Announcements

- Written exam tomorrow (Thursday, Nov 8) – do you know your room assignment?

❑ Upcoming deadlines:

- Study for written exam

Center of Gravity

$$\bar{x} = \frac{\int \tilde{x}\, dW}{\int dW}$$

$$\bar{y} = \frac{\int \tilde{y}\, dW}{\int dW}$$

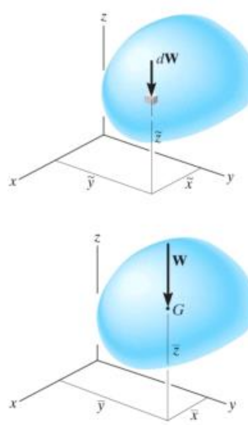
$$\bar{z} = \frac{\int \tilde{z}\, dW}{\int dW}$$

Center of Volume

$$\bar{x} = \frac{\int \tilde{x}\, dV}{\int dV}$$

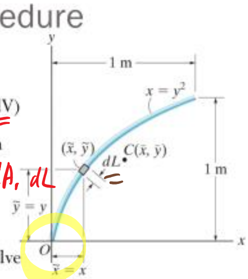
$$\bar{y} = \frac{\int \tilde{y}\, dV}{\int dV}$$

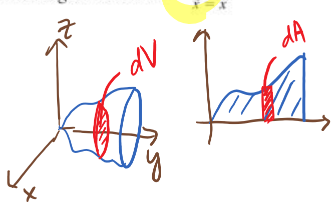
$$\bar{z} = \frac{\int \tilde{z}\, dV}{\int dV}$$



Centroid – Analysis Procedure

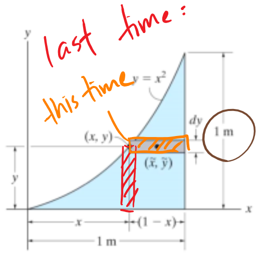
1. Select an appropriate coordinate system
2. Define the appropriate element (dL, dA, or dV)
3. Express (2) in terms of the coordinate system
4. Identify any symmetry $(\tilde{x}, \tilde{y}, \tilde{z})$, dV, dA, dL
5. Express the moment arms (centroid) of (2)
6. Substitute (3) and (4) into the integral and solve



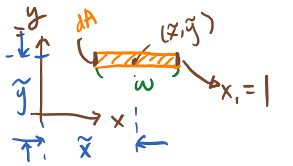


last time:

this time



Locate the centroid of the area.



$$\bar{x} = \frac{\int \tilde{x}\,dA}{\int dA}$$

$$\rightarrow \bar{x} = \frac{\int_0^1 \frac{1}{2}(1+\sqrt{y})(1-\sqrt{y})\,dy}{\int_0^1 (1-\sqrt{y})\,dy}$$

$$\bar{y} = \frac{\int \tilde{y}\,dA}{\int dA}$$

$\tilde{x}$ = $\frac{1}{2}$ way between x_1 & x_2

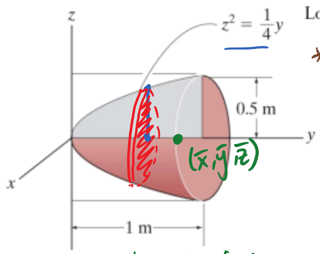
$= \frac{1}{2}(x_1+x_2) = \frac{1}{2}(1+\sqrt{y})$

$y = y$

$dA = wh = (x_1 - x_2)\,dy = (1-\sqrt{y})\,dy$

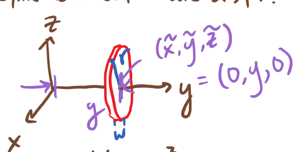
w = difference between x_1 & x_2

$$\bar{y} = \frac{\int \tilde{y}\,dA}{\int dA} = \frac{\int_0^1 y(1-\sqrt{y})\,dy}{\int_0^1 (1-\sqrt{y})\,dy}$$



Locate the centroid of the volume.

* Define element: use disks.



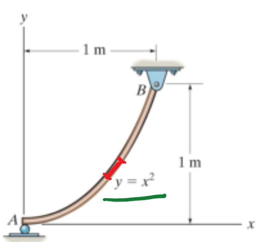
$dV = \pi r^2 w$

$r = z = \sqrt{\frac{1}{4}y}$ $\quad w = dy$

* Use symmetry to find $\bar{x}, \bar{y}, \bar{z}$: $\bar{x} = 0$, $\bar{z} = 0$

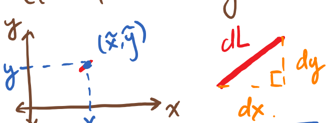
(make sure the shape is homogeneous) $\rightarrow dV = \pi\left(\sqrt{\frac{1}{4}y}\right)^2 dy$

$$\bar{y} = \frac{\int \tilde{y}\,dV}{\int dV} = \frac{\int_0^1 y\left[\pi\left(\sqrt{\frac{1}{4}y}\right)^2\right]dy}{\int_0^1 \left[\pi\left(\sqrt{\frac{1}{4}y}\right)^2\right]dy}$$



Locate the center of gravity of the homogeneous rod. If the rod has a weight per unit length of 100 N/m, determine the reaction supports at A and B.

