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Announcements

- Quiz 1 Next Week!
- If this is your first week – check out the course website for all the logistics you need to know:

    https://courses.engr.illinois.edu/tam210

❑ Upcoming deadlines:

- Friday (9/7 – TODAY!)
    - Writtein Assignment #1
- Tuesday (9/11)
    - PL HW
- Friday (9/14)
    - Writtein Assignment #2

1 L5 - Force along a line Cross product

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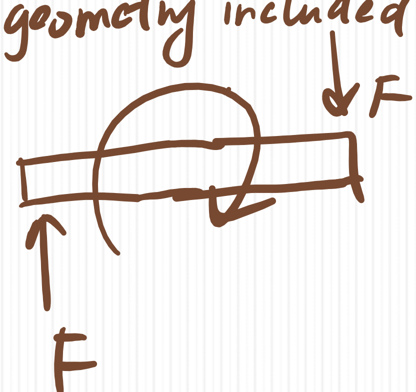
Chapter 3: Equilibrium of a particle

If geometry included



$\Sigma F = 0$: rotation would still occur.

- no geometry

NO Translation

$\Rightarrow \Sigma F = 0$

Goals and Objectives

- Practice following general procedure for analysis.
- Introduce the concept of a free-body diagram for an object modeled as a particle.
- Solve particle equilibrium problems using the equations of equilibrium.

Applications

For a spool of given weight, how would you find the forces in cables AB and AC?

If designing a spreader bar (BC) like this one, you need to know the forces to make sure the rigging (A) doesn't fail.

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General procedure for analysis

1. Read the problem carefully; write it down carefully.

FBD

2. MODEL THE PROBLEM: Draw given diagrams neatly and construct additional figures as necessary.

3. Apply principles needed.

4. Solve problem symbolically. Make sure equations are dimensionally homogeneous

5. Substitute numbers. Provide proper units *throughout*. Check significant figures. Box the final answer(s).

6. See if answer is reasonable.

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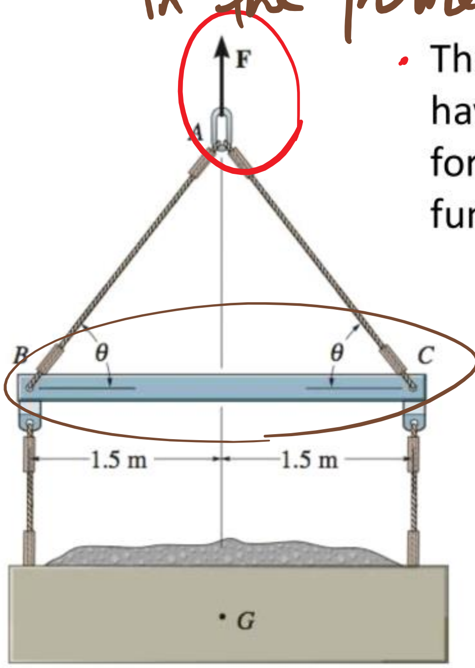
Free body diagram

- Account for all the forces involved in the problem



- The lift sling is used to hoist a container having a mass of 500 kg. Determine the force in each of the cables *AB* and *AC* as a function of θ.

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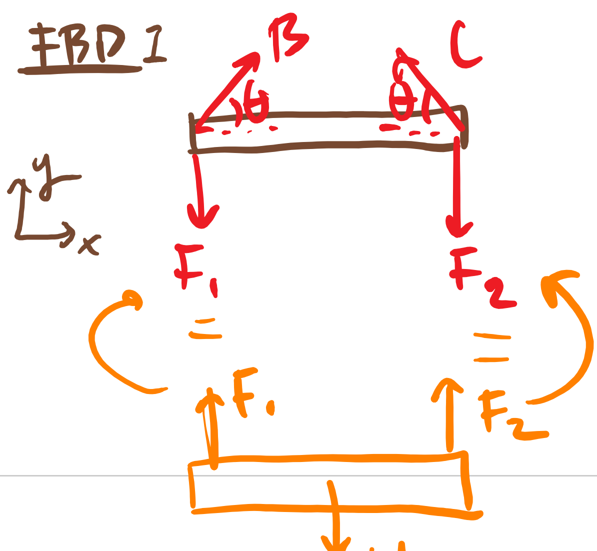
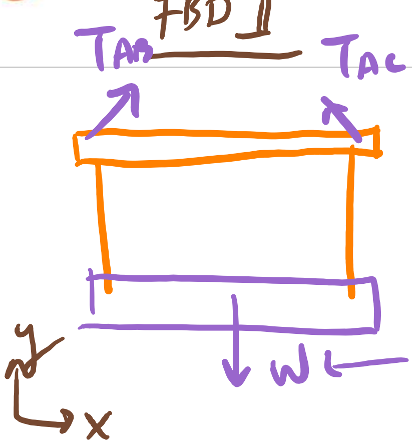
FBD I



