Announcements

- MATLAB Clinics
- Concept Inventory Pre-test
- Office Hours start today (no OH on Labor Day)
- Students in Monday discussions may choose a different section to attend next week (9/3-7)

- Upcoming deadlines:
  - Friday (8/31) – Today!
    - PrairieLearn HW0
  - Next Thursday (9/6)
    - PrairieLearn HW1
  - Next Friday (9/7)
    - Written Assignment 1

National College Color Day

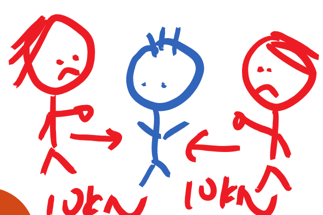
Recap

- Pay attention to units!
- Equations must be dimensionally homogenous
- 1% accuracy
- Scalar – defined by magnitude (negative/positive)
- Vector – defined by magnitude and direction

force needs direction

$$\vec{F} = F\hat{u}$$

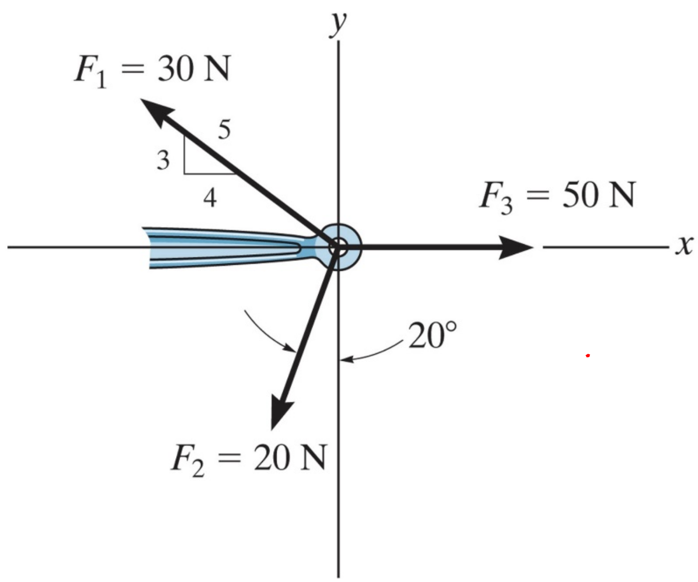
static



Dynamic

Example

Express force vector $\mathbf{F}_1$ using the Cartesian vector form.



Vector form : $\vec{F}_1 = F_1 \, \hat{u}_1$

mag. dir.

Unit vector : $\hat{u}_1 = -\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}$

Force magnitude : $F_1 = 30N$

$\rightarrow \vec{F}_1 = \left[30\left(-\frac{4}{5}\right)\hat{i} + 30\left(\frac{3}{5}\right)\hat{j}\right] N$

$\vec{F}_1 = -24\hat{i} + 18\hat{j} \; N$

$\vec{F}_2 = F_2 \, \hat{u}_2$, $F_2 = 20N$

$\hat{u}_2 = -\sin 20^\circ \hat{i} - \cos 20^\circ \hat{j}$

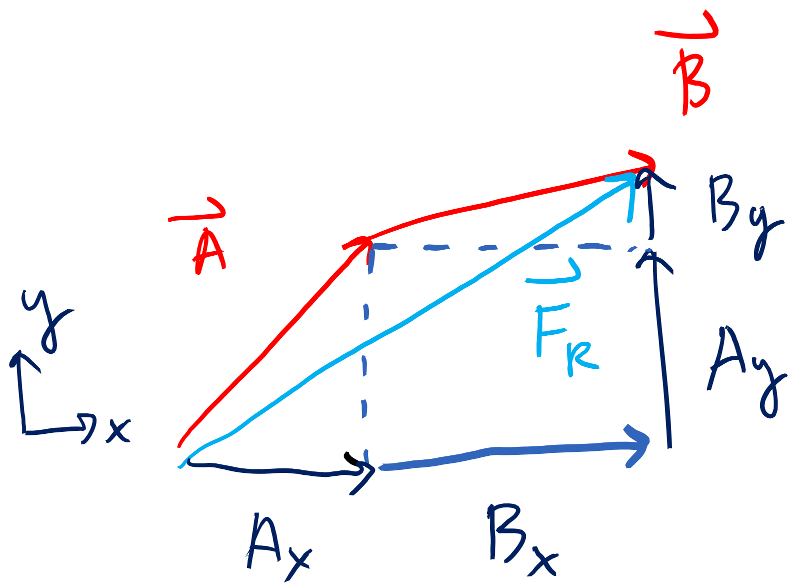
$\Rightarrow \vec{F}_2 = -20N \sin 20^\circ \hat{i} - 20N \cos 20^\circ \hat{j}$

