

Normal-state Properties: Optimal Doping: (+ overdoped regime)

[Caution on rapid evolution of data in some areas.]

1. Elastic properties.¹

Not particularly remarkable. The *density* of a typical cuprate (YBCO) is $\sim 6 - 7$ gm/cc at atmospheric pressure. The *bulk modulus* (isotropic average) is typically $\sim 1.5 \times 10^{12}$ dynes/cm², and the *thermal expansion coefficient* is $\sim 10^{-5} K^{-1}$. The degree of anisotropy is small for LSCO but substantial for some of the bi- and tri-layer cuprates (see Schilling, Table 2): e.g. for Bi-2212 a factor of 3 more compressible² along c-axis. Typically, at 10 GPa (the limit of strictly hydrostatic measurements), $\Delta a/a \sim \Delta b/b \sim 1.5 - 2\%$, $\Delta c/c \sim 3 - 6\%$ (Thermal expansion ratios crudely similar.) A general rule of thumb is that under compression the apical oxygens move further (closer to planes) than would be implied by simple scaling. In some compounds (e.g. RE Ba₂Cu₃O₇) there is an O-T crystallographic transformation under pressure ~ 20 GPa.

The *sound velocity* of the bulk material is not that easy to measure directly, because most ultrasound experiments are conducted on powders with $\lambda \gg$ grain size [this also prevents ultrasound absorption being a particularly useful probe]. But from compressibility and/or neutron scattering, infer for YBCO $v_l \sim 5$ km/sec, $v_t \sim 2.75$ km/sec (typical for solid). This would imply a Debye temperature ~ 300 K; value inferred from specific heat is somewhat greater, ~ 400 K, but anyway fairly typical. The Grüneisen parameter ($-\partial \ln \omega / \partial \ln V$) is ~ 1.75 for both LSCO and YBCO (similar to e.g. SrTiO₃).

In sum, apart from anisotropy, elastic properties not qualitatively different from those of standard cubic perovskites.

2. Electronic specific heat.³

Because of the high T_c 's of the cuprates relative to θ_D , the experimentally observed specific heat even for $T \sim T_c$ tends to be dominated by the phonon contribution, which is usually well fitted by a Debye formula with θ_D ranging from ~ 200 K for Bi-2201 to ~ 400 K for YBCO. Loram + co-workers obtain the electronic contribution by comparing the observed specific heat with that of a reference compound, usually the "parent" (e.g. YBa₂Cu₃O₆), which is an AF insulator and thus is presumed to have a negligible electronic specific heat over the range up to $\sim RT$.

Textbook result: $c_v^{\text{el}} = \gamma T$, $\gamma = (\pi^2/3)k_B^2 (dn/d\epsilon)$, $(dn/d\epsilon) =$ single-electron DOS at Fermi surface.

Actual result (optimal doping): c_v^{el} is indeed fairly rigorously linear in T from T_c up to RT, see Loram, op. cit., Figs. 1, 6a, 7, 8. This result is also found for overdoped Tl-2201 (Fig. 8).

¹Refs.: Allen in G I, Schilling in G III.

²This is consistent with the easy cleavability of Bi-2212 along the Bi-O planes.

³Ref.: J.W. Loram et al., Physica C **235-40**, 134 (1994).

Magnitude: Loram always quotes results for γ as mJ/g atom K², but when converted to “per mole planar CuO₂” it is almost the same for YBCO₇, LSCO and Tl-2201, namely

$$\gamma \cong 6.5 \text{ mJ}/(\text{mol}(\text{CuO}_2) \text{ K}^2) \quad (1)$$

If interpreted in terms of a DOS by the textbook formula, this would give a value ($N(0) \equiv$ DOS for one spin $\equiv \frac{1}{2}(dn/d\epsilon)$)

$$N(0) \cong 1.4 \text{ eV}^{-1} \text{ spin}^{-1} (\text{CuO}_2 \text{ unit})^{-1} \quad (2)$$

which is a factor ~ 4 times the value ($ma^2/2\pi\hbar^2$) predicted for a free-electron model.

