Announcements

- You survived the written exam! Enjoy your weekend ☺

❑ Upcoming deadlines:

- Tuesday (11/13)
  - PL HW

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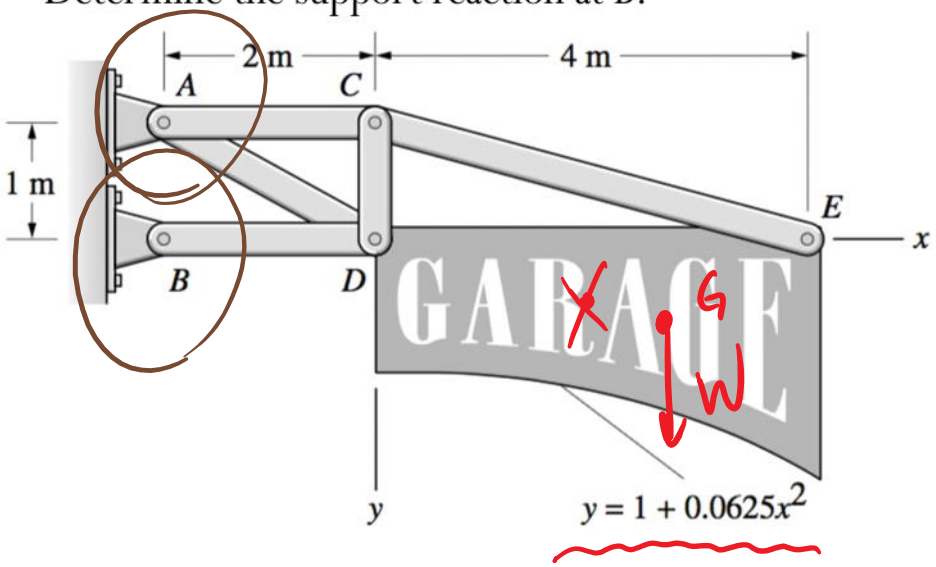
Center of Gravity Application

The suspended sign is a homogeneous flat plate that has a mass of 130 kg. Determine the support reaction at *B*.



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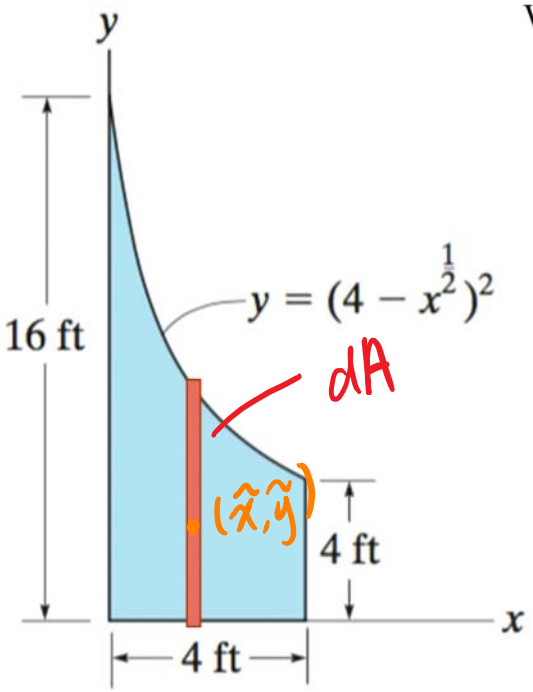
Example



Where is the centroid of the area?

