Announcements

- Quiz next week
- Have you been on Piazza lately?

i-clicker ready?

❑ Upcoming deadlines:

- Friday (9/21) ✓
  - Writing Assignment
- Tuesday (9/25) ✓
  - PL HW

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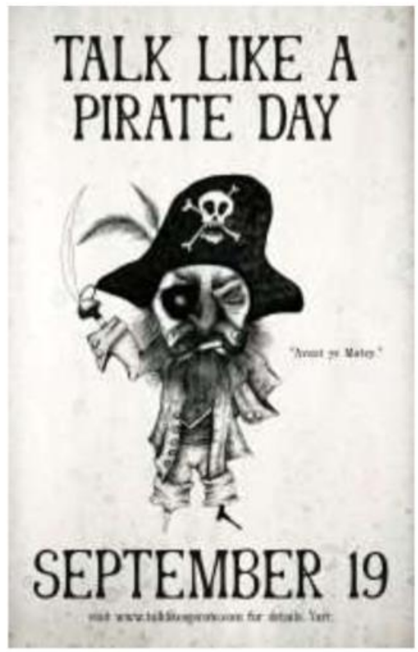
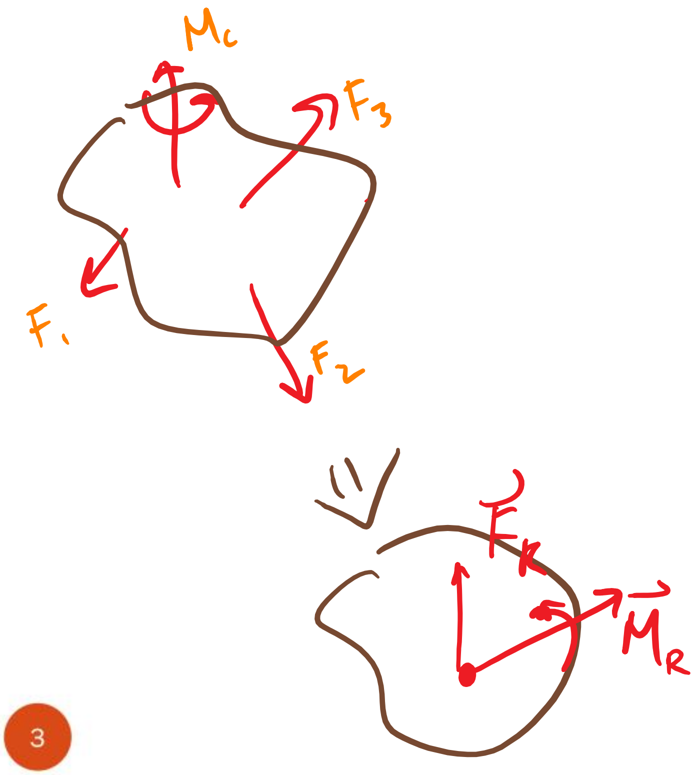
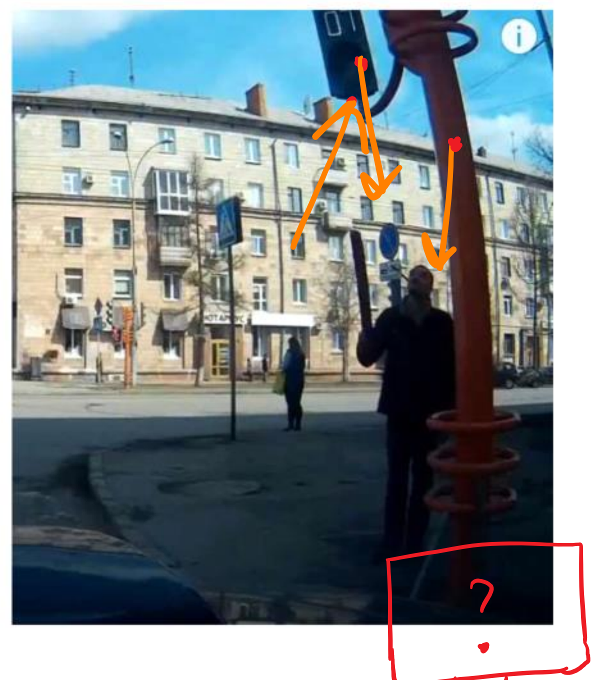


Objective

- Equivalent System





How is the base affected by theses forces?