Thus, the specific heat data at optimal doping (+ overdoped) seem *prima facie* consistent with a simple textbook (“Fermi-liquid”) model of the N state with modest FL enhancement (at least for $T \lesssim \text{R.T.}$)

3. Magnetic properties (static).⁴

Textbook prediction: total susceptibility $\chi \sim \text{const.}$ as $f(T)$, with “Wilson ratio” $(\pi^2(k_B/\mu_B)^2/3)\chi/\gamma \sim 1$. Knight shift $K_s \propto \chi$, so $\sim \text{const.}$: inverse NMR relaxation time $T_1^{-1} \propto cT$ (Korringa law) where $c \propto K_s^2$.

Actual results: Acc. Loram (op. cit., Fig. 5) the *total susceptibility* of optimally doped YBCO is indeed independent of T for $T_c < T \lesssim \text{R.T.}$, with a Wilson ratio close to 1 (i.e. no FL enhancement).

However, most information on magnetic properties obtained from NMR: because “large” samples needed, most work has been done on YBCO. The relevant isotopes are: ⁶³Cu (natural abundance 69%, $I = 3/2$), ⁶⁵Cu (31%, $I = 3/2$), ¹⁷O (0.037%, $I = 5/2$) and ⁸⁹Y (100%, $I = 1/2$). Note the $I > 1/2$ nuclei all have EQM’s (permitting zero-field NQR); however, the ratio of the ⁶⁵Cu/⁶³Cu relaxation rates indicates that relaxation is primarily magnetic rather than electric. Because the in-plane Cu’s (“Cu(2)”) and O’s (“O(2), O(3)”) have an environment of different symmetry from the off-plane Cu’s and O’s, the resonance lines associated with them can be unambiguously identified: below we refer only to these (though the data on the chain Cu’s (“Cu(1)”) is also informative), and to the (intercalant) Y’s.

The analysis of the NMR data is quite complicated and I just summarize the principal conclusions (all for optimally doped YBCO):

- (1) The *Knight shift* is independent of temperature for the in-plane Cu’s, and the data on the in-plane O’s is consistent with T -independence for them also. The Y Knight shift is nearly T -independent but shows some “turn-down” a little above T_c which is not seen in the Cu data. The magnitude of the Cu Knight shift relative to the Y and O ones is much larger than expected from the ratio of the known structure of the electron-nuclear interaction, and this observation gives important information about the structure of the N-state spin fluctuations. (I return to this subject in lecture 13.)

⁴Ref.: Pennington and Slichter, in Ginsberg II.

- (2) The *nuclear relaxation rate* T_1^{-1} satisfies the Korringa relation $T_1^{-1} \propto T$ for the Y and in-plane O. For the in-plane Cu's the relation is not satisfied, rather the data are relatively well fitted by a formula of the form $T_1^{-1} \propto aT + b$. (For $T \sim 200$ K the quantity b itself appears to decrease.)

Thus, *prima facie*, apart from the constant term in the Cu(2) T_1 , the data appear to be in at least qualitative agreement with a FL picture of the N state.

The picture has been considerably complicated by the results obtained by the Slichter group, using a more sophisticated ('SEDOR') technique. The results appear to indicate that the magnetic and electric-quadrupole fields seen by different (crystallographically equivalent) nuclei are not identical, rather there is a distribution of both: it is significant, however, that the deviations of the two fields from the mean appear to be connected, as if there is a single parameter controlling both. The magnitude of the effect increases with decreasing temperature. This phenomenon is not at all understood at present.

4. Transport properties.⁵

Textbook predictions: d.c. resistivity $R \sim T$ for $T \gtrsim \theta_D$, \sim complicated for intermediate regime, for low $T \sim a + bT^5$. (Bloch-Grüneisen).

a.c. conductivity has Drude form, $\sigma(\omega) \sim R^{-1}(1 + i\omega\tau)^{-1}$.

