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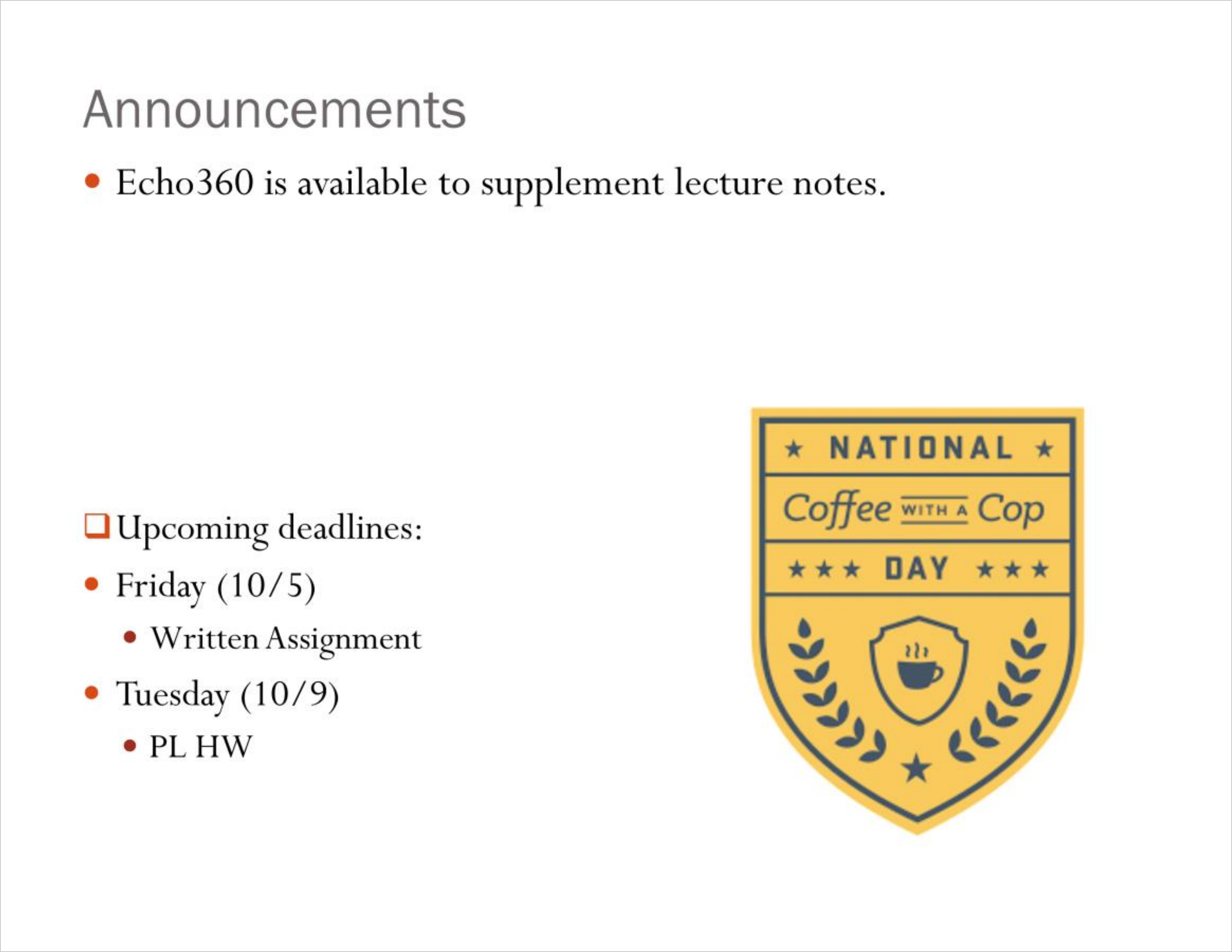

Announcements

- Echo360 is available to supplement lecture notes.