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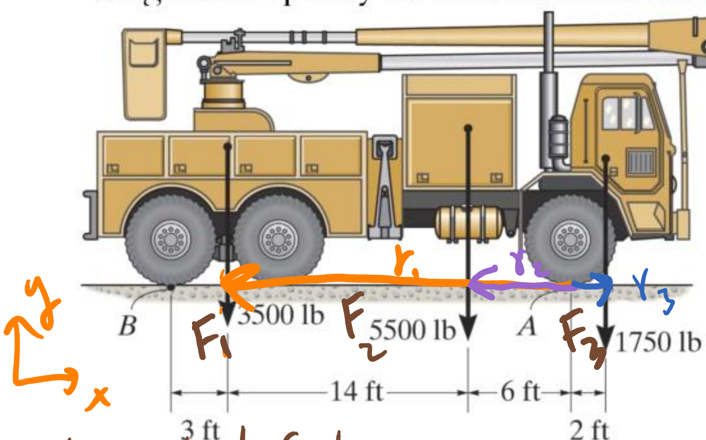
Example – 2D Equivalent System

Replace weights of the components of the truck with a single equivalent weight and specify its location measured from A. =



Governing Equations

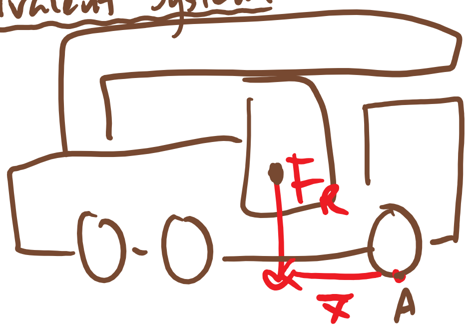
$$\vec{F}_R = \sum \vec{F}$$

None for this problem.

$$\vec{M}_R = \cancel{\sum \vec{M}_c} + \sum \vec{r}_i \times \vec{F}_i$$

$$\vec{M}_{RA} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_3 \times \vec{F}_3$$

Equivalent System



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$\vec{r}_1 = -(6+14)\text{ ft } \hat{\imath} = -20\hat{\imath}\text{ ft}.$

$\vec{r}_2 = -6\hat{\imath}\text{ ft}.$

$\vec{r}_3 = 2\hat{\imath}\text{ ft}.$

$\vec{F}_1 = -3500\hat{\jmath}\text{ lb}$

$\vec{F}_2 = -5500\hat{\jmath}\text{ lb}$

$\vec{F}_3 = -1750\hat{\jmath}\text{ lb}$

$\vec{M}_{1A} = 70000\text{ lb}\cdot\text{ft}\,\hat{k}$

$\vec{M}_{2A} = 33000\text{ lb}\cdot\text{ft}\,\hat{k}$

$+\ \vec{M}_{3A} = -3500\text{ lb}\cdot\text{ft}\,\hat{k}$

$\vec{M}_{RA} = 99500\text{ lb}\cdot\text{ft}\ k$

$$\boxed{\bar{x} = \frac{M_{RA}}{F_R} \quad \frac{99500\text{ lb}\cdot\text{ft}}{(3500+5500+1750)\text{ lb}} \approx 9.26\text{ ft}}$$

* Since $\bar{x}$ and F_R are $\perp$ to each other, $M_{RA} = F_R \cdot \bar{x}$

Example – 2D Equivalent System

What is the resultant force and moment on point A from F_1 and F_2?

