Announcements

- Do NOT discuss quiz material with <u>anyone</u> until after the end of the testing period (Sunday)

❑ Upcoming deadlines:

- Today! (9/14)
  - Written Assignment
- Tuesday (9/18)
  - PL HW
- Friday (9/21)
  - Written Assignment

1

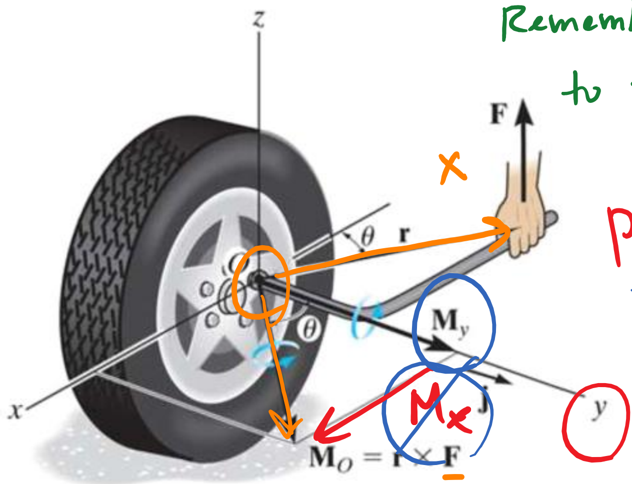
Objective

- Moment of a force about a specific axis
- Couple Moment

2

Moment about a Specific Axis

Remember, the component of a vector, **A**, along the direction of another, **B**, can be determined using the dot product:

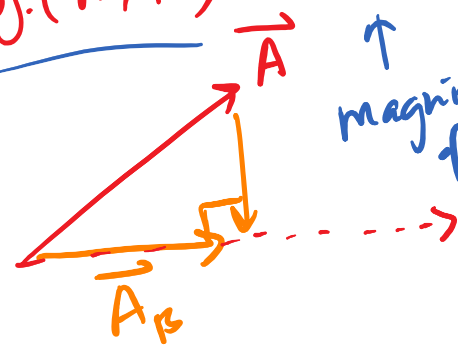


Remember from before, when we want to find a vector component along another vector?

$$\text{proj.}(\vec{A}, \vec{B}) = \vec{A} \cdot \hat{u}_B$$

↑ magnitude

{ projected vector



Now we use it for moment vector.

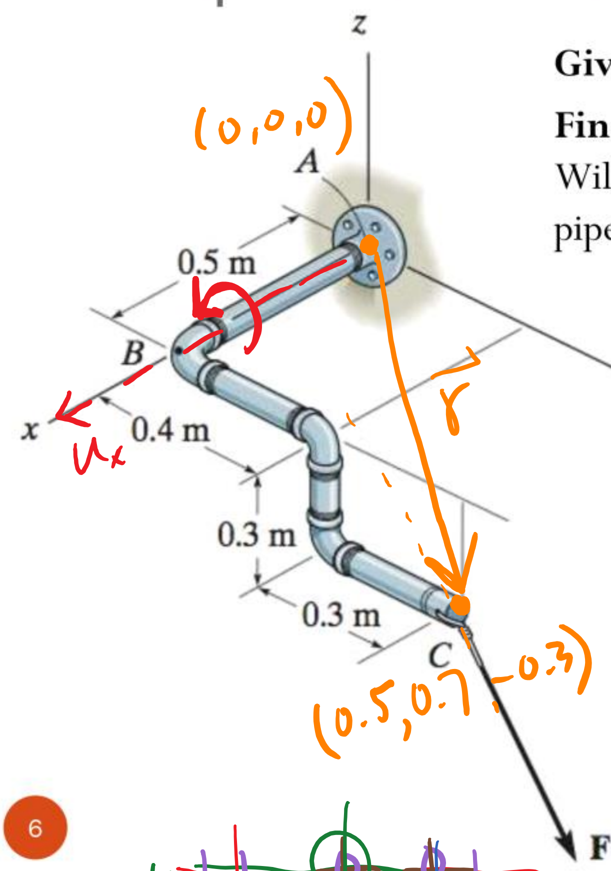
$$\vec{M}_O = \vec{r} \times \vec{F}$$

$$\vec{M}_A = (\vec{M}_O \cdot \hat{u}_A)\,\hat{u}_A$$

$$\vec{M}_A = [(\vec{r} \times \vec{F}) \cdot \hat{u}_A]\,\hat{u}_A$$

5

Example – Vector Formulation



Given: **F** = {600**i** + 800**j** - 500**k**} N

Find: Moment of the force about the *x*-axis. Will the force be tightening or loosening the pipe assembly at *A*?

$$\vec{M} = [\hat{u}_x \cdot (\vec{r} \times \vec{F})]\hat{u}_x$$

triple scalar product.