$\vec{F}_3 = 50\hat{i} \; N$

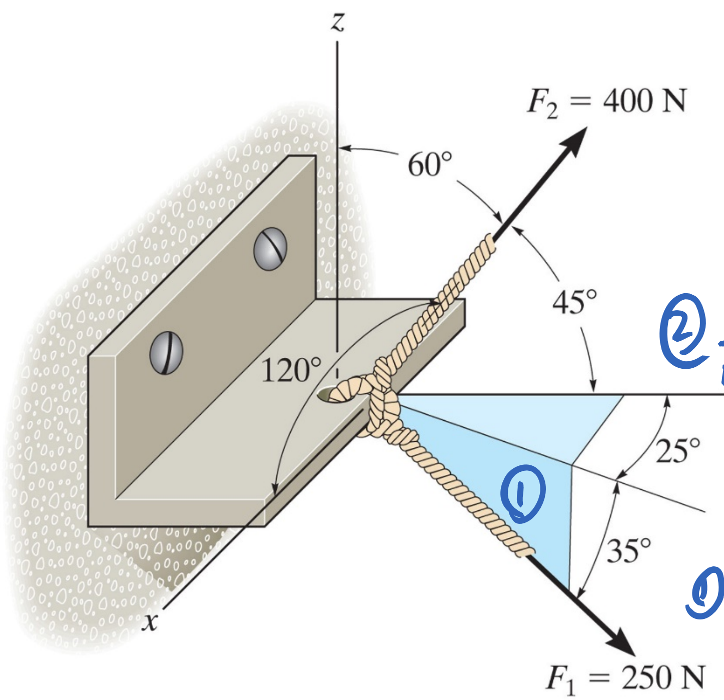
Addition of Cartesian vectors

$$\boldsymbol{R} = \boldsymbol{A} + \boldsymbol{B} = (A_x + B_x)\,\boldsymbol{i} + (A_y + B_y)\,\boldsymbol{j} + (A_z + B_z)\,\boldsymbol{k}$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} \qquad \vec{B} = B_x\hat{i} + B_y\hat{j}$$



Example



The bracket is subjected to the two forces on the ropes. Determine the magnitude and direction cosines of the resultant force vector.

