very scattered, sometimes differing by factors $\sim 5-10$ for a single material (note that this corresponds to a scatter of $25 - 100$ in the more physically meaningful quantity $\rho_s \propto \lambda^{-2}$!). Almost certainly, one major reason for this unsatisfactory situation is that in many of these measurements the raw data is the magnetization or something

[3] Note that we cannot use the T-dependence of the thermodynamic critical field, inter alia because the materials are strongly type-II.

[4] Ref.: Pennington & Slichter, in Ginsberg II.: Tallon et al., in SNS 1997.

[5] Ref.: Bonn & Hardy, in G V.

related (e.g. torque), often of a polycrystalline sample, as a function of field and temperature, and the analysis used to convert this raw data into values of $\lambda$ has often been dubious. In my opinion the most direct and reliable way of measuring the penetration depth is via the microwave surface impedance: a second method that is less direct, and involves some assumptions, but is probably a good deal more reliable than the magnetization method, is muon spin relaxation. To the best of my knowledge results obtained by these two methods are usually mutually consistent; I will discuss mostly the former, which is explained in detail by Bonn and Hardy, op. cit., whom I follow for the next few paragraphs. It should be mentioned that a third method, based on the period of the Fraunhofer diffraction pattern of a Josephson junction, is also quite accurate but has not apparently been widely applied to the cuprates.

What is directly measured in a microwave surface impedance experiment is the complex surface impedance $Z_s(\omega) \equiv R(\omega) + iX_s(\omega)$; for example, in a standard resonant-cavity geometry these quantities are related to the frequency shift and $Q$-factor of the resonance relative to those of the empty cavity by

$$\frac{\delta\omega}{\omega_0} \equiv \frac{\omega - \omega_0}{\omega_0} = -KX_s \tag{1}$$

$$\delta\left(\frac{1}{Q}\right)\frac{1}{Q} - \frac{1}{Q_0} = 2KR_s \tag{2}$$

where $K$ is a calculable factor that depends on the geometry. The above simple formulae implicitly assume that $Z_s$ is isotropic; in general this is not the case for the cuprates, and the particular eigenvalue(s) of $Z_s$ that are measured depend on the sample orientation etc.

The cuprates are almost without exception strongly in the London limit,[6] and thus can be described by a local complex conductivity $\sigma(\omega)$. The relation between $Z_s(\omega)$ and $\sigma(\omega)$ is

$$Z_s(\omega) = \left(\frac{i\mu_0\omega}{\sigma_1(\omega) - i\sigma_2(\omega)}\right)^{1/2} \tag{3}$$

Most microwave measurements are made at frequencies $\lesssim$100GHz, so that in the normal phase the condition $\omega\tau \ll 1$ is well fulfilled. In that case, $\sigma_2 \sim (\omega\tau)\sigma_1 \ll \sigma_1$, so we can write approximately

$$R_s = X_s = \left(\frac{\mu_0\omega}{2\sigma_1}\right)^{1/2} \tag{4}$$

and since $\sigma_1$ is usually known from independent measurements, this allows the calibration of the constant $K$ in the above formulae. In the superconducting state we have a $\delta$-function in $\sigma_1$ at $\omega = 0$, which is not of interest in itself but gives rise to the Meissner term in $\sigma_2$, namely

$$\sigma_2(\omega, T) = 1/\mu_0\omega\lambda^2(T) \tag{5}$$

[6] However, the low-temperature $T$-dependence may be sensitive to "Pippard" effects.