Hall coefficient: $\sim \text{const}$, where in high-field limit ($\omega_c\tau \gg 1$) and in free-electron model the constant is $1/ne$. (n = no. of carriers, e = charge). Note for half-filled tight-binding band in 2D R_H predicted to be zero. (Electrons + holes cancel!) Hall *angle* is defined as $\tan^{-1}\{\sigma_{xy}(B)/\sigma_{xx}\}$ and is predicted to be proportional to τ^{-1} and hence to T .

Thermoelectric power $\sim T$.

Thermal conductivity (mostly due to phonons for not too low T) $\sim \text{const}$. for $T \gtrsim \Theta_D$.

Actual results: (note need to distinguish ab-plane + c-axis coefficients! I refer to the ab-plane values until further notice.)

The d.c. resistivity is one of the most striking features of the N phase of the cuprates. As far as is known, *all* superconducting cuprates at optimal doping show an ab-plane d.c. resistivity that is strictly proportional to T from the highest temperatures (limited by O desorption, but ~ 1000 K for LSCO) down to the lowest above the onset of superconductivity (10 K for some Bi-2201 samples).⁶ There is sometimes a small constant term but it appears to vanish in the limit of high purity. As we go away from optimal doping in the direction of overdoping, the T -dependence gradually crosses over from T towards T^2 (reaching the latter behavior as $T_c \rightarrow 0$ in Tl-2201, see Iye, Fig. 7). On the underdoped side the deviation is in the opposite direction, see lecture 6. When the crystal is orthorhombic (e.g. YBCO) the in-plane resistivity is anisotropic (e.g. in YBCO by a factor ~ 2) (but both components are strictly linear in T).

⁵Refs.: Ong in G II. Iye (and Uher) in G III, Cooper & Gray in G IV ("CG"), Hussey (J. Phys. Cond. Mat. **20**, 123201 (2008))

⁶cf. also data when superconductivity is suppressed by magnetic field (Bockinger et al., PRL **85**, 638 (2000), (BLSCO) and earlier refs. cited therein.

An obvious question is whether, if different compounds are compared at optimal doping, the (2D) resistance per CuO_2 plane is universal? If this is so, then at any given temperature the ratio of resistivities should be the ratio of the mean spacing of CuO_2 planes along the c -axis. Inspection of e.g. Fig. 5 of Iye indicates that at first sight this does not work very well: e.g. LSCO has a much higher resistivity than YBCO, even though the average interplane spacing is only slightly greater⁷ (6.5\AA vs. 5.9\AA). However, if we restrict ourselves to the higher- T_c (say $T_c \gtrsim 80\text{ K}$) compounds it appears to work reasonably well, with an R that at R.T. is $\sim 3\text{k}\Omega$, i.e. about 0.12 of the “quantum unit of resistance” h/e^2 .

The ac resistivity does not appear to have a simple Drude form. It is possible to fit it to a Drude formula, $\sigma(\omega) \sim \frac{ne^2\tau/m}{1+i\omega\tau}$ but only if τ is allowed to be itself a function of ω , with $\tau^{-1}(\omega) \sim \max. \omega, k_B T/\hbar$ (thus giving the dc result $R \sim T$). See also below on the optical properties.

The *Hall effect* is also anomalous. In YBCO (pure or Zn-doped) R_H^{-1} is closely proportional to $a+bT$, but in other superconducting cuprates when it has been measured the dependence is considerably weaker⁸ (and approaches a constant as we overdope). The sign is usually positive.

The *Hall angle* shows a very characteristic behavior, at least in (pure or Zn-doped) YBCO: at $B = 8\text{T}$,

$$\cot \theta_H = aT^2 + b, \quad b = f(\text{doping}), \quad a = \text{independent of doping} \quad (3)$$

The *thermoelectric power* S usually has positive value at T_c and a constant negative slope. A very intriguing observation is that as p is varied, the RT value of S crosses zero at almost exactly the point where $T_c(p)$ has its maximum ($p \cong 0.16$). (Obertelli et al., Phys. Rev. B **46**, 14928 (1992).)