FBD II



T_{AC} ← find

W ← given

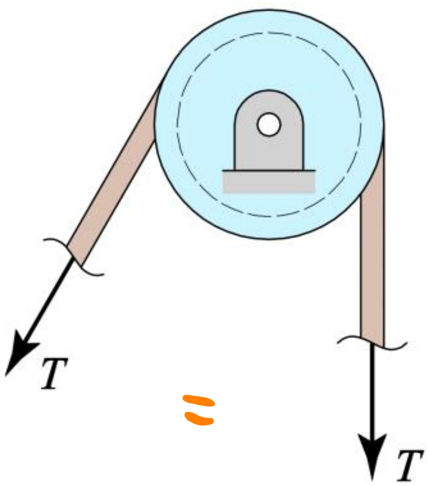
* Be strategic about choosing the right "body" for the problem

* Only include external forces acting on the "body" in the diagram

* Include geometry & coordinate system

Idealizations

Pulleys are (usually) regarded as frictionless; then the tension in a rope or cord around the pulley is the same on either side.



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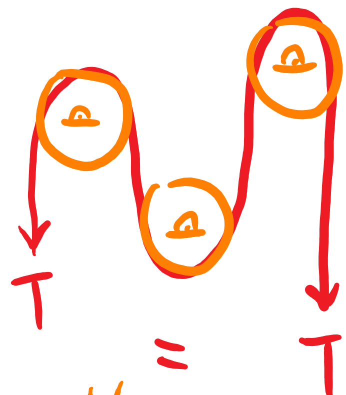
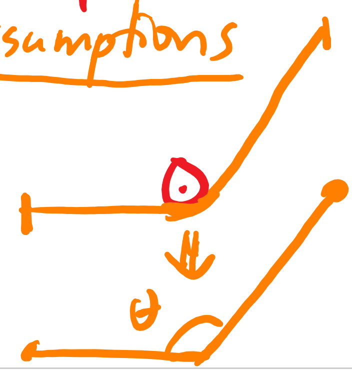
Frictionless pulley



Other pulley assumptions

- massless
- particle

(unless stated otherwise)



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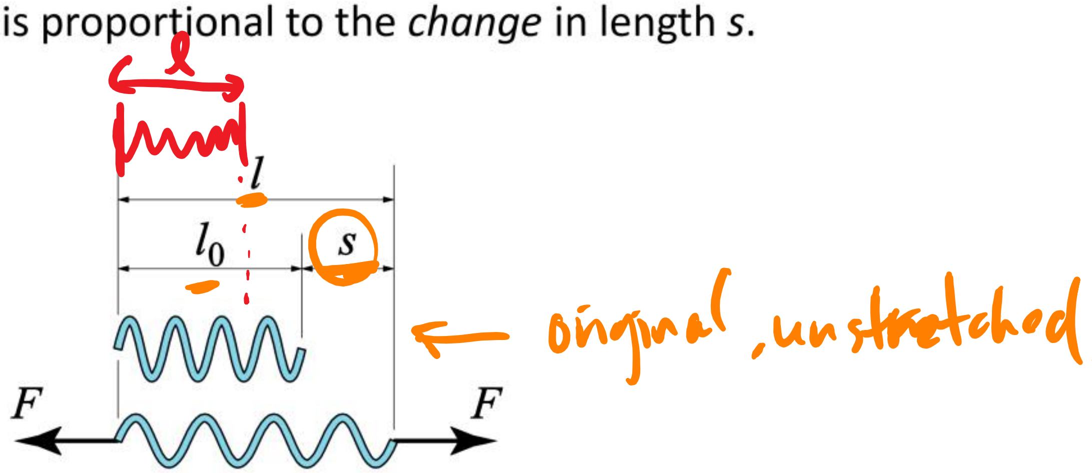
Idealizations

Springs are (usually) regarded as linearly elastic; then the tension is proportional to the *change* in length *s*.