$\vec{F}_R = \sum \vec{F} = \vec{F}_1 + \vec{F}_2 = (-400\hat{j} + 400\hat{j})\,N = 0$

$\vec{M}_R = \sum \vec{r} \times \vec{F} + \sum \vec{M}_c$

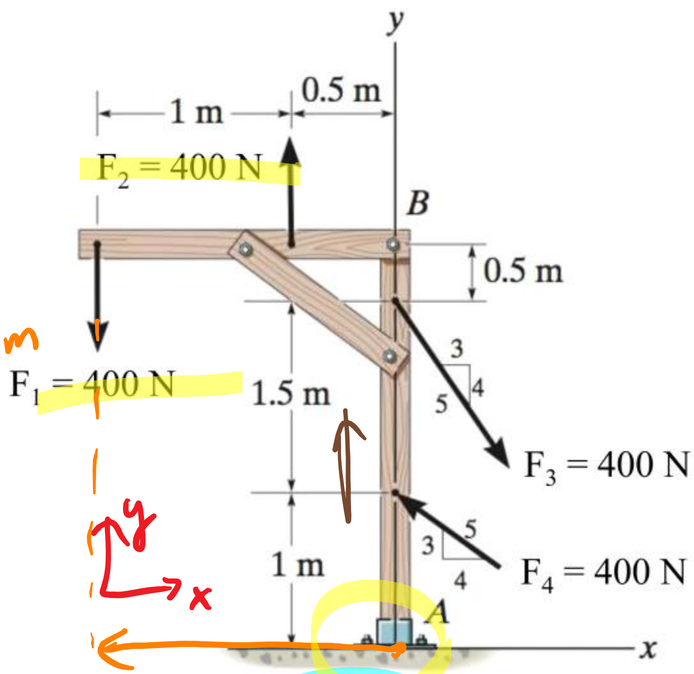
$= \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2$

$= [(-1.5\hat{i}) \times (-400\hat{j})]\,N \cdot m$

$+ [(-0.5\hat{i}) \times (400\hat{j})]\,N \cdot m$

$= (600 - 200)\hat{k}\ N \cdot m$

$= 400\ N \cdot m\ \hat{k}$



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$\vec{M}_c = \vec{r}_{12} \times \vec{F}_2 = (1m\,\hat{i}) \times (400\,N\,\hat{j}) = 400\,\hat{k}\ N \cdot m$

pos. vector from F_1 to F_2 (or F_2 to F_1)

+ F_1 + F_2 makes a couple!

Example – 2D Equivalent System

What is the resultant force and moment on point B from F_1 and F_2?

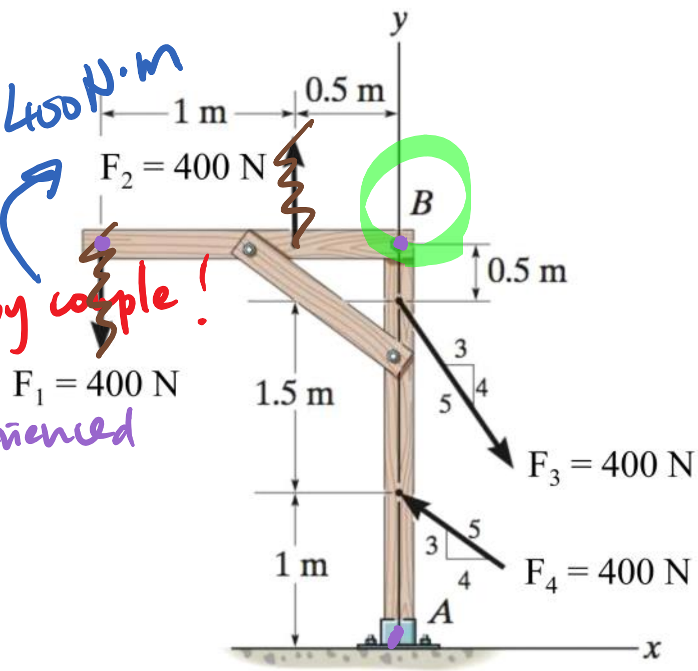
$\vec{F}_R = \sum \vec{F} = 0$

$\vec{M}_R = \sum M_c + \sum \vec{r} \times \vec{F}$

$= 400\ N \cdot m\ \hat{k}$

$(F_1 + F_2)$ makes a happy couple!

400 N·m



+ same couple moment is experienced everywhere (A & B).

