Announcements

- Written Exam: Thursday, November 8, 7-8:50pm
  - If you submitted a request for conflict exam via excused absence form and have not received a conflict exam time, contact the course staff team ASAP via Piazza.

- Upcoming deadlines:
- Study for written exam

Vote Early: Illini Union - Room 404

Center of Gravity and Centroid

Goals and Objectives

- Understand the concepts of center of gravity, center of mass, and centroid.
- Be able to determine the location of these points for a body.

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Center of gravity

To design the structure for supporting a water tank, we will need to know the weight of the tank and water as well as the locations where the resultant forces representing these distributed loads act.

How can we determine these resultant weights and their lines of action?

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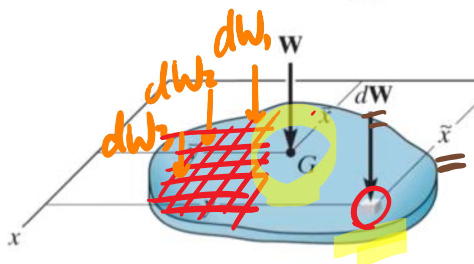
Center of gravity



$$W = \Sigma\, dW_i = \int dW$$

$$M_{Ry} = \Sigma\, \tilde{x}\, dW$$

$$= \int \tilde{x}\, dW$$

← force

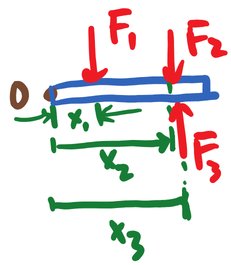
↑ moment arm

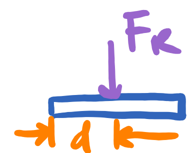
$$\bar{x} = \frac{M_{Ry}}{F_R} = \frac{\int \tilde{x}\, dW}{\int dW}$$

Equivalent Force & Location Review

$$F_R = \Sigma F_i$$

$$M_{RO} = -x_1 F_1 - x_2 F_2 + x_3 F_3$$



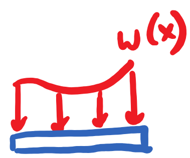


$$d = \frac{M_R}{F_R} = \frac{\Sigma x_i F_i}{F_R}$$

or

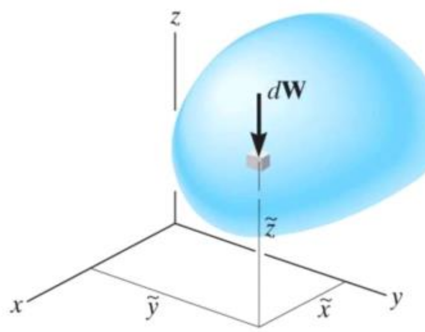
$$= \frac{\int w\, dx}{\int w\, dx}$$

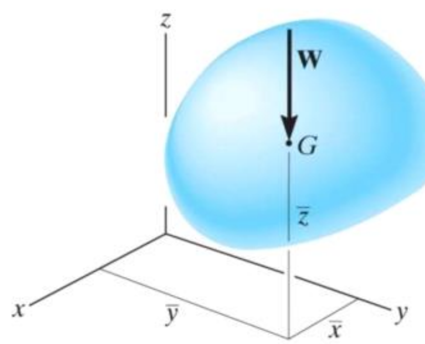
or



$$F_R = \int w\, dx.$$

Center of gravity





A body is composed of an infinite number of particles, and so if the body is located within a gravitational field, then each of these particles will have a weight dW.

The **<u>center of gravity (CG)</u>** is a point, often shown as G, which locates the resultant weight of a system of particles or a solid body.

From the definition of a resultant force, the sum of moments due to individual particle weight about any point is the same as the moment due to the resultant weight located at G.

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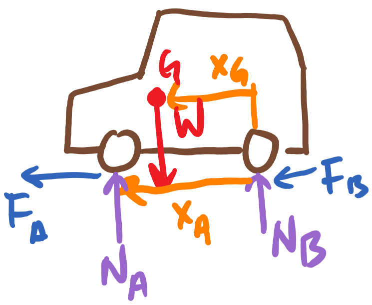
Center of gravity

Is it better to have front wheel or rear wheel drive?