1.1 Express force $\mathbf{F_1}$ in Cartesian vector form.

$\vec{F}_1 = F_1 \hat{u}_1$

② $F_1 \cos 35 \cos 25$

$F_1 \cos\theta$ $\quad F_1 \cos 25$ $\quad 25°$

$F_1 \cos\theta$

① $F_1 \sin\theta$

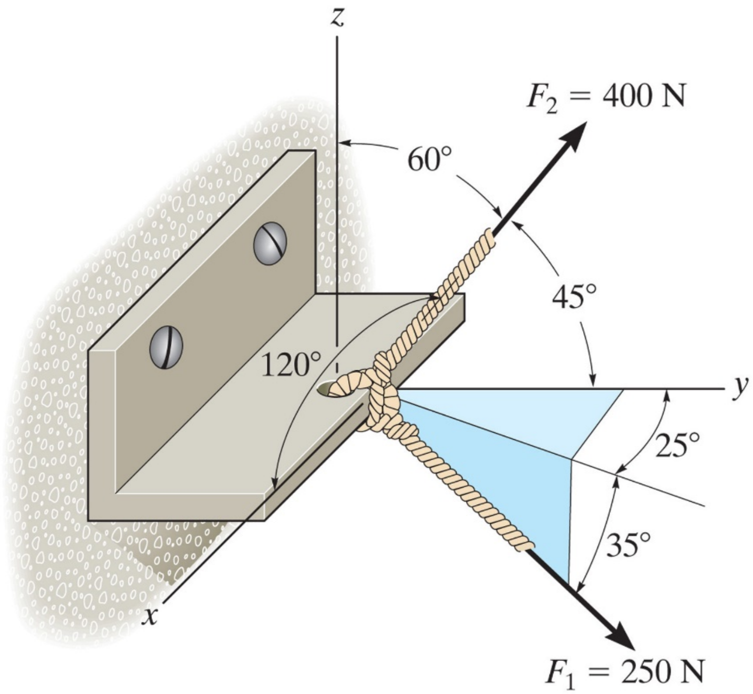
F_1

$\hat{u}_1 = \cos 35° \sin 35 \cos 25$

$+ \cos 35 \cos 25°$

$$\vec{F}_1 = \underbrace{250 \cos 35° \sin 25°}_{F_{1x}} \hat{i} + \underbrace{250 \cos 35° \cos 25°}_{F_{1y}} \hat{j} - \underbrace{250 \sin 35°}_{F_{1z}} \hat{k}$$

Example



The bracket is subjected to the two forces on the ropes. Determine the magnitude and direction cosines of the resultant force vector.

1.2 Express force $\mathbf{F_2}$ in Cartesian vector form.

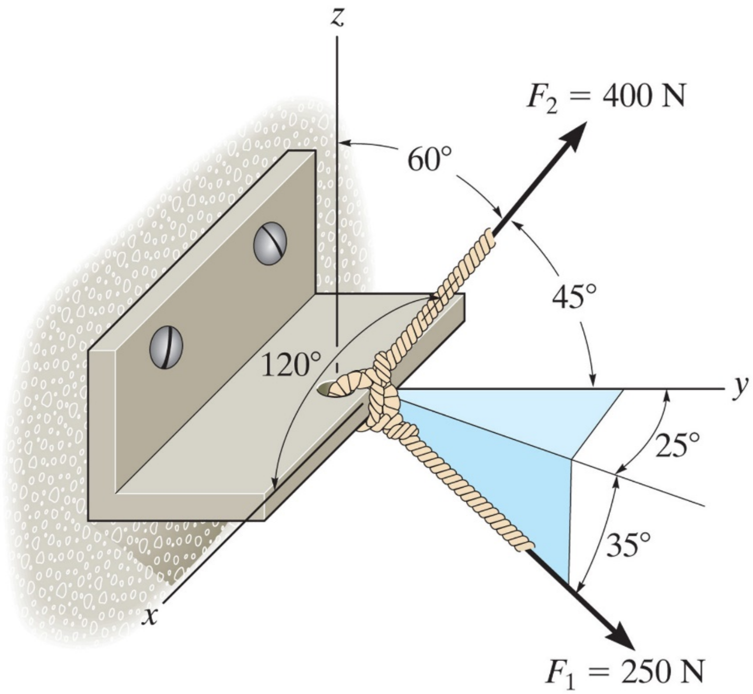
$$\vec{F}_2 = F_2\,\hat{u}_2$$

$$= [F_2(\cos 120°)\hat{i} + F_2(\cos 45°)\hat{j} + F_2(\cos 60°)\hat{k}]\ \text{N}$$

$$= 400\left(-\tfrac{1}{2}\right)\hat{i} + 400\left(\tfrac{\sqrt{2}}{2}\right)\hat{j} + 400\left(\tfrac{1}{2}\right)\hat{k}\ \text{N}$$

$$\vec{F}_2 = \underbrace{-200\,\hat{i}}_{F_{2x}} + \underbrace{200\sqrt{2}\,\hat{j}}_{F_{2y}} - \underbrace{200\,\hat{k}}_{F_{2z}}\ \text{N}$$