☐ Upcoming deadlines:

- Friday (10/5)
  - Written Assignment
- Tuesday (10/9)
  - PL HW

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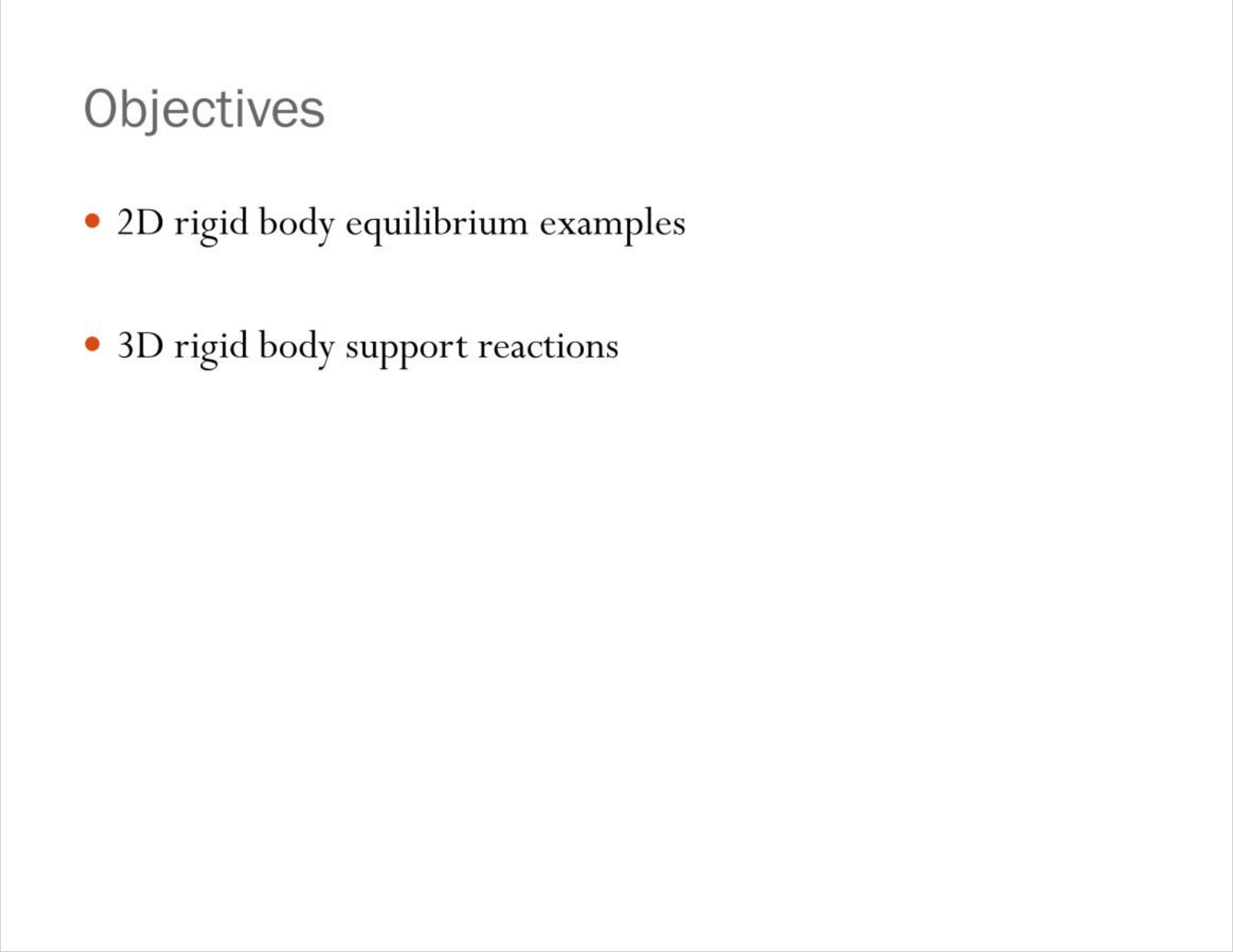
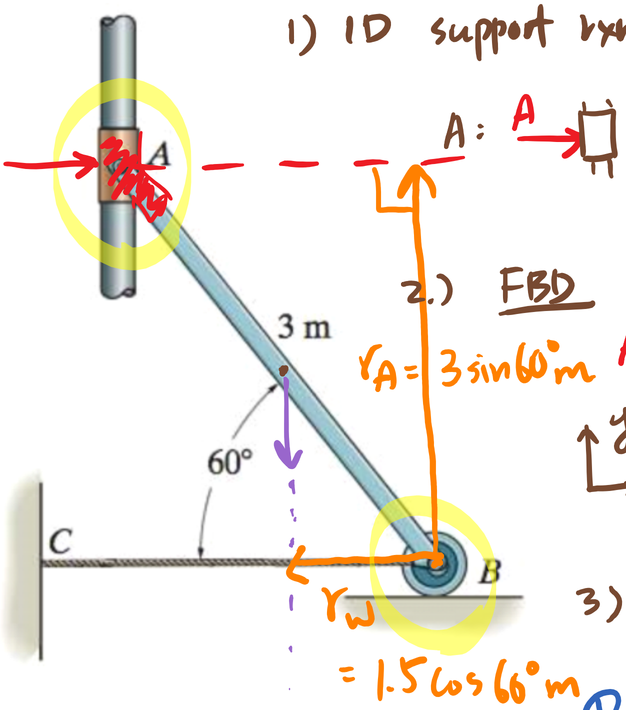


