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Announcements

- CBTF Quiz 5 starts tomorrow (Thu, 11/29)

Q0: Example Quiz

Total points: 0/60 0% Assessment is **closed** and you cannot answer questions.

For this practice quiz you can use this Centroid and Moment of Inertia Table.

Question	Best submission	Available points	Awarded points
Question 1	unanswered	—	0 /10
Question 2	unanswered	—	0 /10
Question 3	unanswered	—	0 /10
Question 4	unanswered	—	0 /10

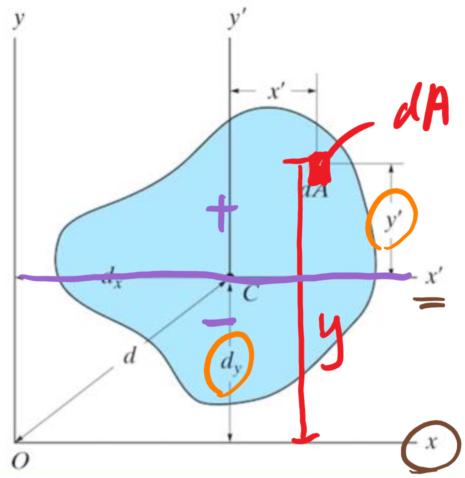
❑ Upcoming deadlines:

- Friday (11/30)
  - Written Assignment
- Tuesday (12/4)
  - PL HW

Parallel axis theorem

- Often, the **moment of inertia** of an area is known for an axis passing through the **centroid**; e.g., *x'* and *y'*:
- The moments around other axes can be computed from the known $I_{x'}$ and $I_{y'}$:

$$I_x = I_{x'} + d_y^2 A.$$



Note: the integral over y' gives zero *when done through the centroid axis.*

Derivation:

$$I_x = \int y^2\, dA, \quad y = d_y + y'$$

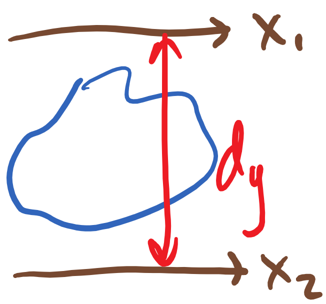
$$= \int (d_y + y')^2\, dA.$$

$$= \int (d_y^2 + 2 d_y y' + y'^2)\, dA.$$

$$= \underbrace{\int d_y^2\, dA}_{d_y^2 \int dA \,=\, d_y^2 A} + \underbrace{2 \int d_y y'\, dA}_{?\;\; 2 d_y \int y'\, dA \,=\, 0} + \underbrace{\int y'^2\, dA}_{I_{x'}} = I_{x'} + d_y^2 A$$

d_y is a constant.

- Does not apply to 2 parallel axes that are both non-centroidal.



$$I_{x2} \neq I_{x1} + d_y^2 A$$

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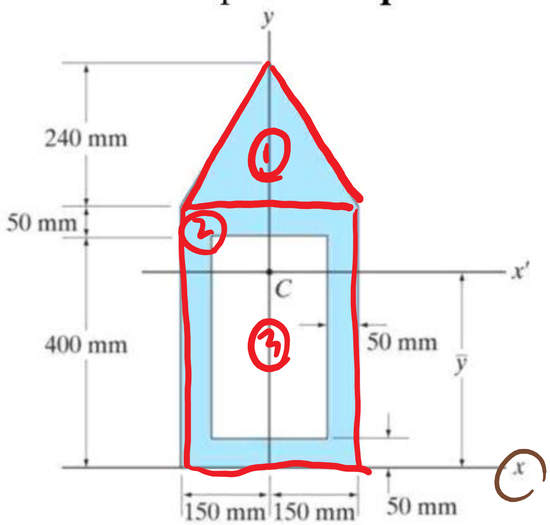
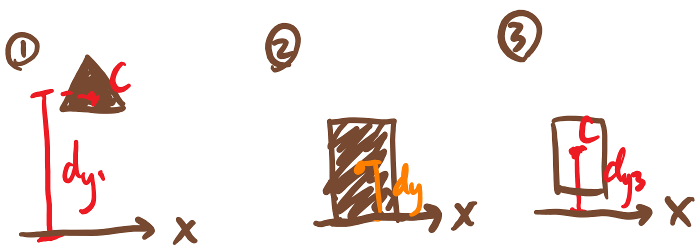
Moment of inertia of composite