Example

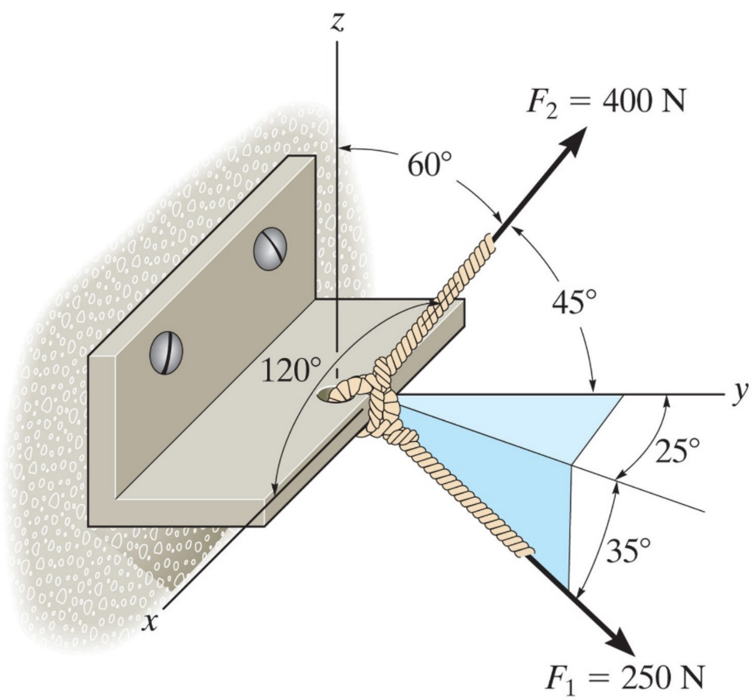


The bracket is subjected to the two forces on the ropes. Determine the magnitude and direction cosines of the resultant force vector.

1. Express forces $\mathbf{F_1}$ and $\mathbf{F_2}$ in Cartesian vector form.
2. Find force $\mathbf{F_R}$ in Cartesian vector form.

$$\vec{F}_R = \overbrace{(F_{1x}+F_{2x})}^{F_{Rx}}\hat{i} + \overbrace{(F_{1y}+F_{2y})}^{F_{Ry}}\hat{j} + \overbrace{(F_{1z}+F_{2z})}^{F_{Rz}}\hat{k}$$

Example



The bracket is subjected to the two forces on the ropes. Determine the magnitude and direction cosines of the resultant force vector.

1. Express forces $\mathbf{F_1}$ and $\mathbf{F_2}$ in Cartesian vector form.
2. Find force $\mathbf{F_R}$ in Cartesian vector form.
3. Determine the magnitude and direction cosines of $\mathbf{F_R}$.

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2 + F_{Rz}^2}$$

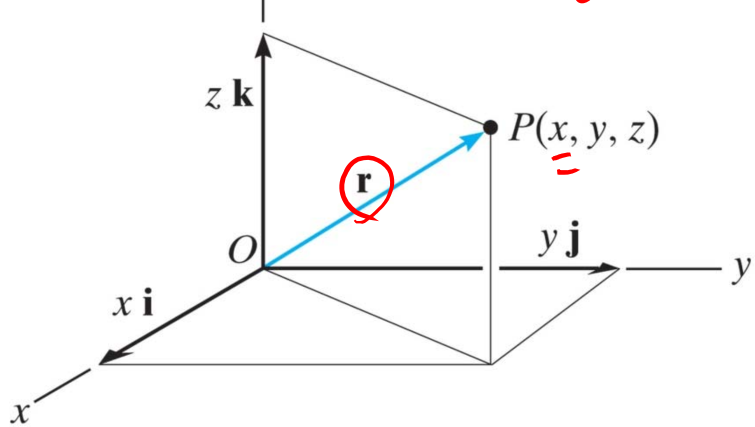
$$\alpha = \cos^{-1}(F_{Rx}/F_R)$$

$$\beta = \cos^{-1}(F_{Ry}/F_R)$$

$$\gamma = \cos^{-1}(F_{Rz}/F_R)$$

Position vectors

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$



A position vector $\boldsymbol{r}$ is defined as a fixed vector which locates a point in space relative to another point. For example,

$$\boldsymbol{r} = x\,\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$$

expresses the position of point $P(x,y,z)$ with respect to the origin O.