It is conventional to subtract this out and write the full complex $\sigma$ in the form

$$\sigma(\omega, T) = \sigma^*(\omega, T) - i/\mu_0\omega\lambda^2(T) \tag{6}$$

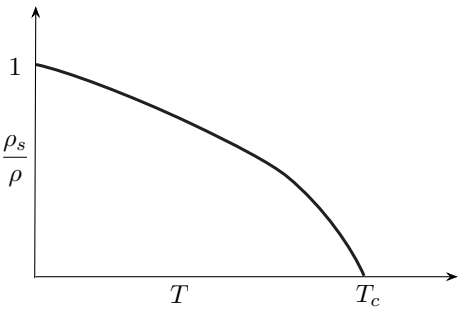
so that $\sigma^*$ represents all the "non-Meissner" contributions, in particular the dissipative contribution, at finite $T$, of the normal quasiparticles. Usually $\sigma^*$ is nearly real for $\omega\tau \ll 1$ and moreover is $\ll$ the reactive $\lambda^2$) term (except close to $T_c$). In this limit we have the simple relation

$$X_s = \mu_0\omega\lambda(T) \tag{7}$$

so that the frequency shift $\delta\omega$ is directly proportional to $\lambda(T)$. Note that none of the above relies in any way on a BCS-type theory being applicable. It is clear that (since the formula for $X_s$ cannot be extrapolated to $T_c$) it is easier to measure *changes* in $\lambda(T)$ than to measure the absolute value e.g. of $\lambda(0)$ (which is actually easier to get from $\mu$SR).

Results: these are most naturally plotted in terms of the quantity $\lambda^2(0)/\lambda^2(T) = \rho_s(T)/\rho$. Of course, $\rho_s$ is a tensor and hence one should distinguish between $\lambda_{ab}\,(\lambda_\parallel)$ and $\lambda_c\,(\lambda_\perp)$, and also, in the case of orthorhombic materials such as YBCO, between $\lambda_a$ and $\lambda_b$: note that the subscripts refer to the direction *of the induced current* (not the direction of penetration!). Results are quoted for optimal doping unless otherwise stated.

ab-plane: Generally, the temperature-dependence of $\rho_s/\rho$ is roughly as shown, i.e. much more $T$-dependent at low temperature than BCS, although in the case of NCCO it is almost BCS-like. In the region close to $T_c$ the behavior is probably not linear but better fitted to $\rho_s/\rho \sim (T_c - T)^{2/3}$ which is the behavior expected[7] for a so-called 3D XY model (single complex order parameter). At low temperatures the behavior is very sensitive to the doping and impurity level; in the purest crystals the evidence is that $\Delta\lambda(T) \equiv \lambda(T) - \lambda(0)$ ($\cong \rho_n(T)$) is linear in temperature, but a quite small concentration ($-0.3\%$) of Zn or (to a lesser extent) Ni impurities is sufficient to convert the temperature-dependence to *quadratic* for a considerable range of $T$ ($\lesssim 0.2\,T_c$).




The temperature-dependence of $\lambda_{ab}(T)$ does not appear to be particularly sensitive to doping, although the zero-temperature value $\lambda_{ab}(0)$ itself does depend on it (cf. below). The absolute value of $\lambda_{ab}(0)$ is subject to much more uncertainty than the temperature-dependence. The following values are quoted by Bonn & Hardy, op. cit.: some are from microwave measurements, others from $\mu$SR.

$$\text{YBCO, optimally doped:} \begin{cases} \lambda_a(0) \cong 1600\text{Å} \\ \lambda_b(0) \cong 1030\text{Å} \end{cases}$$

$$\text{YBCO, underdoped } (x = 0.6) \begin{cases} \lambda_a(0) \cong 2100\text{Å} \\ \lambda_b(0) \cong 1600\text{Å} \end{cases}$$

[7] cf. the behavior of $\rho_s/\rho$ in liquid $^4$He near the $\lambda$-point. The exponent 2/3 is not exact according to RG calculations, but is 0.668...

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|---|---|
| BSCCO-2212 (optimal) | $\lambda_{ab}(0) \cong 2100$Å |
| Tl-2201, overdoped ($T_c \cong 78K$) | $\lambda_{ab}(0) \cong 1650$Å |
| LSCO (optimal) | $\lambda_{ab}(0) \cong 4000$Å |

Values for many other materials are quoted in the literature, but the techniques used are often indirect.

