## Announcements

- CBTF Quiz 5 starts tomorrow (Thu, 11/29)

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Q0: Example Quiz
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Total points: 0/60

## 0\%

Assessment is closed and you cannot answer questions.

For this practice quiz you can use this Centroid and Moment of Inertia Table.

## Question

Best submission (3)
Available points (3)
Awarded points ?
Question 1
Question 2
Question 3
Question 4

| unanswered |
| :---: |
| unanswered |
| unanswered |
| unanswered |

$0 / 10$
$0 / 10$
$0 / 10$
$0 / 10$
$\square$ Upcoming deadlines:

- Friday (11/30)
- Written Assignment
- Tuesday (12/4)
- PL HW



## Parallel axis theorem

- Often, the moment of inertia of an area is known for an axis passing through the centroid; e.g., $x$ ' and $y^{\prime}$ :
- The moments around other axes can be computed from the known $I_{x^{\prime}}$ and $I_{y}$ :


Note: the integral over y gives zero when done through the centroid axis.

## Moment of inertia of composite

- If individual bodies making up a composite body have individual areas $A$ and moments of inertia I computed through their centroids, then the composite area and moment of inertia is a sum of the individual component contributions.
- This requires the parallel axis theorem:


| Rectangle |  | $\begin{aligned} & \bar{I}_{x^{\prime}}=\frac{1}{12} b h^{3} \\ & \bar{I}_{y^{\prime}}=\frac{1}{12} b h \\ & I_{x}=\frac{1}{3} b h^{3} \\ & I_{y}=\frac{1}{3} b^{3} h \\ & J_{C}=\frac{1}{12} b h\left(b^{2}+h^{2}\right) \end{aligned}$ |
| :---: | :---: | :---: |
| Triangle |  | $\begin{gathered} \bar{I}_{x^{\prime}}=\frac{1}{36} b h^{3} \\ I_{x}=\frac{1}{12} b h^{3} \end{gathered}$ |
| Circle |  | $\begin{aligned} & \bar{I}_{x}=\bar{I}_{y}=\frac{1}{4} \pi r^{4} \\ & J_{O}=\frac{1}{2} \pi r^{4} \end{aligned}$ |
| Semicircle |  | $\begin{aligned} & I_{x}=I_{y}=\frac{1}{8} \pi r^{4} \\ & J_{O}=\frac{1}{4} \pi r^{4} \end{aligned}$ |
| Quarter circle |  | $\begin{aligned} & I_{x}=I_{y}=\frac{1}{16} \pi r^{4} \\ & J_{O}=\frac{1}{8} \pi r^{4} \end{aligned}$ |
| Ellipse |  | $\begin{aligned} & \bar{I}_{x}=\frac{1}{4} \pi a b^{3} \\ & \bar{I}_{y}=\frac{1}{4} \pi a^{3} b \\ & J_{O}=\frac{1}{4} \pi a b\left(a^{2}+b^{2}\right) \end{aligned}$ |

Find the moment of inertia about its centroid:


$$
\bar{Y}=\frac{4 t^{2}(3.5 t)+6 t^{2}(1.5 t)}{4 t^{2}+6 t^{2}}=\frac{23 t}{10}
$$

Determine the moment of inertia for the cross-sectional area about the $x$ and $y$ centroidal axes.

$100 \mathrm{~mm}-$



Two channels are welded to a rolled W section as shown. Determine the moments of inertia of the combined section with respect to the centroidal x and y axes.