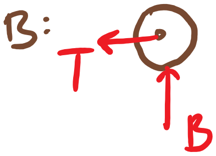
Objectives

- 2D rigid body equilibrium examples
- 3D rigid body support reactions

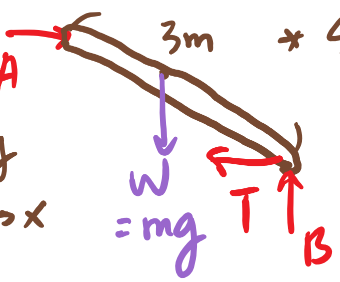
The uniform rod *AB* has a mass of 40 kg. Determine the force in the cable when the rod is in the position shown. There is a smooth collar at *A*.

1) 1D support rxns → forces / moment they provide





2.) FBD



* Statically Determinant ✓

3) EoE

① $\Sigma F_x = A - T = 0$

② $\Sigma F_y = -W + B = 0$

③ $\Sigma M_B = +(1.5\cos 60° m)W - (3\sin 60° m)A = 0$

→ ③

$A = \frac{(1.5\cos 60°)W}{3\sin 60°}$

→ ② $B = W$

→ ③ + ①

$T = A = \frac{\cos 60°}{2\sin 60°} W$

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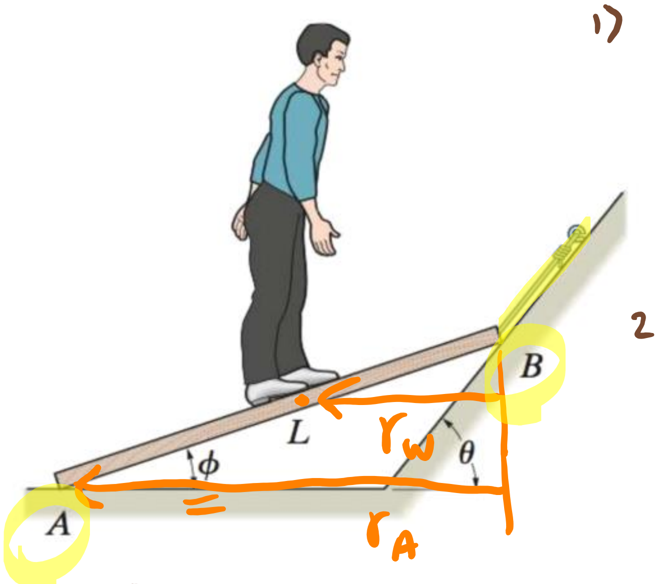
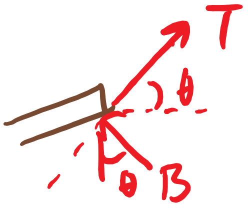
The man has a weight W and stands at the center of a plank with negligible weight. If the planes at A and B are smooth, determine the tension in the cord in terms of W and θ.



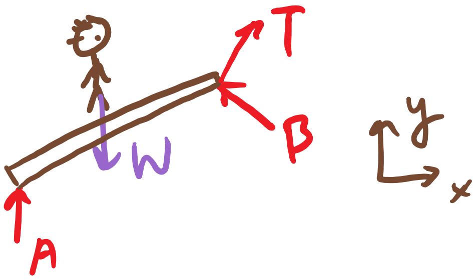
1) ID support

A:

B



2) FBD



3.) EoE

$$\Sigma F_x = -B\sin\theta + T\cos\theta = 0$$

$$\Sigma F_y = A - W + B\cos\theta + T\sin\theta = 0$$

$$\Sigma M_B = +W\left(\tfrac{L}{2}\cos\phi\right) - A\left(L\cos\phi\right) = 0$$