Finally, the thermal conductivity is the one transport property that behaves reasonably “normally”: it is usually either $\sim \text{const}$ or weakly decreasing as a function of T .

Uniqueness: Although it is often stated that the proportionality of resistivity to T in “textbook” metals holds only for $T \gtrsim \theta_D$, in fact some such metals show a linear resistivity down to much lower temperatures: see e.g. the graph for Re, (G I, p. 221), which appears to follow the law $a + bT$ down to $T \sim 50\text{K}$ (despite a $\theta_D \sim 400\text{ K}$!). However, in some cuprates (e.g. BiSrCuO_6) the linear resistivity appears to hold right down to below 10 K, and moreover in the cleanest samples there appears to be little or no offset a . No obvious analogs for the behavior of the Hall effect (or Hall angle) are known.

Unlike the ab -plane properties, the *c-axis transport properties*⁹ are very far from universal (at least at first sight!). The *dc c-axis resistivity* is usually relatively well fitted (at optimal doping) by a power law, $\rho_c(T) \sim T^\alpha$, but the exponent α can range

⁷However, the case of YBCO is complicated because it is not clear whether, when considering the b -axis resistivity, the chains should be counted as an “extra plane”.

⁸Note that Iye’s Figs. 14 and 15 plot R_H while Fig. 17 plots R_1^{-1} .

⁹cf. Takagi review, in 1997 Varenna lectures.

from $\sim +1$ (e.g. YBCO, Tl-2201) through 0 (Hg-1201, LSCO) to ~ -1 (Bi-2212). The absolute magnitude of ρ_c (at $T \sim T_c$, say) is always much larger than that of ρ_{ab} , by a factor that varies from ~ 30 for optimally doped YBCO to $\sim 10^5$ for Bi-2212. The optical conductivity is featureless as a function of ω right up to frequencies $\sim 1\text{eV}$, except for isolated peaks that can be correlated to known phonons. (What is usually measured directly is the reflectance, which depends on the *complex* dielectric constant, but barring pathologies a constant reflectance as measured (see e.g. CG, Fig. 13) implies constant ϵ_1 and ϵ_2 , hence $\sigma(\omega) \sim \omega^{-1}$).

5. Spectroscopic probes.¹⁰

(a) “Surface” probes

*Tunnelling*¹¹ ($I - V$ characteristic). Experiments on LSCO, YBCO, BSCCO ...

This should measure the product of an appropriate squared matrix element times the density of single-particle states at the energy eV .

Textbook prediction: differential conductance $G_n(V) \equiv \partial I / \partial V$ should be flat, with any curvature or structure on a scale $\sim \epsilon_F$ (or some characteristic energy associated with the structure of the junction: temperature-dependence also only on this scale).

Actual result: in all cases where measured, $G_n(V)$ appears to have a characteristic form that is well approximated by

$$G_n(V) = a + b|V| \quad (4)$$

with the constant b virtually independent of T and a decreasing weakly with decreasing T .¹² The absolute values of a and b are clearly junction-specific: the ratio b/a is typically of the order of $50 - 100\text{mV}^{-1}$.

Uniqueness: A similar behavior is found in some conventional NIN junctions, but only at low voltages ($\sim 50\text{mV}$). A closer parallel is found in BKBO, where b/a appears to be very close to constant (Sharifi et al., PRL **67**, 509 (1991)).

Caution: most of the results are obtained not with macroscopic tunnel oxide junctions but with STM, and there is evidence that the form of the $I(V)$ characteristic may be sensitive to the tip-to-surface distance: at short distances the characteristic appears to become more “normal”.

Angle-resolved photoelectron emission spectroscopy (ARPES)

Most experiments done on BSCCO, plus some on YBCO and more recently on LSCO.¹³ Note that BSCCO almost always cleaves between the two BiO planes, so the “nearest” CuO plane is two layers down.