|  | Designation | Area $\mathrm{in}^{2}$ | Depth in. | Width in. | Ads X-X |  |  | Aris $Y$ - $Y$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | $\overline{I_{x}}, \mathrm{in}^{4}$ | $\bar{k}_{x}$, in. | $\bar{y}, \mathrm{in}$. | $\bar{I}_{5}, \mathrm{fn}^{4}$ | $\bar{k}_{\text {y }}, \mathrm{in}$. | $\bar{x}, \mathrm{in}$. |
| W Shapes (Wide-Flange Shapes) | $\begin{aligned} & \text { W } 18 \times 76 \dagger \\ & \text { W } 16 \times 57 \\ & \text { W14 } \times 38 \\ & \text { W } 8 \times 31 \end{aligned}$ | $\begin{gathered} 22.3 \\ 16.8 \\ 11.2 \\ 9.12 \end{gathered}$ | 182 <br> 16.4 <br> 14.1 <br> 800 | $\begin{gathered} 11.0 \\ 7.12 \\ 6.77 \\ 8.00 \end{gathered}$ | $\begin{array}{r} 1330 \\ 758 \\ 385 \\ 110 \end{array}$ | $\begin{aligned} & 7.73 \\ & 6.72 \\ & 5.87 \\ & 3.47 \end{aligned}$ |  | $\begin{aligned} & 152 \\ & 43.1 \\ & 26.7 \\ & 37.1 \end{aligned}$ | $\begin{aligned} & 2.61 \\ & 1.60 \\ & 1.55 \\ & 2.02 \end{aligned}$ |  |
| S Shapes <br> (American Standard Shapes) | $\begin{aligned} & \mathrm{S} 18 \times 54.7 \dagger \\ & \mathrm{~S} 12 \times 31.8 \\ & \mathrm{~S} 10 \times 25.4 \\ & \mathrm{~S} 6 \times 12.5 \end{aligned}$ | $\begin{gathered} 16.0 \\ 9.31 \\ 7.45 \\ 3.66 \end{gathered}$ | $\begin{gathered} 180 \\ 12.0 \\ 10.0 \\ 6.00 \end{gathered}$ | $\begin{aligned} & 6.00 \\ & 5.00 \\ & 4.66 \\ & 3.33 \end{aligned}$ | $\begin{aligned} & 801 \\ & 217 \\ & 123 \\ & 22.0 \end{aligned}$ | $\begin{aligned} & 7.07 \\ & 4.83 \\ & 4.07 \\ & 2.45 \end{aligned}$ |  | $\begin{gathered} 20.7 \\ 9.33 \\ 6.73 \\ 1.80 \end{gathered}$ | $\begin{aligned} & 1.14 \\ & 1.00 \\ & 0.980 \\ & 0.702 \end{aligned}$ |  |
| C Shapes (American Standard Channels) | $\begin{aligned} & \mathrm{C} 12 \times 20.7 \dagger \\ & \mathrm{C} 10 \times 15.3 \\ & \mathrm{C} 8 \times 11.5 \\ & \mathrm{C} 6 \times 8.2 \end{aligned}$ | $\begin{aligned} & 6.08 \\ & 4.48 \\ & 3.37 \\ & 2.39 \end{aligned}$ | $\begin{gathered} 12.0 \\ 10.0 \\ 8.00 \\ 6.00 \end{gathered}$ | $\begin{aligned} & 2.94 \\ & 2.60 \\ & 2.26 \\ & 1.92 \end{aligned}$ | $\begin{gathered} 129 \\ 67.3 \\ 32.5 \\ 13.1 \end{gathered}$ | $\begin{aligned} & 4.61 \\ & 3.87 \\ & 3.11 \\ & 2.34 \end{aligned}$ |  | $\begin{aligned} & 3.86 \\ & 2.27 \\ & 1.31 \\ & 0.687 \end{aligned}$ | $\begin{aligned} & 0.797 \\ & 0.711 \\ & 0.623 \\ & 0.536 \end{aligned}$ | $\begin{aligned} & 0.698 \\ & 0.634 \\ & 0.572 \\ & 0.512 \end{aligned}$ |
|  | $\begin{aligned} & \mathrm{L} 6 \times 6 \times \mathrm{l} \ddagger \\ & \mathrm{~L} 4 \times 4 \times \frac{1}{2} \\ & \mathrm{~L} 3 \times 3 \times \frac{1}{4} \\ & \mathrm{~L} 6 \times 4 \times \frac{1}{2} \\ & \mathrm{~L} 5 \times 3 \times \frac{1}{2} \\ & \mathrm{~L} 3 \times 2 \times \frac{1}{4} \end{aligned}$ | $\begin{gathered} 11.0 \\ 3.75 \\ 1.44 \\ 4.75 \\ 3.75 \\ 1.19 \end{gathered}$ |  |  | 36.4 <br> 5. 52 <br> 1.23 <br> 17.3 <br> 243 <br> 1.09 | $\begin{aligned} & 1.79 \\ & 1.21 \\ & 0.926 \\ & 1.91 \\ & 1.58 \\ & 0.863 \end{aligned}$ | $\begin{aligned} & 1.86 \\ & 1.18 \\ & 0.836 \\ & 1.98 \\ & 1.74 \\ & 0.980 \end{aligned}$ | $3 \%$. <br> 5.52 <br> 1.23 <br> 6.22 <br> 2.55 <br> 0.390 | $\begin{aligned} & 1.79 \\ & 1.21 \\ & 0.926 \\ & 1.14 \\ & 0.824 \\ & 0.569 \end{aligned}$ | $\begin{aligned} & 1.86 \\ & 1.18 \\ & 0.836 \\ & 0.981 \\ & 0.746 \\ & 0.487 \end{aligned}$ |