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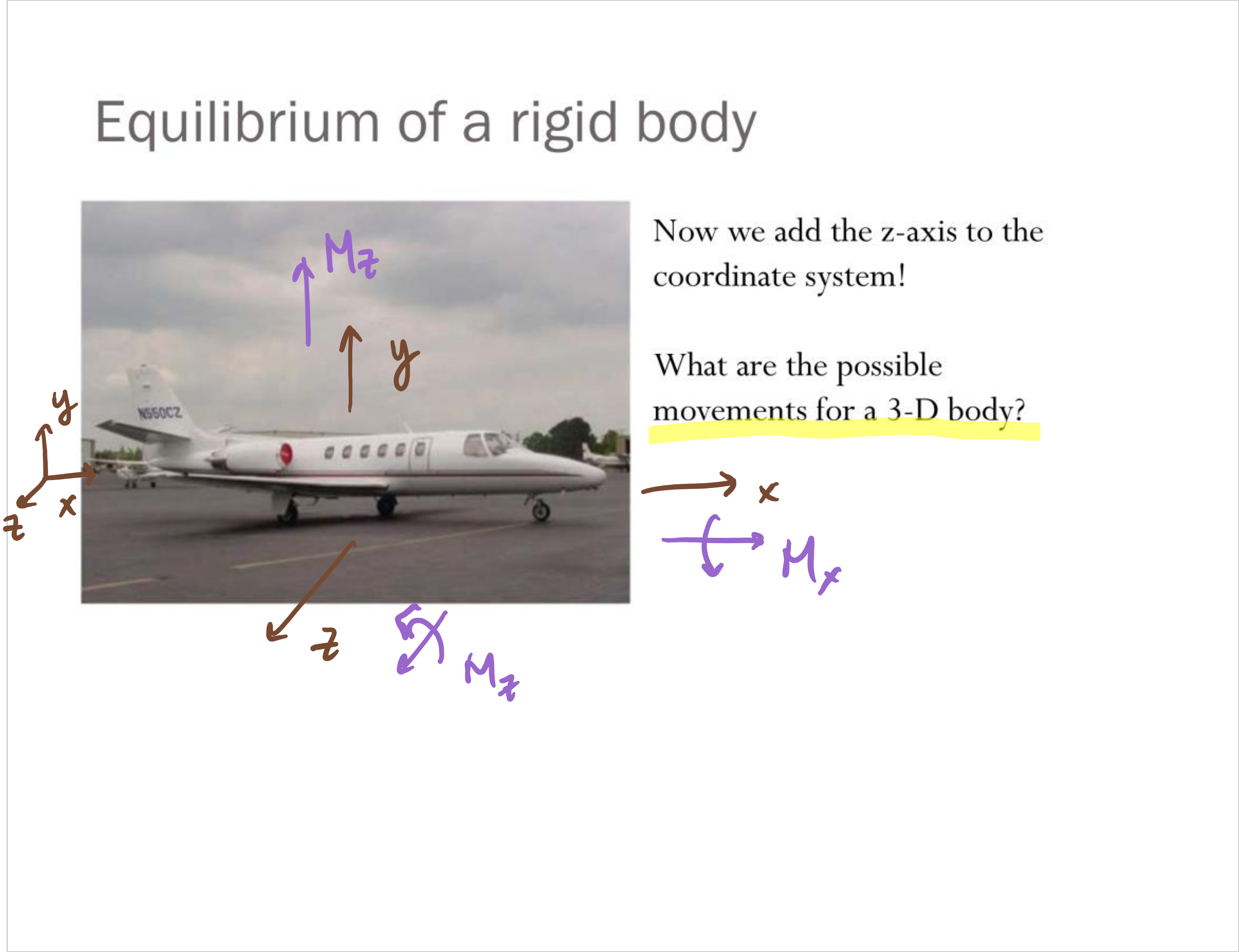

Equilibrium of a rigid body

Now we add the z-axis to the coordinate system!

What are the possible movements for a 3-D body?

Equilibrium of a rigid body

Now we add the z-axis to the coordinate system!