¹⁰A good general reference is the proceedings of the 1997 Conference on Spectroscopies of Novel Superconductors (SNS97), published in J. Phys. Chem. Solids, Vol. **59**, pp. 1675–2236 (1998).

¹¹Ref.: Hasegawa et al., in Ginsberg III, pp. 583–8.

¹²However, above $\sim 200\text{K}$ the curve gradually switches over to near-parabolic.

¹³Fujimori et al., in SNS 1997.

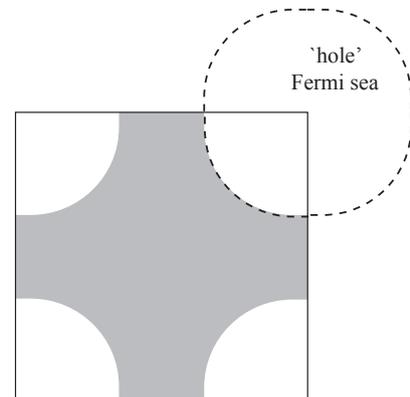
In ARPES, one shines on the sample surface a beam of photons of known energy: the photon is absorbed and kicks out an electron, and the raw data is the number of electrons emitted with energy ϵ' and direction $\hat{\mathbf{n}}$ (the magnitude of $\hat{\mathbf{k}}'$ is of course fixed by ϵ'). In a naïve independent-electron model, conservation of energy implies that the initial electron energy ϵ is $\epsilon' + W - \hbar\omega_{\text{ph}}$ where ω_{ph} is the photon frequency and W the work function, while conservation of transverse momentum (i.e. parallel to surface) implies that $\mathbf{k}' = \mathbf{k}$ where \mathbf{k} is the initial wave vector. There is no conservation of momentum normal to the surface, but fortunately in the case of the cuprates this does not matter much since in view of their highly 2D structure \mathbf{k}_{\perp} is not really defined (or, if it is, the dispersion $\epsilon(\mathbf{k}_{\perp})$ is very flat). Thus in this simple model there should be a unique relation between the energy and transverse momentum of the ejected electron:

$$\epsilon' = \epsilon(\mathbf{k}'_{\parallel}) + \text{const.} \quad (5)$$

where $\epsilon(k)$ is the band-theoretic dispersion relation. Moreover, since the state $\mathbf{k}_{\parallel} = \mathbf{k}'_{\parallel}$ must have been occupied, we expect the flux to fall discontinuously¹⁴ to zero for given $\hat{\mathbf{n}}_{\perp}$ as soon as \mathbf{k}_{\parallel} reaches the Fermi surface.

In real life, if we hold \mathbf{k}'_{\parallel} constant and vary ϵ' , we find the flux varies continuously with ϵ' . This reflects the effect of electron-electron interactions, and a complete theory is complicated. However, there is a strong argument that the photoelectron current $I(\mathbf{k}'_{\parallel}, \epsilon')$ should be approximately proportional to the “spectral function” $A(\mathbf{k}, \epsilon)$, with $\mathbf{k} = \mathbf{k}'$ (all vectors in ab-plane) and $\epsilon = \epsilon' + W - \hbar\omega_{\text{ph}}$ i.e. the probability that an electron has in-plane momentum \mathbf{k} and energy ϵ . If that is so, then by integrating $I(\mathbf{k}', \epsilon')$ over ϵ' we obtain the total occupation $\langle n(\mathbf{k}) \rangle$ of the state \mathbf{k} . If this function shows a discontinuity as $|\mathbf{k}|$ is varied for fixed direction $\hat{\mathbf{n}}$, we identify this value of $|\mathbf{k}|$ with the Fermi surface. Thus we plot out the Fermi surface (actually line) as a function of direction $\hat{\mathbf{n}}$.