$$F = ks = k(l - l_0)$$

Linearly elastic spring

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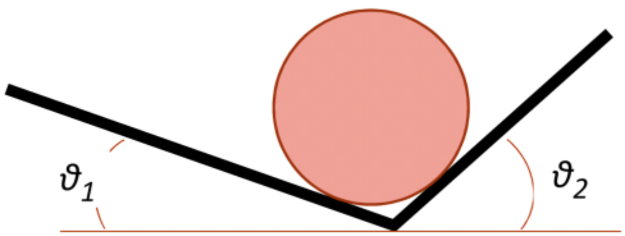
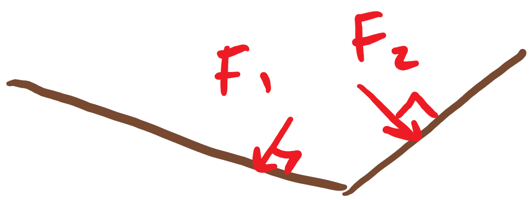
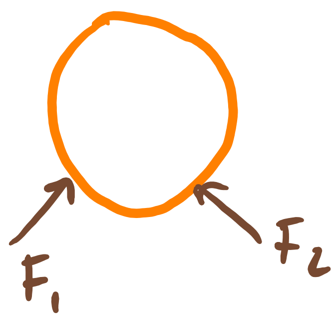
Idealizations



Contact force in smooth surface:

Force (normal) will always be $\perp$ to the surface





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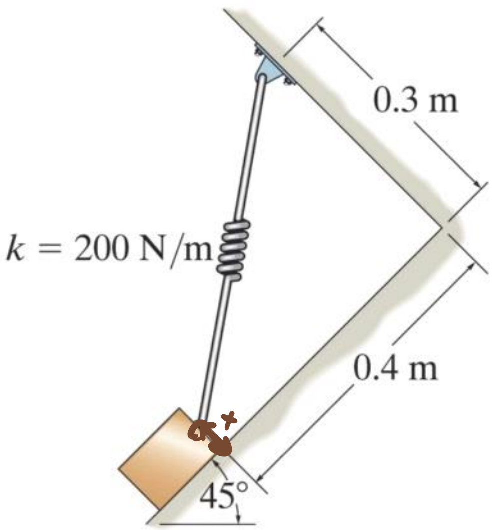
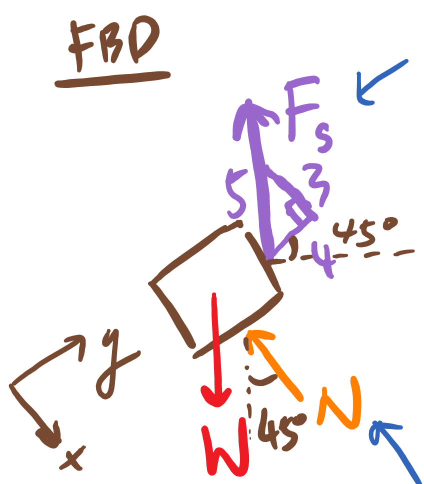
Free Body Diagram Example





1 ~~2~~ unknowns (magnitude & ~~direction~~)

given by particle assumption

1 unknown (magnitude)

a "slanted" coordinate system is preferred

- Assume the box is a particle.

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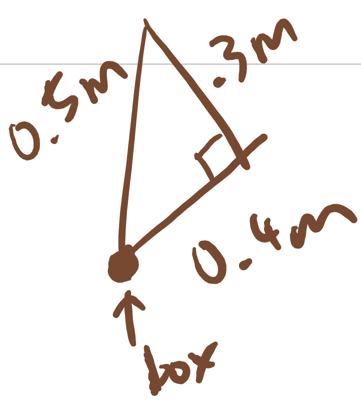
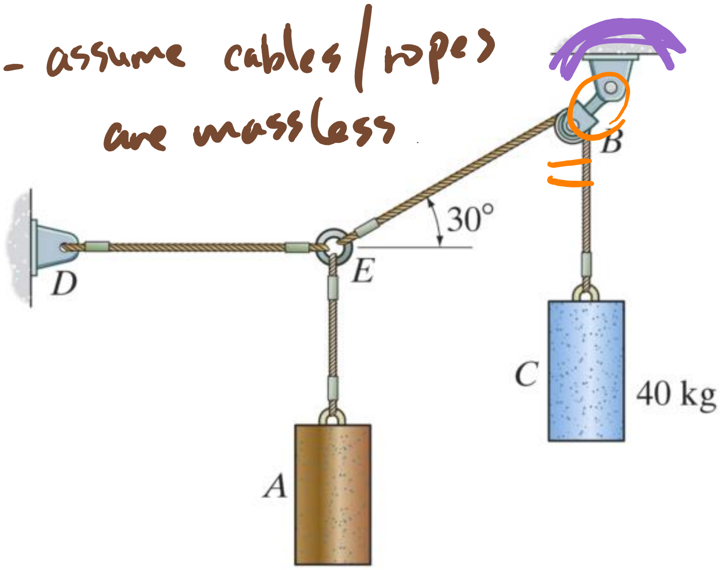
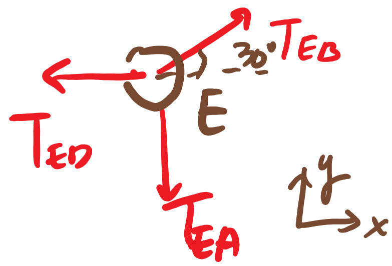