6 Equations of Equilibriums:

$\Sigma F_x = 0$ $\Sigma M_{0x} = 0$

$\Sigma F_y = 0$ $\Sigma M_{0y} = 0$

$\Sigma F_z = 0$ $\Sigma M_{0z} = 0$

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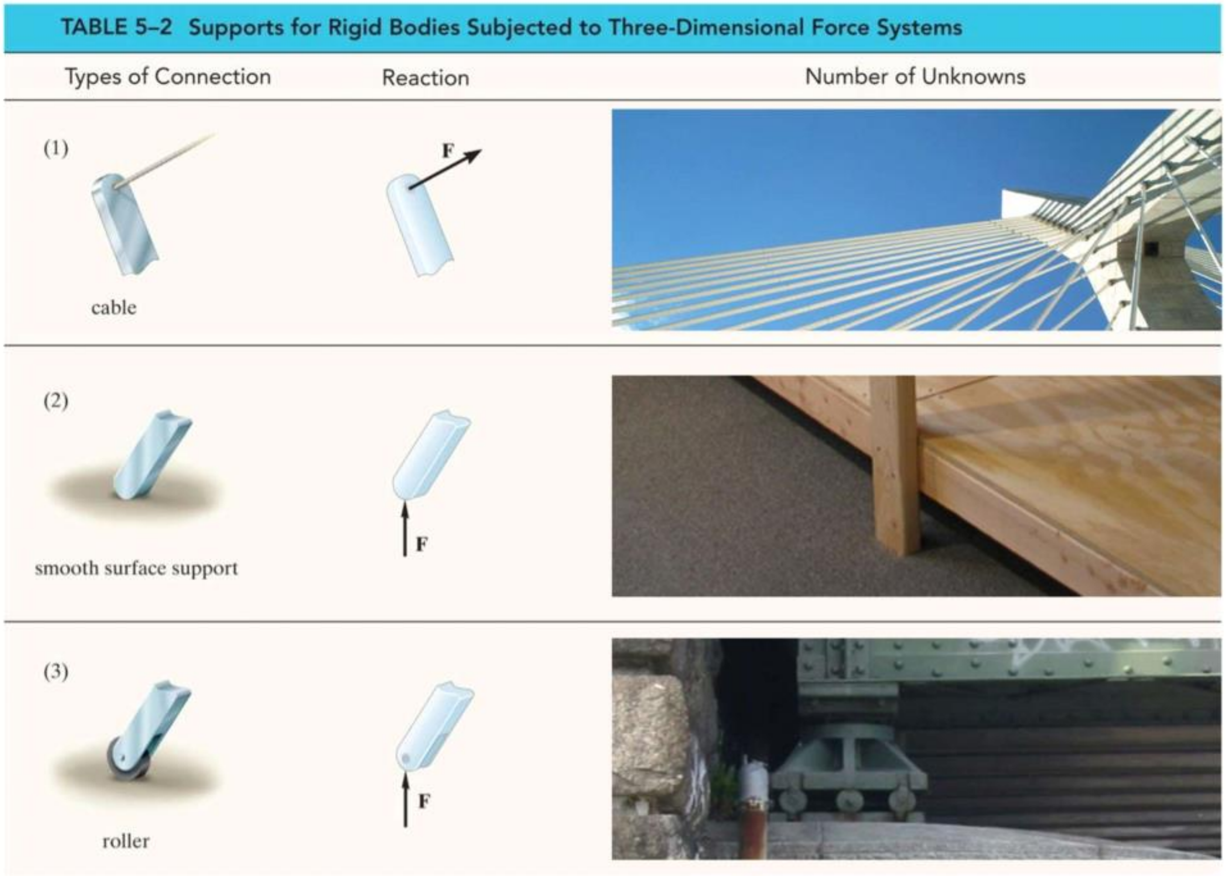

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1) cable	F	
(2) smooth surface support	F	
(3) roller	F	