When this procedure is applied to the cuprates in the N phase at optimal doping or overdoping, the results are complicated but two things stand out. First, in contrast to the textbook prediction that $A(\mathbf{k}, \epsilon)$ should be sharply peaked near a “quasiparticle energy” $\epsilon = \epsilon(\mathbf{k})$, with a width that tends to zero at least as k approaches k_{F} , the spectral function is very smeared-out in ϵ , with at least 90% of the “weight” in the background for all values of \mathbf{k} . Secondly, if we integrate over ϵ to find $\langle n_{\mathbf{k}} \rangle$, we indeed get a sharp drop as a function of $|\mathbf{k}|$ for fixed $\hat{\mathbf{n}}$, though it is again at most $\sim 10\%$ of the mean value. If we use this drop to define a Fermi surface as above, we find that, crudely speaking,¹⁵ we get one of the form shown, so that the “Fermi sea” is hole-like and centered



¹⁴At zero temperature. At finite T there will be a smearing by the Fermi function.

¹⁵There is actually more structure than this, and in particular there is some evidence in the YBCO data for (at least) two Fermi surfaces, corresponding to the expected “even” and “odd” 2-layer bands.

at (π, π) , with a total area that corresponds roughly to $(1+p)$ ($\cong 1.19$ for optimal doping) holes per CuO_2 unit (note not 0.19!). It is somewhat reassuring that this is just what is predicted by a phenomenological band-structure calculation based on a tight-binding model with reasonable values of the hopping parameters.

(b) *Volume (bulk) probes*

Typically, these probes couple to the phonon degrees of freedom as well as the electrons. It is often possible to distinguish peaks due to phonons by using symmetry arguments, etc., and I shall assume these have been subtracted out of the data.¹⁶

Neutron scattering (LSCO, some YBCO¹⁷)

Neutrons couple negligibly to the electronic charge degree of freedom: the principal coupling to the electrons is via their spin¹⁸, so the neutron scattering cross-section $\sigma(\mathbf{q}, \omega)$ measures (after subtraction of phonons) the spectrum of spin fluctuations of wave vector \mathbf{q} and frequency ω . Neutron scattering experiments need large crystals and even then are very time-consuming, so that the error bars are often comparable to the real data. If for optimally doped YBCO in the normal phase we plot $\sigma(\mathbf{q}, \omega)$ as a function of ω for fixed (in-plane) wave vector \mathbf{q} , there is no particularly significant structure (though cf. below). If on the other hand we fix ω and plot $\sigma(\mathbf{q}, \omega)$ as a function of \mathbf{q} , then there is a marked maximum close to the “commensurate” value (in units of π/a) $(0.5, 0.5)$, this is exactly the point at which in the AF phase of the present compound we get magnetic “superlattice” scattering, see lecture 6. It is debated whether at optimal doping the peak is a single, exactly commensurate one (i.e. exactly at $(0.5, 0.5)$) or whether it is really four peaks at $(0.5 \pm \delta, 0.5 \pm \delta)$ where δ is small (for *underdoped* YBCO it seems almost certain that the latter assignment is correct). At RT the cross-section at $(0.5, 0.5)$ has very little energy-dependence, but as T is lowered there are some indications of a broad peak centered at 34meV.

Raman scattering¹⁹ (YBCO, BSCCO)

In Raman scattering, one shines on the system light of frequency ω_i and definite polarization $\hat{\epsilon}_i$, and detects the light scattered with frequency ω_f and polarization $\hat{\epsilon}_f$, in general different from $\hat{\epsilon}_i$. If the *difference* in wave vector of the incident and scattered light is \mathbf{q} (note that \mathbf{q} is almost invariably very small on the scale of the reciprocal lattice, etc.) and ω the difference $\omega_i - \omega_f$, and if we assume that the intermediate state (wave vector \mathbf{q}_i , energy ω_i) is not too close to a resonance, then what Raman scattering essentially measures is the fluctuations of the dielectric constant tensor, $\langle \epsilon_{\alpha\beta} : \epsilon_{\gamma\delta} \rangle(\mathbf{q}, \omega)$

¹⁶However, there are a few cases when the question of assignment of a particular peak to phonon or electron degrees of freedom is still controversial.

¹⁷Mook et al., in SNS 1997, and references cited therein.