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Example – 2D Equivalent System

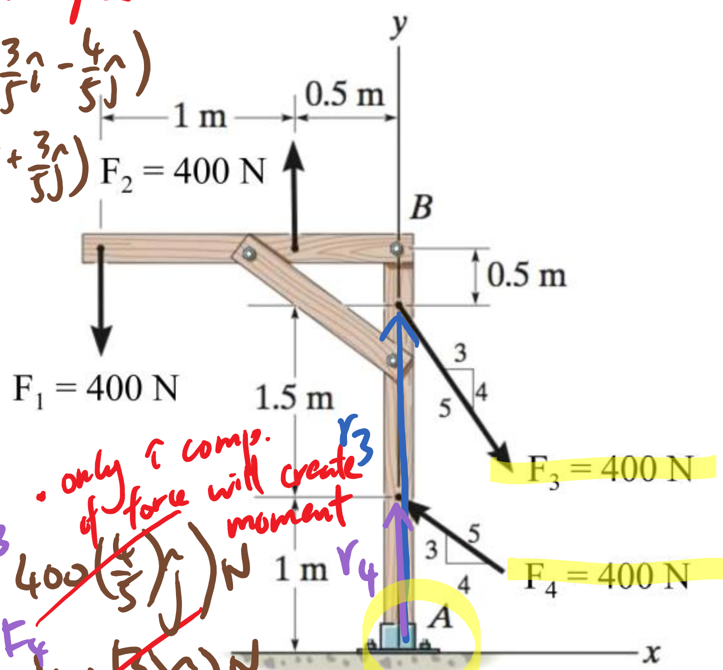
What is the resultant force and moment on point *A* from F_3 and F_4?

F_3 & F_4 ≠ happy couple

$\vec{F}_R = \sum \vec{F} = \vec{F}_3 + \vec{F}_4$, $\vec{F}_3 = 400N\left(\frac{3}{5}\hat{i} - \frac{4}{5}\hat{j}\right)$

$\vec{F}_4 = 400N\left(-\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}\right)$

$$\boxed{F_R = -80\hat{i} - 80\hat{j}\ N}$$



$$\vec{M}_{RA} = \underbrace{\sum \vec{r}_i \times \vec{F}_i} + \cancel{\sum M_c}$$

$$= \vec{r}_3 \times \vec{F}_3 + \vec{r}_4 \times \vec{F}_4$$

• only $\hat{i}$ comp. of force will create moment

$$= \underbrace{(2.5m\hat{j})}_{r_3} \times \underbrace{\left(400\left(\tfrac{3}{5}\right)\hat{i} - \cancel{400\left(\tfrac{4}{5}\right)\hat{j}}\right)N}_{F_3}$$

$$\underbrace{(1m\hat{j})}_{r_4} \times \underbrace{\left(-400\left(\tfrac{4}{5}\right)\hat{i} + \cancel{400\left(\tfrac{3}{5}\right)\hat{j}}\right)N}_{F_4}$$

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$$= (-600 + 320)\hat{k}\ N\cdot m$$

$$\boxed{\vec{M}_{RA} = -280\ \hat{k}\ N\cdot m}$$

Example – 2D Equivalent System

What is the resultant force and moment on point *B* from F_3 and F_4?

$\vec{F}_R = \vec{F}_3 + \vec{F}_4 = (-80\hat{i} - 80\hat{j})$ N ← independent of the reference point (A or B)