dL, dV, dm, dW

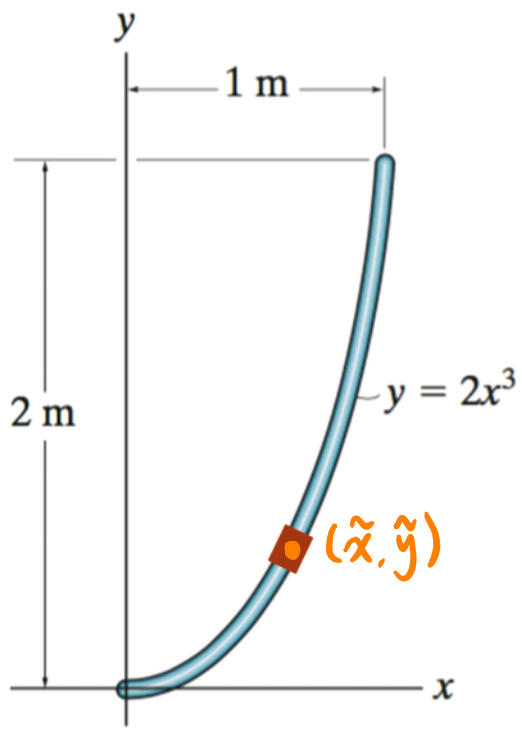
$$\bar{x} = \frac{\int \tilde{x}\, dA}{\int dA}$$

$$\bar{y} = \frac{\int \tilde{y}\, dA}{\int dA}$$

$\tilde{x} = x$, $\tilde{y} = y = (4 - \sqrt{x})^2$

$dA = (4 - \sqrt{x})^2 dx$

Example



Where is the centroid of the bar?

$\tilde{x} = x$

$\tilde{y} = y = 2x^3$

$dL = \sqrt{dx^2 + dy^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$= \sqrt{1 + (6x^2)^2}\, dx$

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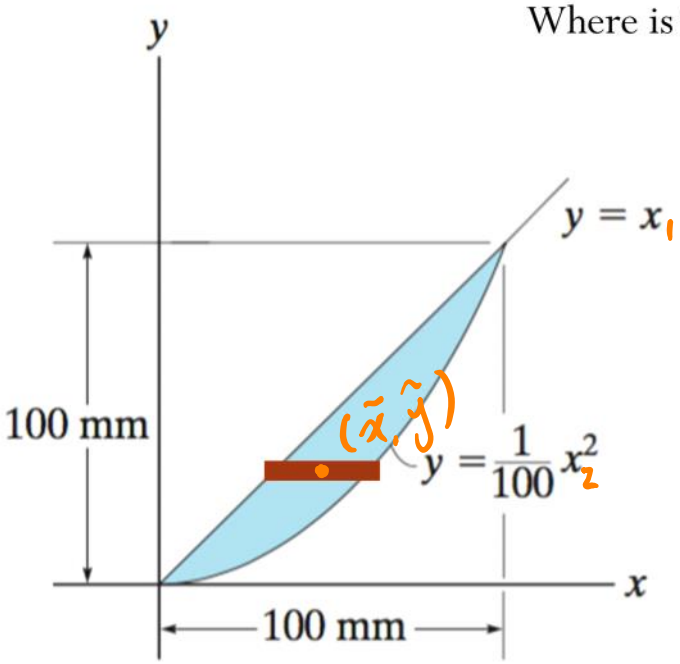
Example

Where is the centroid of the area?



$\tilde{x} = \frac{1}{2}(x_1 + x_2)$

$= \frac{1}{2}(y + \sqrt{100y})$

$\tilde{y} = y$

$dA = (\sqrt{100y} - y)\,dy$

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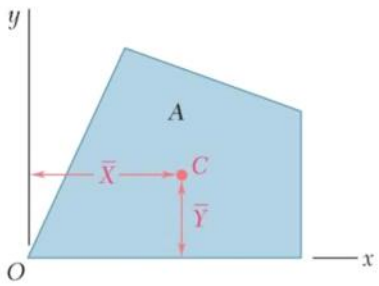
Composite bodies

A composite body consists of a series of connected simpler shaped bodies.

Such body can be sectioned or divided into its composite parts and, provided the weight and location of the center of gravity of each of these parts are known, we can then eliminate the need for integration to determine the center of gravity of the entire body.