① $\hat{u}_x = \hat{i}$

② $\vec{r}_{AC} = (0.5\hat{i} + 0.7\hat{j} - 0.3\hat{k})$ m

③ $\vec{F} = (600\hat{i} + 800\hat{j} - 500\hat{k})$ N

$$M_A = \begin{vmatrix} u_x: & 1 & 0 & 0 \\ r: & 0.5 & 0.7 & -0.3 \\ F: & 600 & 800 & -500 \end{vmatrix} = 1[0.7(-500) - 800(-0.3)]\ N \cdot m$$

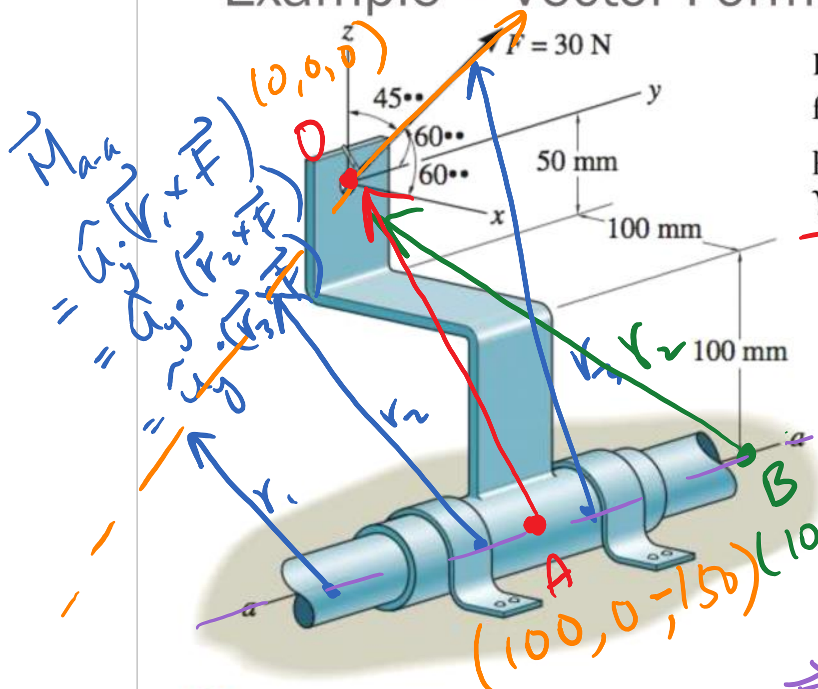
$$- 0(0.5(-500) - 600(-0.3))$$

$$+ 0(0.5(800) - 0.7(600))$$

$$= -110\ N \cdot m$$

$$\vec{M}_A = M_A \hat{u}_x = -110\hat{i}\ N \cdot m$$

Example – Vector Formulation



Determine the moment of the force about the *a-a* axis of the pipe. ($\alpha = 60°$, $\beta = 60°$, and $\gamma = 45°$)

$\vec{M}_{a\text{-}a} = \hat{u}_y \cdot (\vec{r}_1 \times \vec{F}) = \hat{u}_y \cdot (\vec{r}_2 \times \vec{F}) = \hat{u}_y \cdot (\vec{r}_3 \times \vec{F})$

$$\vec{M}_{a\text{-}a} = \hat{u}_y \cdot (\vec{r}_{AO} \times \vec{F}) = \hat{u}_y \cdot (\vec{r}_{BO} \times \vec{F})$$

$$\vec{F} = F(\cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k}) = 30\,\text{N}(\cos 60°\,\hat{i} + \cos 60°\,\hat{j} + \cos 45°\,\hat{k})$$

$\vec{r}$: from any point on the rotation axis to any point of the line of action of $\vec{F}$:

$$r_{AO} = -100\hat{i} + 150\hat{k}\ \text{mm}$$

$$r_{BO} = -100\hat{i} - 50\hat{j} + 150\hat{k}\ \text{mm}$$

$$\vec{M}_{a\text{-}a} = \hat{u}_y \cdot (r_{AO} \times \vec{F}) = \hat{u}_y \cdot (\vec{r}_{BO} \times \vec{F})$$

$$= \begin{vmatrix} 0 & 1 & 0 \\ -100 & 0 & 150 \\ 30c60° & 30c60° & 30c45° \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ -100 & -50 & 150 \\ 30c60° & 30c60° & 30c45° \end{vmatrix}$$

$$= -[-100(30\cos 45°) - 150(30\cos 60°)]\ \text{N}\cdot\text{mm} = -[-100(30\cos 45°) - 150(30\cos 60°)]\ \text{N}\cdot\text{mm}$$