$$\vec{M}_{RB} = \vec{r}_3 \times \vec{F}_3 + \vec{r}_4 \times \vec{F}_4$$

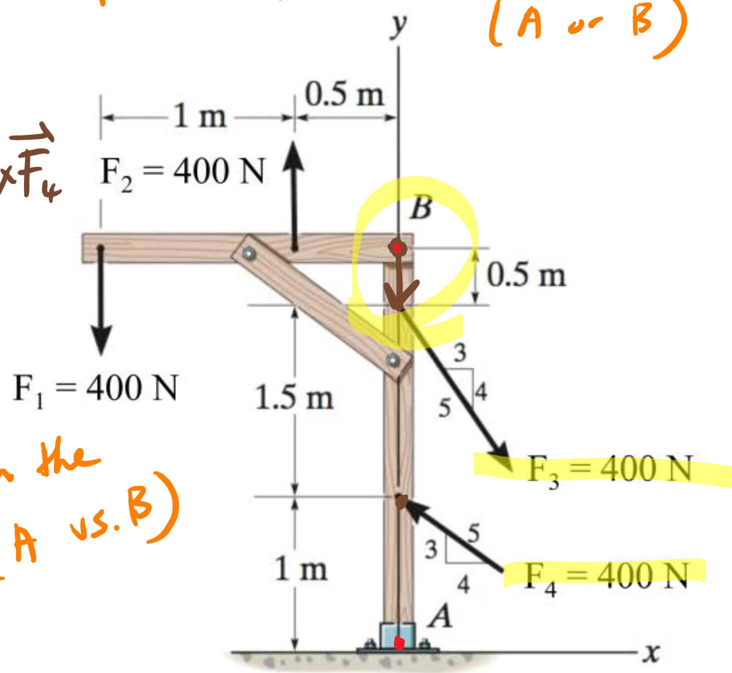
$$= (-0.5\hat{j}\,m) \times \vec{F}_3 + (2\hat{j}\,m) \times \vec{F}_4$$

$$= (120 - 640)\hat{k} \text{ N}\cdot\text{m}$$

$$\boxed{\vec{M}_{RB} = -520\hat{k} \text{ N}\cdot\text{m}}$$

specify where the reference pt. is.

changes depending on the reference point (A vs. B)



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Example – 2D Equivalent System

Replace the loading on the frame by a single resultant force. Specify where its line of action intersects a vertical line along member AB, measured from A.

couples

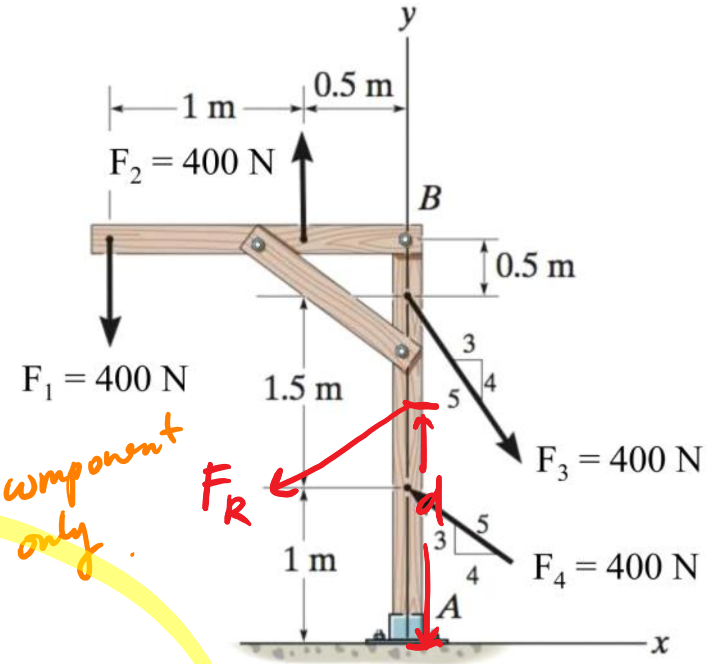
$$\vec{F}_R = \cancel{\vec{F}_1} + \cancel{\vec{F}_2} + \vec{F}_3 + \vec{F}_4$$

$$= (-80\hat{i} - 80\hat{j})\ N.$$

$$\vec{M}_{RA} = \Sigma \vec{M}_c + \Sigma M_A$$

$$= (400 - 280)\hat{k}\ N\cdot m$$

$$\vec{M}_{RA} = 120\hat{k}\ N\cdot m$$



y-component only

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$$\vec{M}_{RA} = \vec{d} \times \vec{F}_R, \quad \vec{d} = d\hat{j}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & d & 0 \\ -80 & -80 & 0 \end{vmatrix}$$