¹⁸Actually, what neutrons are directly sensitive to is the magnetic field, so in the superconducting state they can also be used to probe the structure of vortices, etc.

¹⁹Refs.: Blumberg et al., in SNS 1997: Rübhausen, *ibid*.

where the indices $\alpha, \beta, \gamma, \delta$ depend on the polarizations. Very often in the literature, it is assumed that particular choices of $\alpha, \beta, \gamma, \delta$ (e.g. the so-called B_{1g} geometry) correspond to particular mechanisms of scattering (e.g. the so-called “2-magnon” scattering), but this may be dangerous. Like neutrons, Raman is sensitive to phonons as well as electrons and those must be subtracted.

The Raman spectra of optimally doped normal YBCO and BSCCO appears to be almost totally featureless for ω up to at least 2eV, and moreover at RT to be essentially identical in the A_{1g} and B_{1g} channels; however, for $T \lesssim 2T_c$ some difference appears (and the intensity increases somewhat overall). Note that this is far from what would be *prima facie* expected for the “textbook” model of a metal, where the fluctuations of $\epsilon(\mathbf{q}, \omega)$ should depend on those of conserved physical quantities such as the charge and spin density, and thus *prima facie* be limited to a frequency regime $\sim v_F q \ll 2eV$.

Optical reflectivity

Optical reflectance measurements²⁰ actually probe not just the surface, but the bulk of the metal to a depth that is typically of the order of the high-frequency skin depth; they make minimal demands on crystal size or surface cleanness, and hence have been done on a very wide variety of cuprates, in many cases not only in the “obvious” geometry (surface parallel to ab-plane, so ϵ in plane) but also with $\epsilon \parallel \hat{c}$.

A simple reflectance experiment at normal incidence measures the reflection coefficient $R(\omega)$, which is related to the complex dielectric constant $\epsilon(\omega)$ (which in general is a tensor: we assume a “simple” geometry so that only one element is relevant) by the standard formula

$$R(\omega) = \left| \frac{\sqrt{\epsilon(\omega)} - 1}{\sqrt{\epsilon(\omega)} + 1} \right|^2. \quad (6)$$

Because $\epsilon(\omega)$ is a complex quantity, it is not uniquely determined by $R(\omega)$. However, the analyticity properties of ϵ enable it to be obtained, in principle, by measuring $R(\omega')$ for all relevant ω' and using a Kramers-Kronig relation. This procedure is subject to various sources of error, and a better method is ellipsometry, which measures the complex reflection amplitude $r(\omega) \equiv (\sqrt{\epsilon(\omega)} - 1)/(\sqrt{\epsilon(\omega)} + 1)$ directly; such experiments have been done by the van der Marel group in Groningen (now Geneva) and the Rübhausen group in Hamburg²¹.

In comparing the reflectance data on different cuprates, it is essential to bear in mind that $\epsilon(\omega)$ is a “3D” quantity; thus, even if at (say) optimal doping the behavior of the individual CuO_2 planes is identical in different materials, the fact that the density of planes differs from material to material means that $\epsilon(\omega)$ will for that reason alone be different, and this will be reflected in $R(\omega)$ in a complicated way. A further real-life complication that is easy to forget is that while the (mostly insulating) “reservoir” layers between the CuO_2 planes are unlikely to contribute much to the conductivity at frequencies: $\lesssim 3 - 4\text{eV}$ (i.e. to the imaginary part of ϵ) they will contribute very

²⁰Refs.: Timusk and Tanner in G I, Tanner and Timusk in G III.