FBD * Note: max friction $F_s = \mu_s N$



EoE

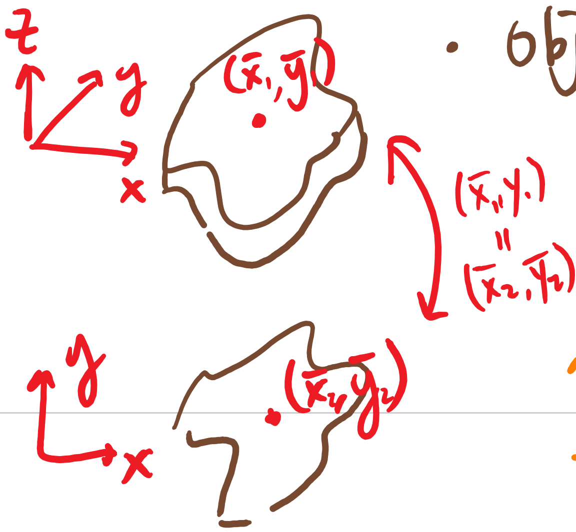
- Find N_A using $\Sigma M_B = +W x_G - N_A x_A = 0$
- Find N_B using $\Sigma F_y = 0 = N_A + N_B - W = 0$.

$$\longrightarrow N_A = \frac{W x_G}{x_A}, \quad N_B = W - N_A = W - W\left(\frac{x_G}{x_A}\right) = W\left(1 - \frac{x_G}{x_A}\right)$$

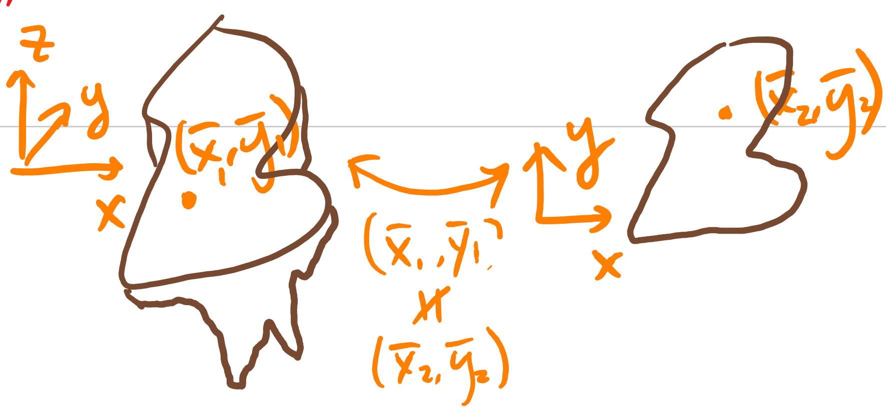
$$\frac{x_G}{x_A} > \left(1 - \frac{x_G}{x_A}\right) \rightarrow N_A > N_B$$

$\therefore F_{sA} > F_{sB}$, front wheels can create more friction.

Center of Area



- Object w/ constant depth will have center of area coincide with center of gravity.



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Center of Mass

"moment" / "force"