- If individual bodies making up a **composite** body have individual areas *A* and moments of inertia *I* computed through their centroids, then the **composite area** and **moment of inertia** is a sum of the individual component contributions. $I_x = I_{x1} + I_{x2} + I_{x3} \cdots = \Sigma I_{xi}$
- This requires the **parallel axis theorem**: (subtract out hollow shapes) or negative area



$I_x = I_{x1} + I_{x2} - I_{x3}$



$I_{x1} = I_{x1}' + d_{y1}^2 A_1$ $\quad I_{x2} = \frac{1}{3} bh^3$ $\quad I_{x3} = I_{x3}' + d_{y3}^2 A_3$

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$I_{x2} = I_{x2}' + d_{y2}^2 A_2$

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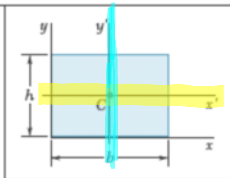
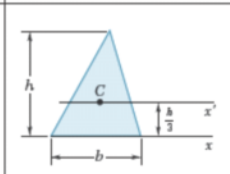
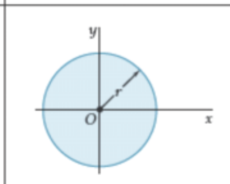
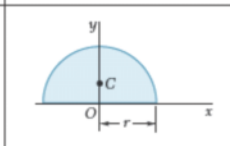
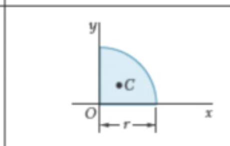
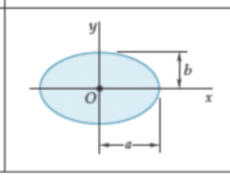
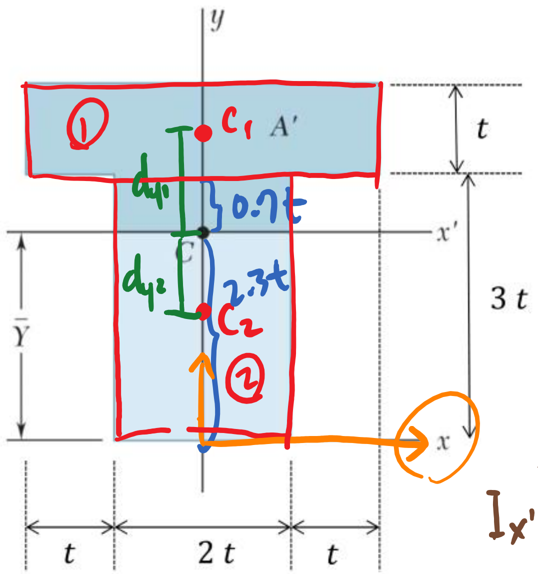
Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ Parallel Axis Thrm. $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ Not these $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ ✓ $I_x = \frac{1}{12}bh^3$ X
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$
Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$
Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$

Find the moment of inertia about its centroid:



Find centroid:

$$\bar{Y} = \frac{\underbrace{4t^2}_{A_1}\underbrace{(3.5t)}_{\tilde{y}_1} + \underbrace{6t^2}_{A_2}\underbrace{(1.5t)}_{\tilde{y}_2}}{\underbrace{4t^2}_{A_1} + \underbrace{6t^2}_{A_2}} = \frac{23t}{10}$$

	①	②
$\tilde{y}$	3.5t	1.5t
A	4t(t)	2t(3t)

$$\bar{Y} = \frac{\Sigma \tilde{y}A}{\Sigma A}$$

Find MoI about x':

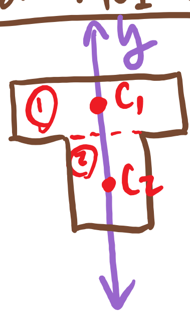
$$I_{x'} = I_{x1} + I_{x2}$$
$$= (I_{x1'} + d_{y1}^2 A_1) + (I_{x2'} + d_{y2}^2 A_2)$$