²¹Molegraaf et al., Science 295, 2239 (2002); Rübhausen et al., PR **B63**, 224514 (2001)

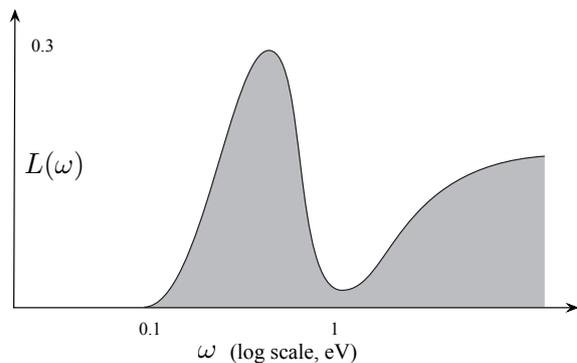
importantly to the *real* part and hence to the reflectance.²² In view of these complications it is perhaps at first sight surprising that the ab-plane optical properties of the cuprates show any “universality” at all.

The most dramatic “universal” property comes out most clearly if we plot not the reflectance or the real part of the conductivity, but the so-called loss function $L(\omega) \equiv -\text{Im} \epsilon^{-1}(\omega)$. If we consider a simple textbook model of a metal and for the moment neglect band-structure effects, then $\epsilon(\omega)$ has the simple form

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i/\tau)} \quad (7)$$

where $\omega_p \equiv (ne^2/m\epsilon_\infty)^{1/2}$ is the (3D) plasmon frequency, τ is the (ω -independent) Drude relaxation time and ϵ_∞ the “high-frequency” dielectric constant. Thus in the usual limit $\omega_p\tau \gg 1$, $L(\omega)$ would consist of a sharp peak of width $\sim 1/\tau$ at the plasma frequency, plus a small background. By contrast, all the superconducting cuprates (indeed, to my knowledge, all the metallic ones even when nonsuperconducting) have a strong and broad spectrum of $L(\omega)$, typically extending from ~ 0.1 eV to a fairly sharp

upper cutoff at a value of ω which varies from ~ 1 to 2 eV depending on the cuprate. This “midinfrared peak” is one of the most striking generic properties of the cuprates, and attracted attention from an early stage. Also characteristic is the near-zero of $L(\omega)$ which occurs at the upper edge of the MIR peak; at higher energies $L(\omega)$ again has some weight, but it is not so spectacular and the detailed form tends to be material-specific as one might expect.



A remarkable conclusion can be drawn from the reflectance measurements of El-Azrak et al. (PR B49, 9846 (1994)) on thin YBCO films and their interpretation (via Kramers-Kronig analysis) in terms of the complex dielectric constant $\epsilon(\omega)$: If one looks at fig. 4 of this reference, it is obvious (though not commented on by the authors!) that throughout the whole infrared regime the phase of $\epsilon(\omega)$ is very close to $-\pi/4$. This is exactly what would be predicted by eqn. (2) if we let $\tau^{-1}(\omega)$ be equal to ω . (However, more recent experiments appear to give a value closer to $-\pi/3$ and moreover suggest the frequency-dependence $|\epsilon(\omega)| \propto \omega^{-2/3}$.)

Electron-energy-loss spectroscopy²³ (EELS)

The simplest kind of EELS experiments, “transmission EELS” measures the fluctuations of the charge density in the scattering system, and thus should be closely related to the

²²However, it may be possible to take this effect into account semiquantitatively for members of the same homologous series.

²³Ref.: N. Nücker et al., Phys. Rev. B **39**, 12379 (1989).

optical experiments. In fact, for an isotropic normal 3D system in the limit $\mathbf{q} \rightarrow 0$ the EELS cross-section $\sigma(\mathbf{q}, \omega)$ should be directly proportional to $L(\omega)$. In a strongly layered system this equivalence holds only for $qd \ll 1$ where d is the interlayer spacing, so one must employ caution in applying it to the cuprates. However, it is reassuring that experiments in BSCCO (and, to a large extent, in YBCO) with $\mathbf{q} \parallel ab$, $|\mathbf{q}| \sim 0.05 - 0.1 \text{ \AA}^{-1}$ (so that $qd \lesssim 1$) do show a spectrum that appears to be consistent with the optical data, in particular they show a strong MIR peak. For larger values of q (up to $\sim 0.5 \text{ \AA}^{-1}$, the upper limit for practical reasons) the peak persists but is somewhat attenuated relative to the background.