It is a very interesting question whether the ab-plane penetration depth data are compatible with the hypothesis of universal behavior in the $CuO_2$ planes at the same level of doping. Recall that $\lambda_{ab}^{-2}$ measures the 3D superfluid density; thus if the hypothesis of universality is correct, one would expect the relation

$$\lambda_{ab}^2(0)/\,\bar{d} = \text{const.} \tag{8}$$

to hold, where $\bar{d}$ is the average distance between $CuO_2$ planes. While the microwave data alone are hardly sufficient to test this hypothesis, we can try to compare the values inferred from $\mu$SR[8]; the ratios may be hoped to be given by this technique more reliably than absolute values. The data of Uemura et al. (op. cit). appear compatible with the hypothesis as regards the higher-$T_c$ materials, i.e. the ratio is the same[9] within the error bars for optimally doped Tl-2223 and (near)-optimally doped YBCO, and if we take the a-values for the latter from the $\mu$-wave data the constant comes out to be near $5 \times 10^5$ Å. For LSCO the number is quite different, about a factor of 2 larger.

The data of Uemura et al. were actually presented as evidence of an intriguing correlation between $\lambda_{ab}^{-2}(0)$ and the transition temperature $T_c$; for doping *below* optimal the relationship, for the nine different systems measured, appears to be rather convincingly linear. However, their Fig. 2 also shows that the increase of $\lambda_{ab}^{-2}(0)$ with doping persists *beyond* the maximum in $T_c$.

One may ask how well the data fit a naïve picture, in which the superfluid density per plane is simply expressed as $nm^2/m^*$, where $n$ is the number of carriers per unit area and $m^*$ ($\sim 4m$) the effective mass inferred from the specific heat measurements, i.e. the quantity $\lambda_{ab}^{-2}(0) = n_{3D}e^2\mu_0/m^*$. For optimally doped YBCO, $n_{3\text{D}} \cong p_{\text{eff}} \times 10^{22}$ $\text{cm}^{-3}$, where $p_{\text{eff}}$ is the effective number of carriers per $CuO_2$ unit (see below), and the quantity $\lambda_{ab}^{-2}(0)$ is therefore approximately $1.5\ p_{\text{eff}}(m/m^*) \cdot 10^{-6} \cdot \text{Å}^{-2}$. We can therefore fit the a-axis data $\lambda_{ab}^{-2}(0) \cong 0.4 \times 10^{-6}\text{Å}^{-2}$) with an $m/m^*$ ratio of 0.25 as indicated by the normal-state specific heat, provided we take $p_{\text{eff}}$ at optimal doping to be not $p$ but rather $(1 + p)$, i.e. all the holes in the $Cu^3d^9$ band and not just the excess ones over the parent compound contribute. Note that to get the b-axis value right (assuming the behavior of the planes themselves is nearly isotropic) we need to assume that the chains contribute 3/2 as much as *both* planes! That this may indeed be so is indicated

[8] Uemura et al., PRL **62**, 2317 (1989).

[9] Actually, the values of $\bar{d}$ and $\lambda_{ab}^{-2}(0)$ separately are closely similar for the two materials, but this is not particularly significant since the multi-layering structure is different.

by the even smaller $\lambda_b$ for 1248 ($\sim$800 Å) (and by the anisotropy of the normal-state resistivity).

Now let's turn to the c-axis penetration depth $\lambda_c$(T). In addition to the methods (microwave, $\mu$SR, powder magnetization ...) used for $\lambda_{ab}$, there is now the possibility of measuring this, at least within $\sim$10–20% accuracy, by STM studies of the structure of vortices (see lecture 13; the reason this technique does not work for $\lambda_{ab}$ is insufficient resolution).