	①	②
$I_{x'}$	$\frac{1}{12}(4t)(t)^3$	$\frac{1}{12}(2t)(3t)^3$
d_y	1.2t	0.8t
A	$4t^2$	$6t^2$

$$= \left[\frac{4t^4}{12} + (1.2t)^2(4t^2)\right] + \left[\frac{54t^4}{12} + (0.8t)^2(6t^2)\right]$$

$$= 14.4t^4$$

Find MoI about y:

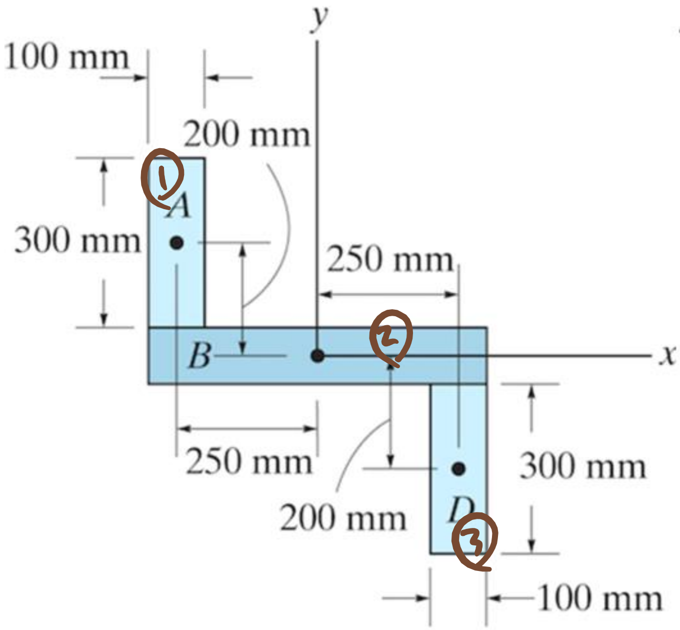


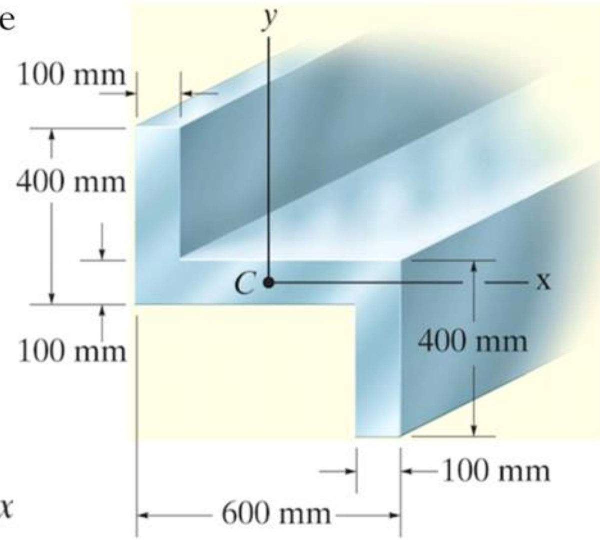
$$I_y = I_{y1} + I_{y2}$$ (no need for parallel axis thm)

$$= \frac{1}{12}(4t)^3(t) + \frac{1}{12}(2t)^3(3t)$$ $\{ I_{y'} = \frac{1}{12}b^3h \}$ from table

$$= \frac{22}{3}t^4$$

Determine the moment of inertia for the cross-sectional area about the x and y centroidal axes.





→ reduce to 2D

x is on its centroid, so no need for parallel axis thm.

$$I_x = I_{x1} + I_{x2} + I_{x3}$$

$$= (I_{x1'} + d_{y1}^2 A_1) + I_{x2'} + (I_{x3'} + d_{y3}^2 A_3) = 0.0021\ m^4$$

	1	2	3	
$I_{x'}$	$0.000225\ m^4$	$0.00005\ m^4$	$0.000225\ m^4$	: $\left(\frac{1}{12}bh^3\right)$
d_y	$0.2\ m$		$0.2\ m$	
A	$0.02\ m^2$		$0.02\ m^2$	: bh