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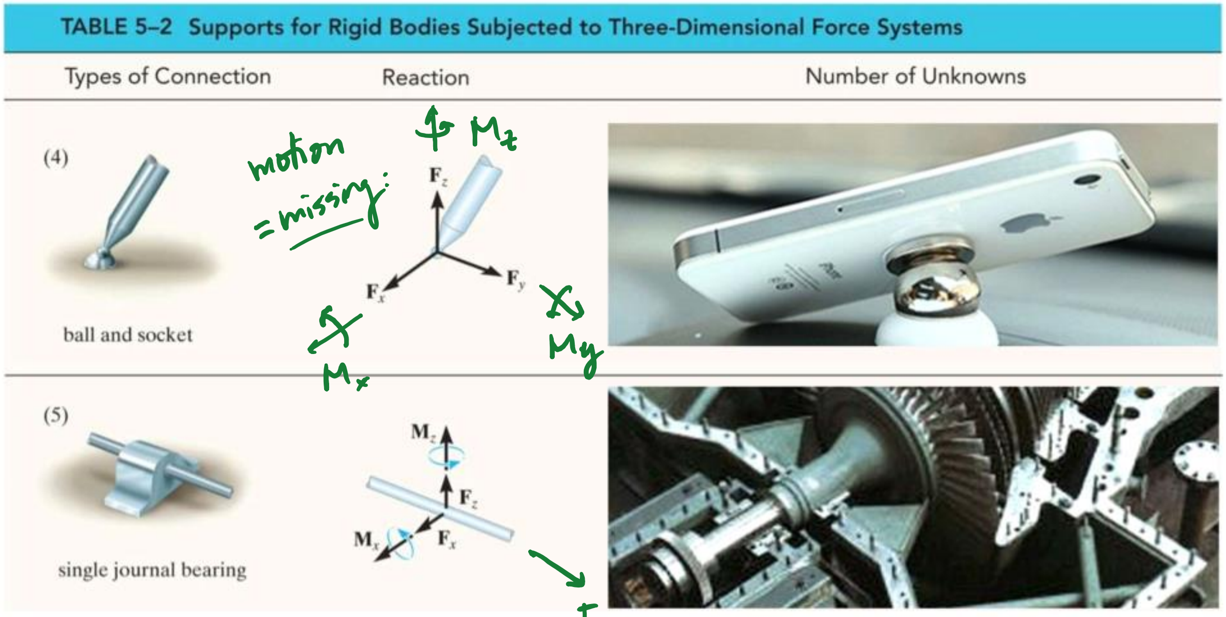

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(4) ball and socket		
(5) single journal bearing		