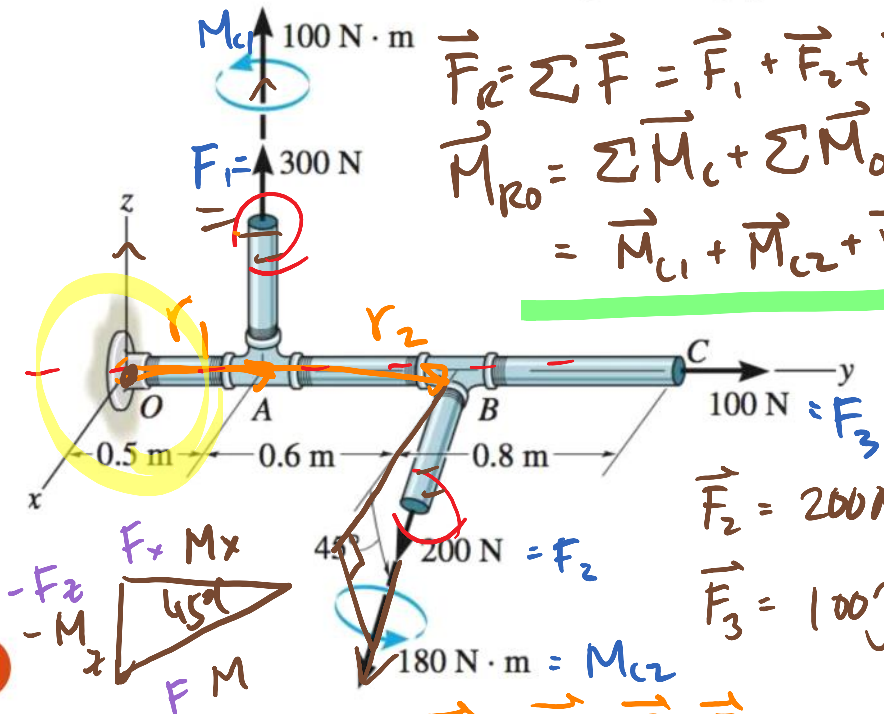
only the x-component contributes to the equivilent M_{RA} from F_R.

$$= [0(-80) - d(-80)]\hat{k}\ N\cdot m = 120\hat{k}\ N\cdot m$$

$$\rightarrow 80d = 120\ N\cdot m, \quad \boxed{d = 1.5 m}$$

Example – 3D Equivalent System

Find the equivalent resultant force and couple moment at point O as the the two wrenches and the force acting on the pipe assembly below.



$$\vec F_R = \sum \vec F = \vec F_1 + \vec F_2 + \vec F_3$$

$$\vec M_{RO} = \sum \vec M_c + \sum \vec M_o$$

$$= \vec M_{c1} + \vec M_{c2} + \vec r_1 \times \vec F_1 + \vec r_2 \times \vec F_2 + \cancel{\vec r_3 \times \vec F_3}$$

$\vec F_1 = 300\hat k$ N

$\vec F_2 = 200\text{N}(\cos 45^\circ \hat i - \sin 45^\circ \hat k)$

$\vec F_3 = 100\hat j$ N

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$$\Rightarrow \vec F_R = \vec F_1 + \vec F_2 + \vec F_3$$

$$= 100\sqrt{2}\,\hat i + 100\hat j + (300 - 100\sqrt{2})\hat k \text{ N.}$$

$\vec M_{c1} = 100\hat k$ N·m, $\vec M_{c2} = 180(\cos 45^\circ \hat i - \sin 45^\circ \hat k)$ N·m

$\vec r_1 = 0.5\hat j$ m, $\vec r_2 = 1.1\hat j$ m, $\vec r_3 = 1.9\hat j$ m.

$$\vec M_{RO} = \left[(100\hat k) + 90\sqrt{2}\,\hat i - 90\sqrt{2}\,\hat k + \overset{(\vec r_1 \times \vec F_1)}{150\hat i} + (-110\sqrt{2}\,\hat i - 110\sqrt{2}\,\hat k)\right] \text{N·m}$$

$$= \left[(150 - 20\sqrt{2})\hat i + 0\hat j + (100 - 200\sqrt{2})\hat k\right] \text{N·m}$$