Free Body Diagram Example

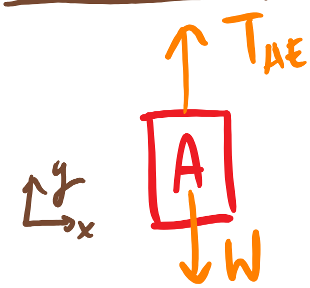
- assume cables/ropes are massless



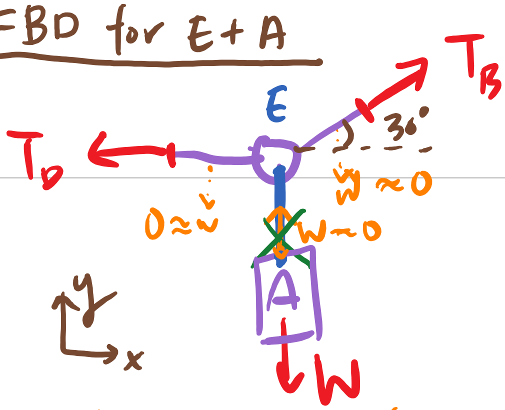
FBD for E



FBD for A



FBD for E + A



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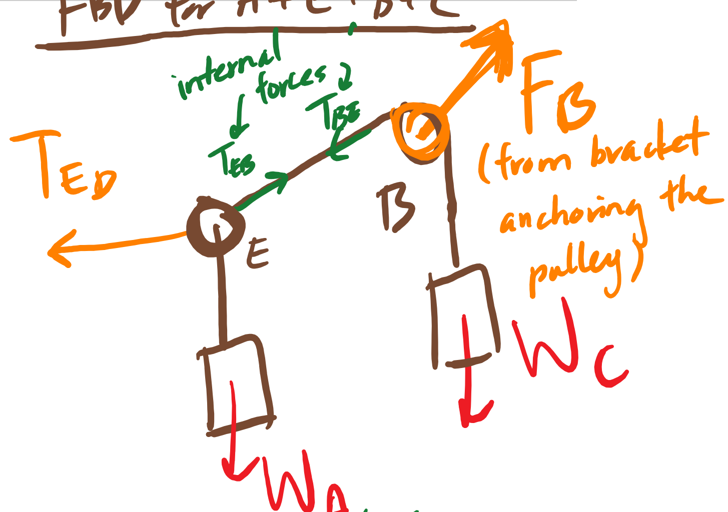
Note: Cables have negligible mass assumption is usually applied for this course unless specified otherwise.

FBD for A+E+B+C



* Do NOT include internal forces

Equilibrium of a particle

According to Newton's first law of motion , a particle will be in **equilibrium** (that is, it will remain at rest or continue to move with constant velocity) if and only if

$$\Sigma \vec{F} = 0$$

In three dimensions, equilibrium requires:

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0$$

Coplanar forces: if all forces are acting in a single plane, such as the "xy" plane, then the equilibrium condition becomes (2D)