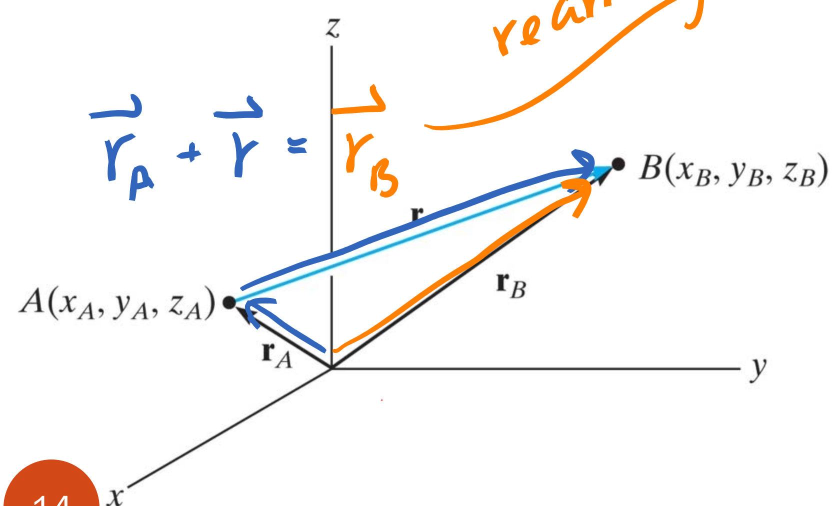
The position vector $\boldsymbol{r}$ of point $\boldsymbol{B}$ with respect to point $\boldsymbol{A}$ is obtained from

$\boxed{\vec{r} = \vec{r}_B - \vec{r}_A}$ (end - beginning)

$\vec{r} = (x_B - x_A)\hat{i} + (y_B - y_A)\hat{j} + (z_B - z_A)\hat{k}$

rearrange

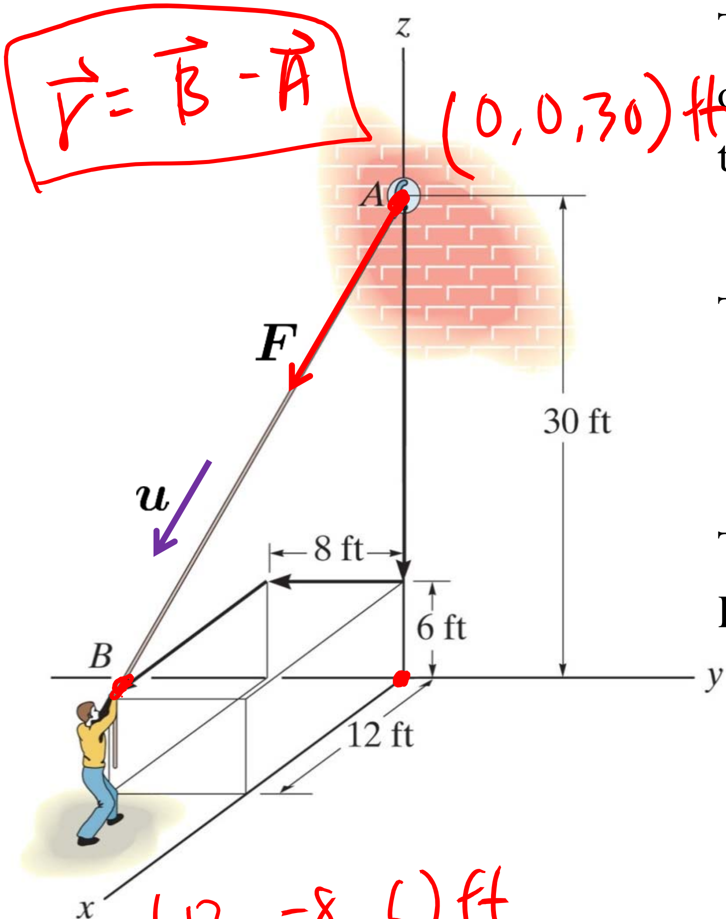
$\vec{r}_A + \vec{r} = \vec{r}_B$



Thus, the $(\boldsymbol{i},\ \boldsymbol{j},\ \boldsymbol{k})$ components of the positon vector $\boldsymbol{r}$ may be formed by taking the coordinates of the tail (point A) and subtracting them from the corresponding coordinates of the head (point B).