$= -[-100(30\cos 45°)$

$-150(30\cos 60°)]$ N·mm

$-150(30\cos 60°)]$ N·mm

same results

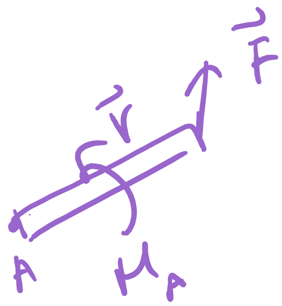
Page 9

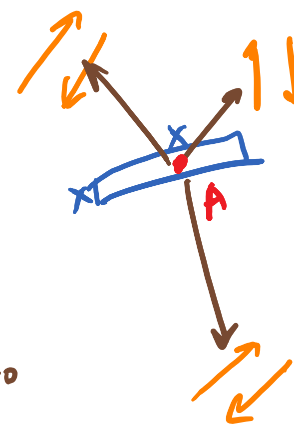
Friday, September 14, 2018 11:07 AM

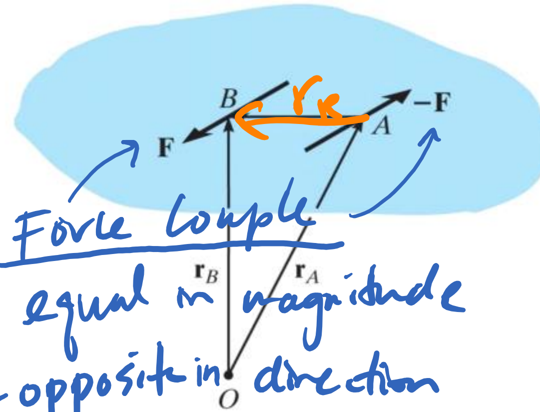
Couple Moment

9

Moment of a couple







Force couple
- equal in magnitude
- opposite in direction

- moment from a force: proportional to distance from the force

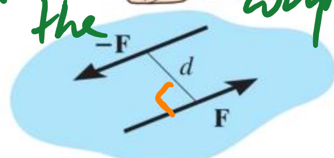
$$\vec{M}_O = \vec{M}_A + \vec{M}_B$$

$$= [\vec{r}_A \times (\vec{F})] + [\vec{r}_B \times \vec{F}]$$

$$= (-\vec{r}_A + \vec{r}_B) \times (-\vec{F})$$

$$\vec{M}_O = \vec{r}_R \times \vec{F}$$

- free vector (moment from a force couple): applies everywhere in the system independent of the distance from the couple.

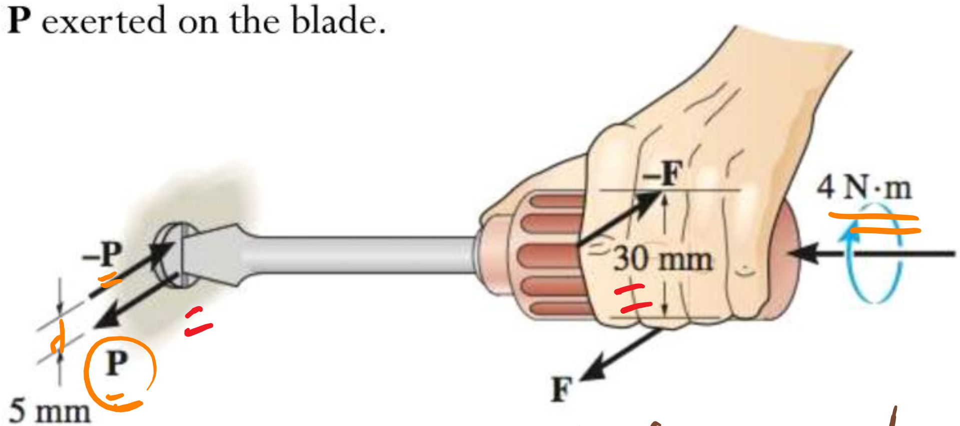


11

distance between the couple

$$M_O = F \cdot d$$

A twist of 4 N-m is applied to the handle of the screwdriver. Resolve this couple moment into a pair of couple forces **F** exerted on the handle and **P** exerted on the blade.



$M = d \cdot P$ ← moment of a couple

$$P = \frac{M}{d} = \frac{4\ N \cdot m}{5\ mm}\left(\frac{10^3\ mm}{1\ m}\right) = \frac{4000\ N \cdot mm}{5\ mm}$$

$d = 5\ mm$

$M = 4\ N \cdot m$

$$P = 800\ N$$

12

$$M = d \cdot F \longrightarrow F = \frac{M}{d} = \frac{4\ N \cdot m}{30\ mm}\left(\frac{10^3\ mm}{m}\right) = \frac{400}{3}\ N$$