Element = line segment.



$dL = \sqrt{dx^2 + dy^2}$

$\left(dx: \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \right.$

$\left(dy: \sqrt{\left(\frac{dx}{dy}\right)^2+1}\,dy \right.$

$\frac{dy}{dx}$: differentiate y with respect to x

$$\bar{x} = \frac{\int \tilde{x}\,dL}{\int dL} = \frac{\int x\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx}{\int \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx}$$

$\frac{dy}{dx} = \frac{d}{dx}(x^2) = 2x$

$$\bar{x} = \frac{\int_0^1 x\sqrt{1+(2x)^2}\,dx}{\int_0^1 \sqrt{1+(2x)^2}\,dx} = [m]$$

$$\bar{y} = \frac{\int \tilde{y}\,dL}{\int dL}.$$

① Use dx.

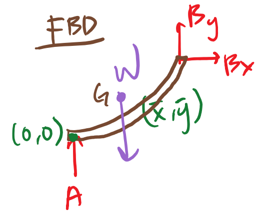
$$\bar{y} = \frac{\int_0^1 y\sqrt{1+(2x)^2}\,dx}{\int_0^1 \sqrt{1+(2x)^2}\,dx}$$

$$= \frac{\int_0^1 (x^2)\sqrt{1+(2x)^2}\,dx}{\int_0^1 \sqrt{1+(2x)^2}\,dx} = \int dL = L$$

② Use dy, $x = \sqrt{y}$, $\frac{dx}{dy} = \frac{1}{2}y^{-1/2}$

$$\bar{y} = \frac{\int_0^1 y\sqrt{\left(\frac{dx}{dy}\right)^2+1}\,dy}{\int_0^1 \sqrt{\left(\frac{dx}{dy}\right)^2+1}\,dy} = \frac{\int_0^1 y\sqrt{\left(\frac{1}{2}y^{-1/2}\right)^2+1}\,dy}{\int_0^1 \sqrt{\left(\frac{1}{2}y^{-1/2}\right)^2+1}\,dy} = \int dL = L$$

FBD



$W = ? = \left(100\frac{N}{m}\right)(L)$

$= \left(100\frac{N}{m}\right)\left(\int dL\right)$

EoE

$\Sigma M_B = W(1-\bar{x}) - A(1\,m) = 0$

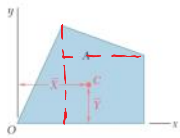
$\Sigma F_y = 0 = B_y + A - W = 0$

$\Sigma F_x = 0 = B_x$

Composite bodies

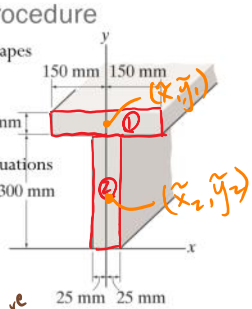
A composite body consists of a series of connected simpler shaped bodies.

Such body can be sectioned or divided into its composite parts and, provided the weight and location of the center of gravity of each of these parts are known, we can then eliminate the need for integration to determine the center of gravity of the entire body.



Composite bodies – Analysis Procedure

1. Divide the body into finite number of simple shapes
2. Consider "holes" as "negative" parts
3. Establish coordinate axes
4. Determine centroid location by applying the equations



$$\bar{x} = \frac{\sum \tilde{x}W}{\sum W} = \frac{\tilde{x}_1 W_1 + \tilde{x}_2 W_2}{W_1 + W_2}$$

$$\bar{y} = \frac{\sum \tilde{y}W}{\sum W} = \frac{\tilde{y}_1 W_1 + \tilde{y}_2 W_2}{W_1 + W_2}$$

if there are only 2 parts.

$$\bar{z} = \frac{\sum \tilde{z}W}{\sum W} = \frac{\tilde{z}_1 W_1 + z_2 W_2}{W_1 + W_2}$$

Example

Locate the centroid of the cross section area.

$$\bar{x} = \frac{\tilde{x}_1 L_1 + \tilde{x}_2 L_2 + \tilde{x}_3 L_3 + x_4 L_4}{L_1 + L_2 + L_3 + L_4}$$

	1	2	3	4
$\tilde{x}$	100 mm	200 mm	100 mm	0
L	$200\sqrt{2}$ mm	100 mm	200 mm	300 mm