Force vector directed along a line

$\boxed{\vec{r} = \vec{B} - \vec{A}}$



(0, 0, 30) ft

(12, −8, 6) ft

The force vector $\boldsymbol{F}$ acting a long the rope can be defined by the unit vector $\boldsymbol{u}$ (defined the direction of the rope) and the magnitude of the force.

$$\boldsymbol{F} = F\,\boldsymbol{u} \qquad (\vec{F} \neq F\vec{r})$$

magnitude would be wrong

The unit vector $\boldsymbol{u}$ is specified by the position vector:

$$\hat{u} = \frac{\vec{r}}{r} = \frac{\vec{B} - \vec{A}}{|\vec{B} - \vec{A}|}$$

The man pulls on the cord with a force of 70 lb. Represent the force F as a Cartesian vector.

$$\vec{r} = (12-0)\hat{i} + (-8-0)\hat{j} + (6-30)\hat{k} \text{ ft}$$

$$= 12\hat{i} - 8\hat{j} - 24\hat{k} \text{ ft}$$

$$r = \sqrt{12^2 + 8^2 + 24^2} \text{ ft} = 28 \text{ ft}$$

$$\hat{u} = \frac{\vec{r}}{r} = \frac{12}{28}\hat{i} - \frac{8}{28}\hat{j} - \frac{24}{28}\hat{k} = \frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} - \frac{6}{7}\hat{k}$$

$$\vec{F} = 70\,\text{lb}\left(\frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} - \frac{6}{7}\hat{k}\right) = 30\hat{i} - 20\hat{j} - 60\hat{k} \text{ lb}$$

- What are the units?

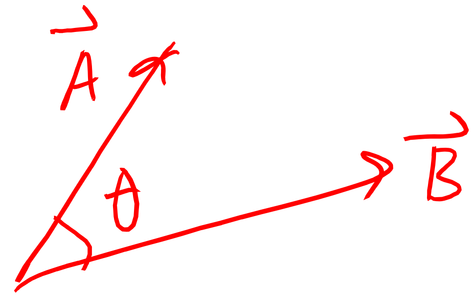
$\vec{F} = F\hat{u}$ ← unitless →

$[N] = [N]\,(1)$

Dot (or scalar) product

The dot product of vectors **A** and **B** is defined as such

$\vec{A} \cdot \vec{B} = AB\cos\theta \longleftrightarrow$ scalar

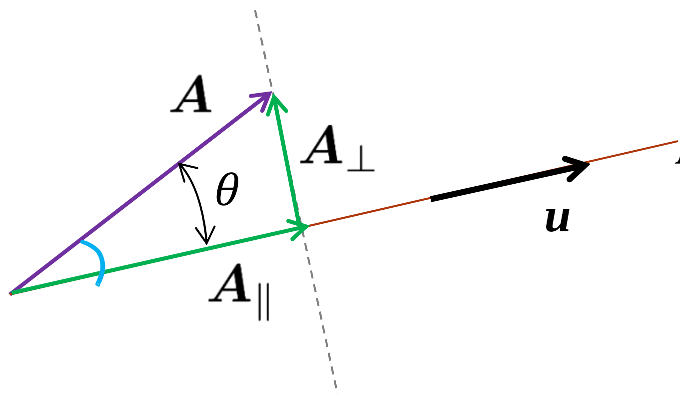


Cartesian vector formulation:

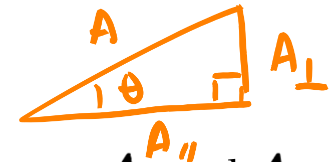
$$\boldsymbol{A} \cdot \boldsymbol{B} = (A_x B_x) + (A_y B_y) + (A_z B_z)$$

Projections

The scalar component $A_{\parallel}$ of a vector $\boldsymbol{A}$ along (parallel to) a line with unit vector $\boldsymbol{u}$ is given by:



$$A_{\parallel} = A\cos\theta$$



And thus the <u>vector</u> components $\boldsymbol{A}_{\parallel}$ and $\boldsymbol{A}_{\perp}$ are given by:

$$\boldsymbol{A}_{\parallel} = A_{\parallel}\,\hat{u}_{\parallel} = \underbrace{A\cos\theta}\,\hat{u} = \underbrace{(\vec{A}\cdot\hat{u})}_{A\cdot 1\cos\theta = \vec{A}\cdot\hat{u}}\,\hat{u}$$

$$\left(\boldsymbol{A}_{\perp} = \vec{A} - \vec{A}_{\parallel}\right)$$

$$\vec{A}_{\parallel} + \vec{A}_{\perp} = \vec{A}$$

rearrange