In general, measured values of $\lambda_c(0)$ are much larger than those of $\lambda_{ab}(0)$, ranging from $\sim 11,000$Å for slightly overdoped YBCO to the enormous value of $\sim 100\mu$ (0.1mm!) for Bi-2212. Not surprisingly, as YBCO is underdoped $\lambda_c$ increases much faster than $\lambda_{ab}$; this presumably reflects the fact that the chain O's, which are the ones removed when the sample is underdoped, play an important role in the contrast between neighboring bilayers.

Although the general shape of $\lambda_c(T)$ on a scale $T \sim T_c$ is qualitatively similar to that of $\lambda_{ab}(T)$, the first corrections to the $T = 0$ value are much smaller, and in fact are often approximated by a $T^5$ form. (However, they do *not* seem well fitted by an exponential).

An obvious question, given the very weak contact between neighboring multilayers in most cuprates, is how well the value of $\lambda_c(0)$ is reproduced by modelling each inter-multilayer link as an independent Josephson junction and applying to it the standard formulae, in particular the Ambegaokar-Baratoff relation

$$I_c(0)R_N = \pi\Delta(0)/2e \tag{9}$$

Since the c-axis resistivity $\rho_c$ is $R_N/d_{\rm int}$ ($d_{\rm int}$ = mean spacing between multilayers) and in a simple Josephson model $\rho_s^{(c)}(0) \sim \lambda_c^{-2}(0)$ is proportional to $I_c(0)$, the predicted relation is

$$\lambda_c^{-2}(0)\rho_c \; d_{\rm int}/\Delta(0) \sim \text{const.} \tag{10}$$

Since $\rho_c$ is often strongly temperature-dependent, a quantitative test of this prediction is somewhat ambiguous. However, Basov et al.[10] (Phys. Rev. B (1994)) have shown that the values of $\lambda_c^{-2}\rho_c(T_c)$ for 8 different systems measured fit the prediction within a factor $\sim$2–3.

## 5. ac conductivity ($\omega \lesssim k_{\rm B}T_c/\hbar$)

Let's first briefly recap the principal features of the low-frequency (let's say $\omega \lesssim 10k_{\rm B}T_c/\hbar$) conductivity in a classic (BCS) superconductor. The most general quantity characterizing the EM response is the complex dielectric constant $\epsilon(\omega) \equiv \epsilon(\omega) + i\epsilon_2(\omega)$, but one often concentrates on the (real part of the) conductivity $\sigma_1(\omega)$, defined by

$$\sigma_1(\omega) = \omega\epsilon_0\epsilon_2(\omega) \tag{11}$$

This quantity obeys the f-sum rule (in the limit of $\mathbf{q} \to 0$, for *arbitrary* polarization of $\mathbf{E}$ relative to $\mathbf{q}$) [SI units]

$$\int_0^\infty \sigma_1(\omega)d\omega = \pi\epsilon_0\omega_p^2/2 \tag{12}$$

[10]Ref: Tanner and Timusk, in G. III. section V (ab-plane): Cooper & Gray, in G. IV (c-axis)

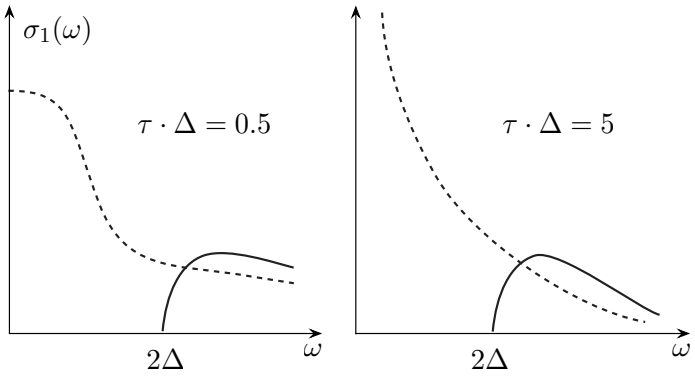
where $\omega_p \equiv (ne^2/m\epsilon_0)^{1/2}$ is the plasma frequency ($n$ = total no. of electrons: in practice, one often replaces this by an effective "number of conduction electrons" and restricts the integral to frequencies low enough that hopefully the contribution of the "core" electrons is excluded). In the normal phase, the sum rule is typically satisfied by a spectrum of $\sigma_1(\omega)$ which extends over a range $\sim 1/\tau$ where $\tau$ is some relaxation time (usually $\gg \omega_p^{-1}$): e.g. for a simple Drude model we have