$$\Sigma F_x = 0 \quad \Sigma F_y = 0$$

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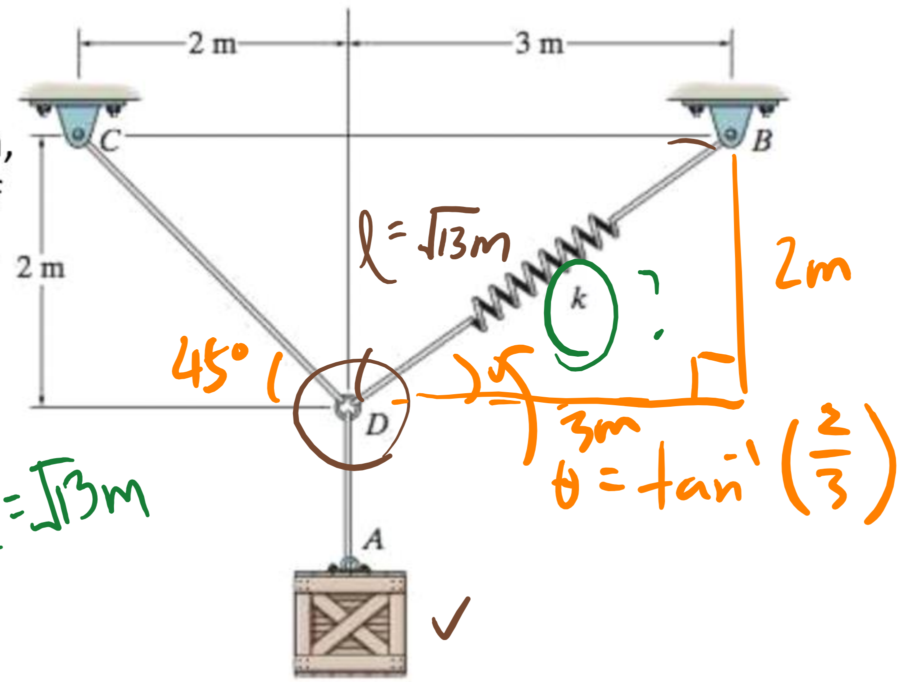
Example

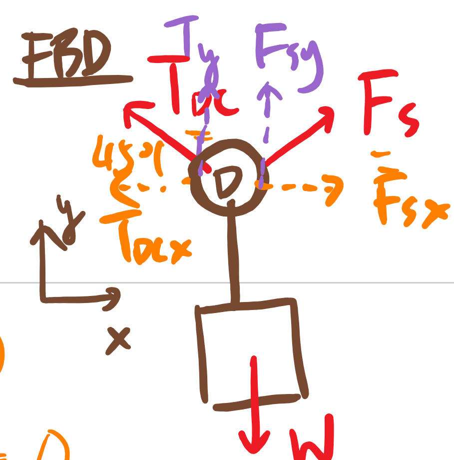
If the spring *DB* has an unstretched length of 2 m, determine the stiffness of the spring to hold the 40-kg crate in the position shown.



Given: $l_0 = 2m$, $l = \sqrt{13}m$

$m = 40$ kg

Find: k



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EoE

$\Sigma F_x = 0 = -T_x + F_{sx}$

$= -T\cos 45 + F_s \cos\theta$

or $= -T\left(\frac{\sqrt{2}}{2}\right) + F_s\left(\frac{3}{\sqrt{13}}\right) = 0$

$\rightarrow T = F_s\left(\frac{3}{\sqrt{13}}\right)\left(\frac{2}{\sqrt{2}}\right) = F_s\left(\frac{6}{\sqrt{26}}\right)$ ①

$\Sigma F_y = 0 = T_y + F_{sy} - W$

$= +T\sin 45° + F_s \sin\theta - W$

or $= T\left(\frac{\sqrt{2}}{2}\right) + F_s\left(\frac{2}{\sqrt{13}}\right) - W = 0$ ②

Substitute ① → ②

$$F_s\left(\frac{6}{\sqrt{26}}\right)\left(\frac{\sqrt{2}}{2}\right) + F_s\left(\frac{2}{\sqrt{13}}\right) - W = 0$$

$$F_s\left(\frac{3}{\sqrt{13}} + \frac{2}{\sqrt{13}}\right) = W \longrightarrow F_s = \frac{W}{\left(\frac{5}{\sqrt{13}}\right)} \quad ③$$

Apply linear spring assumption : $F_s = ks$.

$$s = l - l_0 = \sqrt{13}m - 2m$$

$$F_s = k(\sqrt{13}m - 2m) \quad ④$$

Substitute ④ → ③

$$k(\sqrt{13}m - 2m) = W\left(\frac{\sqrt{13}}{5}\right) \longrightarrow$$

$$\boxed{k = (W\sqrt{13}) / [5(\sqrt{13}m - 2m)]}$$

Substitute in numbers

$$k = \frac{(40\,kg)(9.81\,\frac{m}{s^2})\sqrt{13}}{5(\sqrt{13} - 2)\,m}.$$

$$\boxed{k = 176\ N/m}$$