motion = missing: M_z M_x M_y

missing: F_y M_y

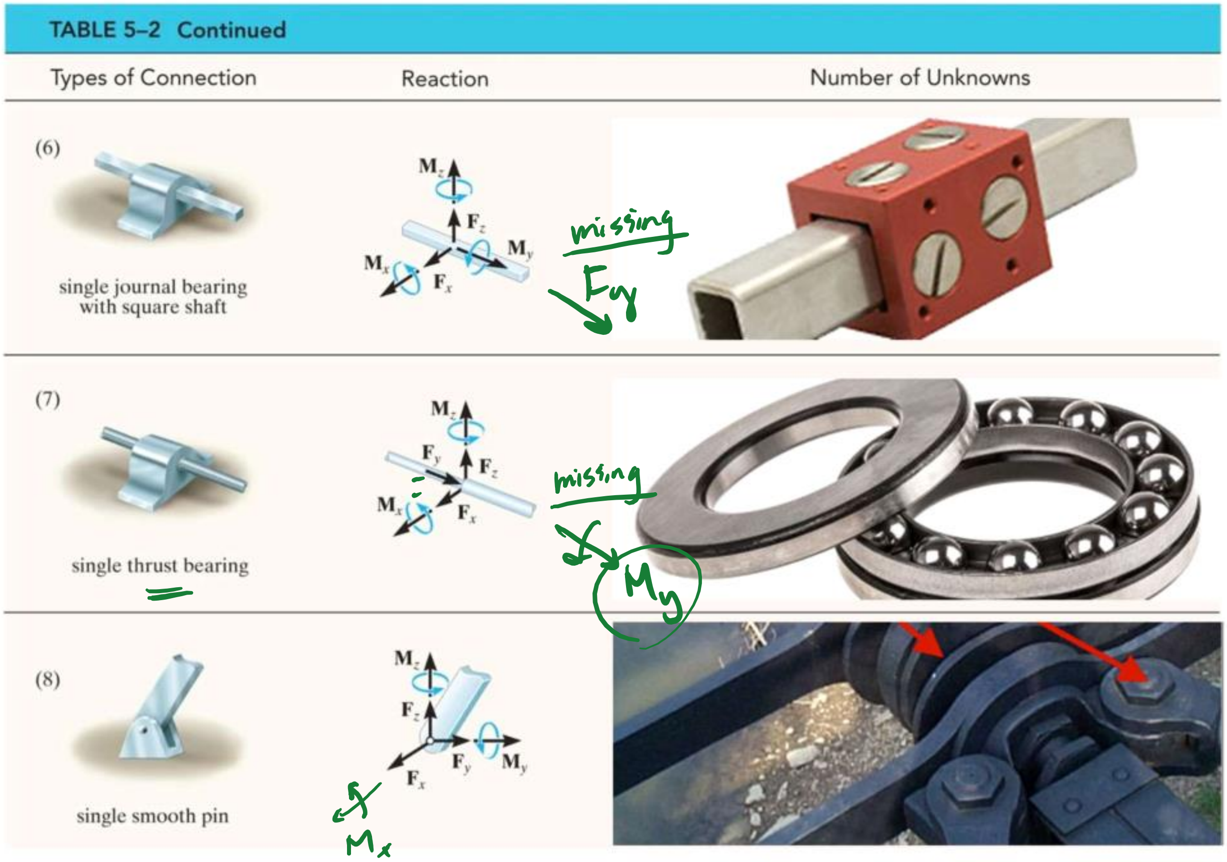

TABLE 5–2 Continued

Types of Connection	Reaction	Number of Unknowns
(6) single journal bearing with square shaft	M_z, F_z, M_y, M_x, F_x	
(7) single thrust bearing	M_z, F_y, F_z, M_x, F_x	
(8) single smooth pin	M_z, F_z, F_y, M_y, F_x	

missing F_y

missing M_y

M_x missing

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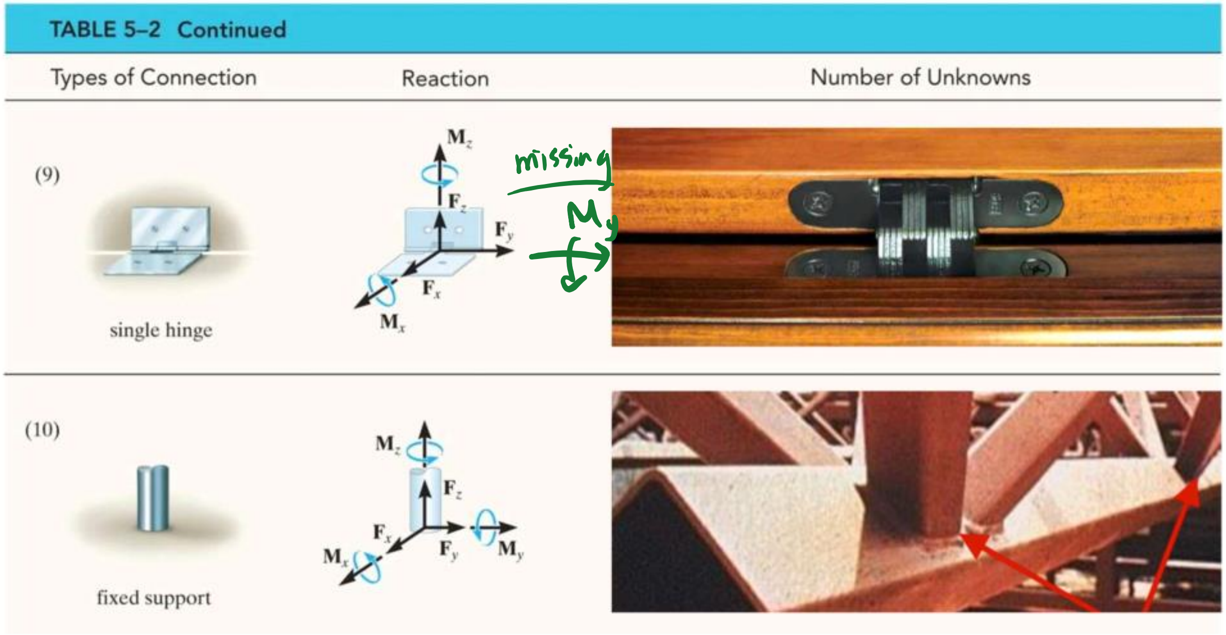