$$\sigma_1(\omega) = \frac{\epsilon_0 \omega_p^2 \tau}{1 + \omega^2 \tau^2} \tag{13}$$

which evidently satisfies eqn. (12).

In the superconducting state the Meissner effect corresponds to a $\delta$-function in $\sigma_1$ at $\omega \to 0$ ($d\mathbf{J}/dt \propto \mathbf{E}$!) with a weight of $(\pi/2)(ne^2/m)(\rho_s/\rho) \equiv (\pi\epsilon_0\omega_p^2/2) \times \rho_s/\rho$. We see that at $T = 0$ for a very clean superconductor ($\rho_s = \rho$) this exhausts the sum rule, so in this case $\sigma_1(\omega)$ is zero at all finite frequencies (but, actually, this would also be true in the normal state!) In practice, $\rho_s$ is always slightly different from $\rho$ even at $T = 0$. Now we further know that in a BCS superconductor (uniform gap $\Delta$) no real excitation can take place at $T = 0$ until $\omega > 2\Delta$. Thus, any spectral weight missing from the Meissner $\delta$-function must be pushed up to frequencies $\geq 2\Delta$. For a clean superconductor ($l \gg \xi_0$, or equivalently $\tau\Delta \gg 1$) this is a small fraction ($\mathcal{O}(\xi_0/l)$), but for a dirty one ($l \ll \xi_0$ or $\tau\Delta \ll 1$) it is most of it. However, in this case the normal-state Drude weight lies mainly above $2\Delta$ anyway, so the *fractional* change in $\sigma_1(\omega)$ for $\omega \geq 2\Delta$ is not large. See Tanner & Timusk, op. cit., Fig. 53 (reproduced approximately).

At finite temperatures, there is a contribution to $\sigma_1(\omega)$ even for $\omega < 2\Delta$ from scattering of the normal component. If we neglect subtleties connected with the coherence factors, this is simply proportional to $\rho_n/\rho$ and its shape is still essentially Drude with the same value of $\tau$ as in the N phase (since in a typical BCS superconductor by the time $T \lesssim T_c$, $\tau$ is usually due mainly to impurity scattering).




In the cuprates the behavior of $\sigma_1(\omega)$ in the superconducting state is rather different: consider first the ab-plane $\sigma_1(\omega)$ at $T \ll T_c$, as shown for six different materials in Fig. 56 of Tanner et al., op. cit. In all cases, $\sigma_1(\omega)$ is appreciable at frequencies well below 3.5 $k_B T_c$ (= $2\Delta$ in BCS theory: note that this is 220 cm$^{-1}$ for $T_c$ = 90 K), although it is smaller than in the normal state (see Fig. 27) and does show signs of dropping towards zero at the lowest frequency (note that the "data" are obtained by KK transformation of the reflectance). Equally striking is the fact that if we sit at a fixed frequency $< 2\Delta(0)$ and lower the temperature through $T_c$, $\sigma_1(\omega)$ actually *increases*

initially before eventually dropping well below the N-state value.[11] This suggests strongly that the effective scattering rate which enters the (pseudo-) Drude formula is primarily due to electrons, which are gapped below $T_c$.

The question of whether one can ever actually "see" the superconducting gap in the optical conductivity at all is a vexed one, see Tanner et al., op. cit.