|  | $\begin{array}{ll}\text { Designation } & \begin{array}{c}\text { Area } \\ \mathrm{mm}^{2}\end{array}\end{array}$ |  | Ads X-X |  |  | Axis $Y$ - $Y$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{array}{cc}\text { Depth } & \text { Width } \\ \mathrm{mm} & \mathrm{mm}\end{array}$ | $\begin{gathered} \bar{I}_{\mathrm{x}} \\ 10^{0} \mathrm{~mm}^{4} \end{gathered}$ | $\begin{gathered} \bar{k}_{x} \\ \mathrm{~mm} \end{gathered}$ | $\underset{\mathrm{mm}}{\bar{y}}$ | $\begin{gathered} \bar{I}_{y} \\ 10^{0} \mathrm{~mm}^{4} \end{gathered}$ | $\begin{gathered} \bar{k}_{y} \\ \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \bar{x} \\ \mathrm{~mm} \end{gathered}$ |
| W Shapes (Wide-Flange Shapes) | W460 $\times 113 \dagger$ 14400 <br> W410 $\times 85$ 10800 <br> $W 360 \times 57.8$ 7230 <br> W $200 \times 46.1$ 5890 | 462 279 <br> 417 181 <br> 358 172 <br> 203 203 | $\begin{gathered} 554 \\ 316 \\ 160 \\ 45.8 \end{gathered}$ | $\begin{gathered} 196 \\ 171 \\ 149 \\ 88.1 \end{gathered}$ |  | $\begin{aligned} & 63.3 \\ & 17.9 \\ & 11.1 \\ & 15.4 \end{aligned}$ | $\begin{aligned} & 66.3 \\ & 40.6 \\ & 39.4 \\ & 51.3 \end{aligned}$ |  |
| S Shapes (American Standard Shapes) | S460 $\times 81.4 \dagger$ 10300 <br> S310 647.3 <br> S $250 \times 37.8$ 6010 <br> S $150 \times 18.6$ 2360 | 457 152 <br> 305 127 <br> 254 118 <br> 152 84.6 | 333 <br> 90.3 <br> 51.2 <br> 9.16 | $\begin{gathered} 180 \\ 123 \\ 103 \\ 62.2 \end{gathered}$ |  | $\begin{aligned} & 8.62 \\ & 3.88 \\ & 2.80 \\ & 0.749 \end{aligned}$ | $\begin{aligned} & 29.0 \\ & 25.4 \\ & 24.1 \\ & 17.8 \end{aligned}$ |  |
| C Shapes (American Standard Channels) | C310 $\times 30.81$ 3920 <br> C250 $\times 22.8$ 2590 <br> C200 $\times 17.1$ 2170 <br> C150 $\times 12.2$ 1540 | 305 74.7 <br> 254 66.0 <br> 203 57.4 <br> 152 48.8 | $\begin{gathered} 53.7 \\ 28.0 \\ 13.5 \\ 5.45 \end{gathered}$ | $\begin{gathered} 117 \\ 98.3 \\ 79.0 \\ 59.4 \end{gathered}$ |  | $\begin{aligned} & 1.61 \\ & 0.945 \\ & 0.845 \\ & 0.256 \end{aligned}$ | $\begin{aligned} & 20.2 \\ & 18.1 \\ & 15.8 \\ & 13.6 \end{aligned}$ | $\begin{aligned} & 17.7 \\ & 16.1 \\ & 14.5 \\ & 13.0 \end{aligned}$ |
|  | $\mathrm{L} 152 \times 152 \times 25.4 \ddagger$ 7100 <br> $\mathrm{~L} 102 \times 102 \times 12.7$ 2420 <br> $\mathrm{~L} 76 \times 76 \times 6.4$ 929 <br> $\mathrm{~L} 152 \times 102 \times 12.7$ 3060 <br> $\mathrm{~L} 127 \times 76 \times 12.7$ 2420 <br> $\mathrm{~L} 76 \times 51 \times 6.4$ 768 |  | 14.7 <br> 2.30 <br> 0.512 <br> 7.20 <br> 3.98 <br> 0.454 | $\begin{aligned} & 45.5 \\ & 30.7 \\ & 23.5 \\ & 48.5 \\ & 40.1 \\ & 24.2 \end{aligned}$ | $\begin{aligned} & 47.2 \\ & 30.0 \\ & 21.2 \\ & 50.3 \\ & 44.2 \\ & 24.9 \end{aligned}$ | $\begin{aligned} & 14.7 \\ & 2.30 \\ & 0.512 \\ & 2.59 \\ & 1.06 \\ & 0.162 \end{aligned}$ | $\begin{aligned} & 45.5 \\ & 30.7 \\ & 23.5 \\ & 29.0 \\ & 20.9 \\ & 14.5 \end{aligned}$ | $\begin{aligned} & 47.2 \\ & 30.0 \\ & 21.2 \\ & 24.9 \\ & 18.9 \\ & 12.4 \end{aligned}$ |