TABLE 5–2 Continued

Types of Connection	Reaction	Number of Unknowns
(9) single hinge	$\mathbf{M}_z$, $\mathbf{F}_z$, $\mathbf{F}_y$, $\mathbf{F}_x$, $\mathbf{M}_x$	
(10) fixed support	$\mathbf{M}_z$, $\mathbf{F}_z$, $\mathbf{F}_x$, $\mathbf{F}_y$, $\mathbf{M}_x$, $\mathbf{M}_y$	

missing

M_y

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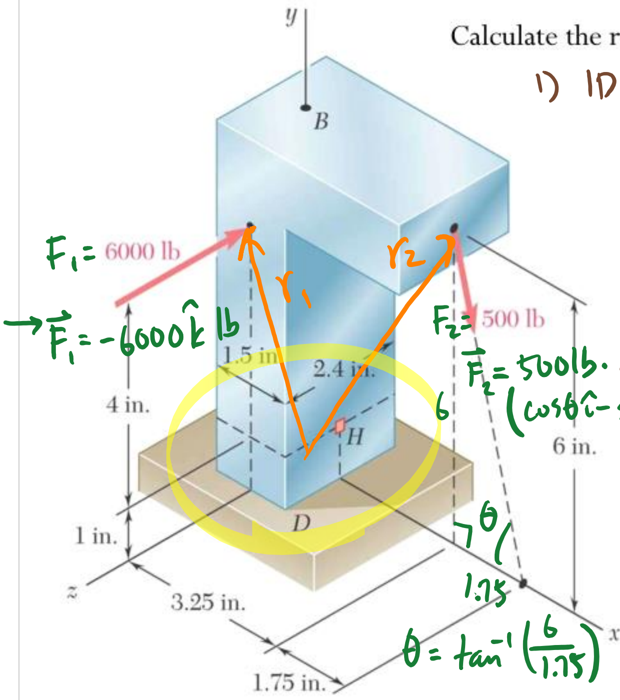
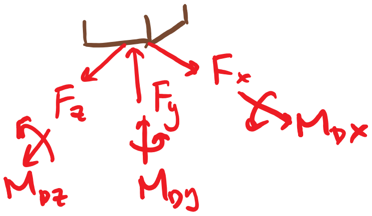
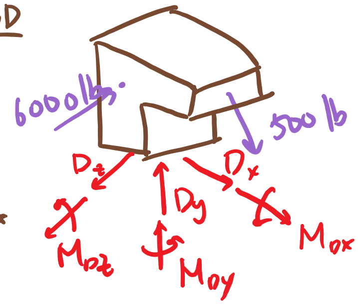
Calculate the reaction forces at the support.

1) 1D Rxn : D is a fixed support.





2) FBD



3) EoE

$\Sigma F_x = 0 = F_2\cos\theta + D_x$

$\Sigma F_y = 0 = -F_2\sin\theta + D_y$

$\Sigma F_z = 0 = -F_1 + D_z$

(assume negligible mass)

$\Rightarrow D_x = -F_2\cos\theta$

$D_y = F_2\sin\theta$

$D_z = F_1$

$\left.\begin{matrix}\Sigma M_{xD}\\ \Sigma M_{yD}\\ \Sigma M_{zD}\end{matrix}\right\}\ \Sigma\vec M_D = \vec r_1\times\vec F_1 + \vec r_2\times\vec F_2 + \vec M_D = 0$

moment arms for F_1 & F_2

$\vec r_1 = (5\hat\jmath + 1.2\hat k)$ in

$\vec r_2 = (3.25\hat\imath + 6\hat\jmath)$ in

$$\vec M_1 = \begin{vmatrix}\hat\imath & \hat\jmath & \hat k\\ 0 & 5 & 1.2\\ 0 & 0 & -6000\end{vmatrix} = -30000\hat\imath \text{ lb}\cdot\text{in}$$

$$\vec M_2 = \begin{vmatrix}\hat\imath & \hat\jmath & \hat k\\ 3.25 & 6 & 0\\ 500c\theta & -500s\theta & 0\end{vmatrix} = [(-1625\sin\theta) - (3000\cos\theta)]\hat k$$

$$\Rightarrow \Sigma\vec M_D = \underbrace{(-30000 + M_{Dx})}_{0}\hat\imath + \underbrace{(M_{Dy})}_{0}\hat\jmath + (\underbrace{-1625\sin\theta - 3000\cos\theta + M_{Dz}}_{0})\hat k$$

$\Rightarrow M_{Dx} = 30000$ lb·in $\Rightarrow M_{Dy} = 0$ lb·in $\Rightarrow M_{Dz} = +1625\sin\theta + 3000\cos\theta$.