Turning to the c-axis conductivity, this typically *decreases*, in the superconducting state, typically in the frequency region below $\sim$150 cm$^{-1}$ for optimally doped samples but up to $\sim$500 cm$^{-1}$ in underdoped ones. A peculiar characteristic of the c-axis electrodynamics of the cuprates is that the plasma frequency (which is proportional to $I_c^{1/2}$) is so low that it may actually be $< 2\Delta$; thus, while the "plasmon" is strongly damped and hence invisible in the normal state, in the superconducting state it may sharpen up enough to be seen. Indeed, evidence for such c-axis plasmons from the infrared reflectivity has been reported for LSCO and BSCCO.

## 6. Thermal conductivity and Nernst coefficient[12]

In a typical metal, the heat is carried both by electrons and by phonons. To estimate their relative contribution to the thermal conductivity $K$, it is adequate to use the "classical" formula

$$K \sim \frac{1}{3} c_v \, \overline{v} \, l \tag{14}$$

where $c_v$ = specific heat of carriers, $\overline{v}$ = mean velocity, $l$ = mean free path. Since for $T \ll \theta$, and $\sim 1$ free electron per unit cell we have $c_v^{\rm el} \sim n k_{\rm B}(T/T_{\rm F})$, $c_v^{\rm ph} \sim n k_{\rm B}(T/\theta_{\rm D})^3$, $\overline{v}_{\rm el} = v_{\rm F}$, $\overline{v}_{\rm ph} = c_s$, and $c_s/v_{\rm F} \sim \theta_{\rm D}/T_{\rm F}$, the ratio is of order

$$K_{\rm ph}/K_{\rm el} \sim (T/\theta_{\rm D})^2 (l_{\rm ph}/l_{\rm el}). \tag{15}$$

For conventional superconductors this ratio is usually $\ll 1$ by the time $T_c$ is reached, but for the cuprates, which simultaneously have much higher $T_c$'s, lower electron densities and shorter mean free paths, the phonon contribution may be dominant in the N phase, or at least comparable to the electronic one.[13]

Since in a conventional superconductor the heat transport for $T \lesssim T_c$ is overwhelmingly electronic (carried by the normal component), and the electronic mean free path is usually predominantly due to impurity scattering, we expect that below $T_c$ $K(T)$ would drop off approximately as $\rho_n(T)/\rho$, and this seems consistent with most of the data.

In the superconducting cuprates, almost universally, $K(T)$ *rises* below $T_c$, peaking in the cleanest samples at a value $\sim 2K(T_c)$ at a temperature $\sim 0.5T_c$, thereafter falling to zero as $T \to 0$. In many cases the low-temperature $K(T)$ appears to be linear in $T$, though in BSCCO it looks like $T^2$ without any visible $T$ region. The rise below $T_c$ indicates a decrease in scattering mechanisms, but does not by itself distinguish between electron and phonon contributions. However, the fact that in conventional

[11] Bonn & Hardy, PRL (single-crystal YBCO).

[12] Ref.: Uher, in G III.

[13] The ratio may be expected to vary from material to material because of the different phonon spectra.

YBCO crystals $K_a$ and $K_b$ are different, with a ratio close to the normal-state ratio of $R_a^{-1}$ and $R_b^{-1}$, indicates that at least in this material the mechanism may be primarily electronic.

The c-axis thermal conductivity has not been much studied but appears to drop off smoothly below $T_c$ (Uher, Figs. 20 and 21). Presumably this is almost entirely due to phonons, and the scattering mechanism seems likely to be primarily by impurities although this is not completely clear.

Ong and co-workers have reported a truly enormous increase (by a factor $\sim 10^3$) in the thermal Hall coefficient that is the off-diagonal ($K_{xy}$) element of the thermal conductivity tensor in a magnetic field, of pure crystal YBCO. As the fields in question were $\gg H_{c1}$ this may be a "vortex" effect; at any rate, it is currently a major puzzle.