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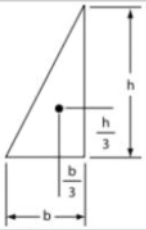
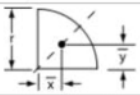
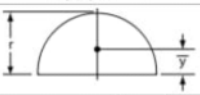
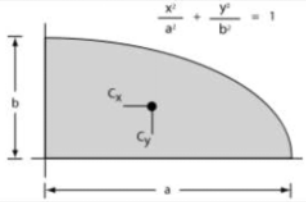
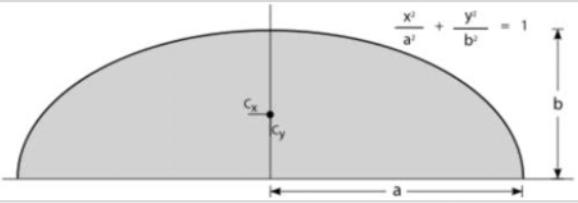
Centroid of typical 2D shapes

Shape	Figure	$\bar{x}$	$\bar{y}$	Area
Right-triangular area		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$

http://en.wikipedia.org/wiki/List_of_centroids

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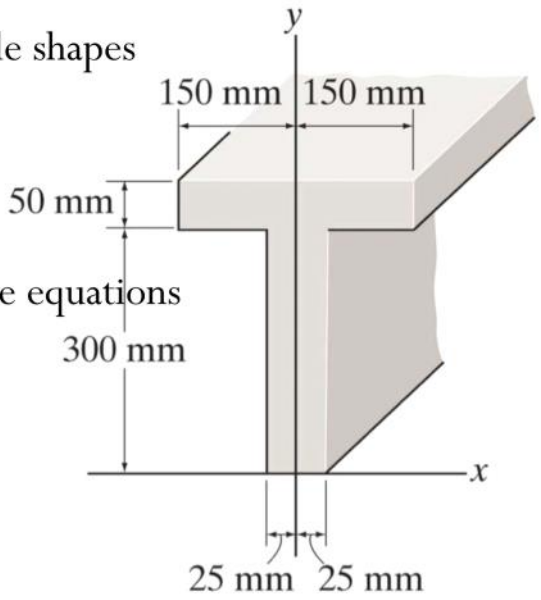
Composite bodies – Analysis Procedure

1. Divide the body into finite number of simple shapes
2. Consider "holes" as "negative" parts
3. Establish coordinate axes
4. Determine centroid location by applying the equations

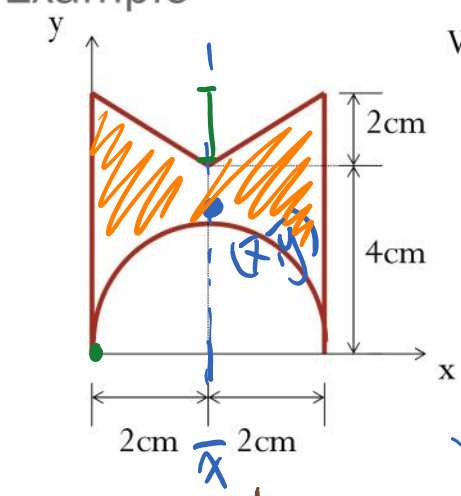


$$\bar{x} = \frac{\sum \tilde{x}W}{\sum W} \qquad \bar{x} = \frac{\sum \tilde{x}A}{\sum A}$$

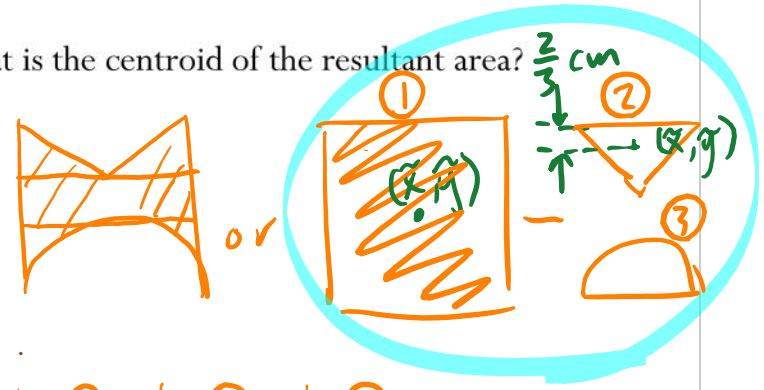
$$\bar{y} = \frac{\sum \tilde{y}W}{\sum W} \qquad \bar{y} = \frac{\sum \tilde{y}A}{\sum A}$$