$$\bar{x} = \frac{\int \tilde{x}\,dm}{\int dm}$$

$$\bar{y} = \frac{\int \tilde{y}\,dm}{\int dm}$$

$$\bar{z} = \frac{\int \tilde{z}\,dm}{\int dm}$$

Center of Volume

$$\bar{x} = \frac{\int \tilde{x}\,dV}{\int dV}$$

$$\bar{y} = \frac{\int \tilde{y}\,dV}{\int dV}$$

$$\bar{z} = \frac{\int \tilde{z}\,dV}{\int dV}$$

Center of Area

$$\bar{x} = \frac{\int \tilde{x}\,dA}{\int dA}$$

$$\bar{y} = \frac{\int \tilde{y}\,dA}{\int dA}$$

$$\bar{z} = \frac{\int \tilde{z}\,dA}{\int dA}$$

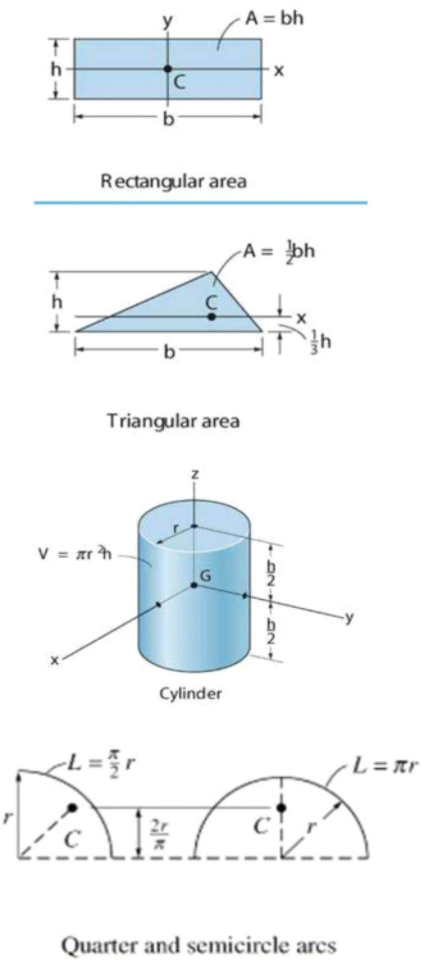
Centroid

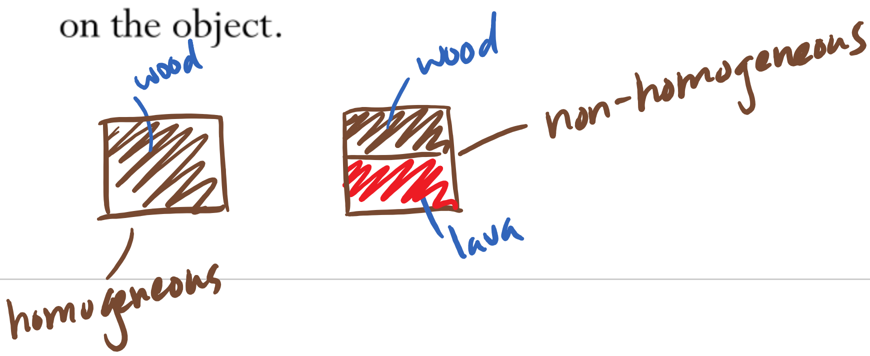
The centroid, C, is a point defining the geometric center of an object.

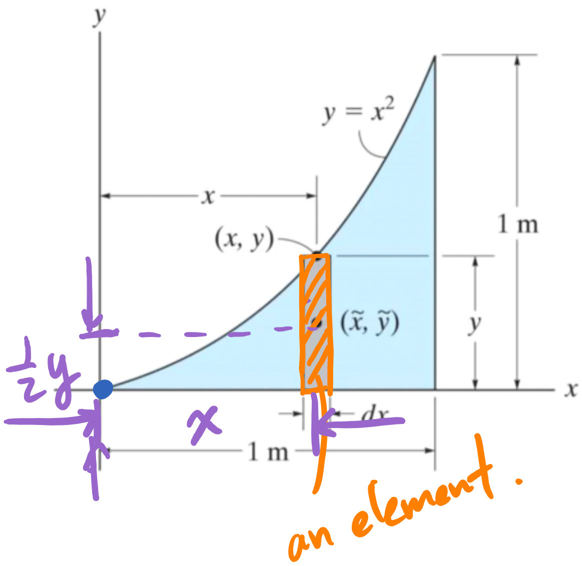
The centroid coincides with the center of mass or the center of gravity only if the material of the body is homogeneous (density or specific weight is constant throughout the body).

If an object has an axis of symmetry, then the centroid of object lies on that axis.

In some cases, the centroid may not be located on the object.







Locate the centroid of the area.

$$\bar{x} = \frac{\int \tilde{x}\, dA}{\int dA}, \quad \bar{y} = \frac{\int \tilde{y}\, dA}{\int dA}$$

$\tilde{x}$ = x-centroid location of the element

$\tilde{y}$ = y-centroid location of the element

dA = area of the element.

* Use rectangle as the shape of an element because we know $(\tilde{x}, \tilde{y})$ and dA.

$b = dx$, $h = y = x^2$

$\rightarrow dA = y\,dx$ or $x^2 dx$

$\tilde{x} = x$

$\tilde{y} = \frac{1}{2}y = \frac{1}{2}x^2$

$$\bar{x} = \frac{\int_0^1 x\,(x^2 dx)}{\int_0^1 x^2 dx}$$

$$\bar{y} = \frac{\int (\frac{1}{2}x^2)(x^2 dx)}{\int_0^1 x^2 dx}$$

$\frac{x^2}{2} = \frac{y}{2} = \frac{h}{2}$