$$\bar{z} = \frac{\sum \tilde{z}W}{\sum W} \qquad \bar{z} = \frac{\sum \tilde{z}A}{\sum A}$$

Example



What is the centroid of the resultant area?

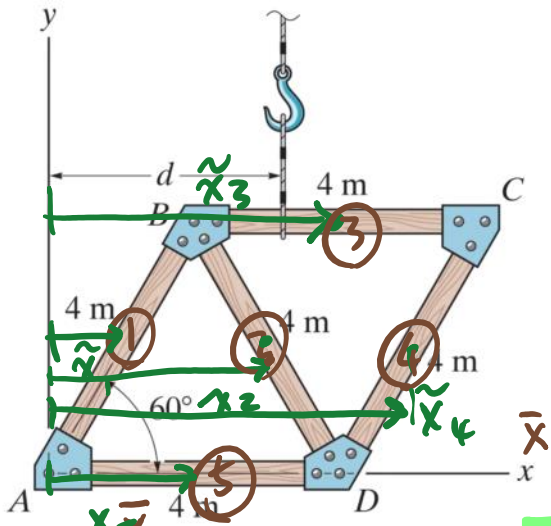


- From symmetry

$\bar{x} = 2\,cm$

	1	2	3
$\tilde{x}$			
$\tilde{y}$	$3\,cm$	$(6-\frac{3}{3})cm$	$\frac{4(2)}{3\pi}cm$
A	$24\,cm^2$	$-4\,cm^2$	$-2\pi\,cm^2$

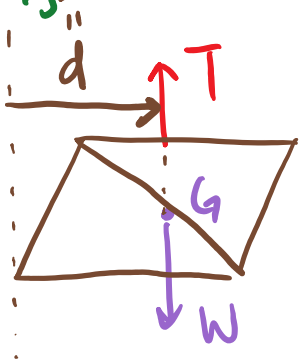
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{(3\,cm)(24\,cm^2) + (6-\frac{2}{3}cm)(-4\,cm^2) + (\frac{8}{3\pi}cm)(-2\pi\,cm^2)}{(24-4-2\pi)\,cm^2}$$



The truss is made from five members, each having a length of 4 m and a mass of 7 kg/m. Determine the distance d to where the hoisting cable must be attached, so that the truss does not tip (rotate) when it is lifted.

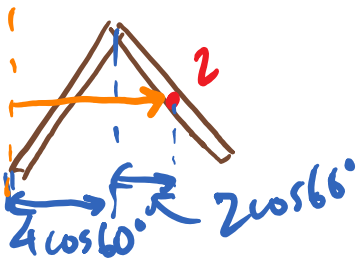
$$\bar{x} = \frac{\Sigma \tilde{x}_i m_i}{\Sigma m_i}, \quad m = \left(7 \frac{kg}{m}\right)(4m) = 28\ kg \text{ (per member)}$$

	1	2	3	4	5	
$\tilde{x}$	$2\cos 60°$	$6\cos 60°$	$4\cos 60° + 2$	$4 + 2\cos 60°$	2	[m]



$$\Sigma m = (28\ kg)(5) = 140\ kg$$

$$\bar{x} = \frac{(1m)(28\ kg) + (3m)(28\ kg) + (4m)(28\ kg) + (5m)(28\ kg) + (2m)(28\ kg)}{140 kg}$$



$$\bar{x} = 3m$$