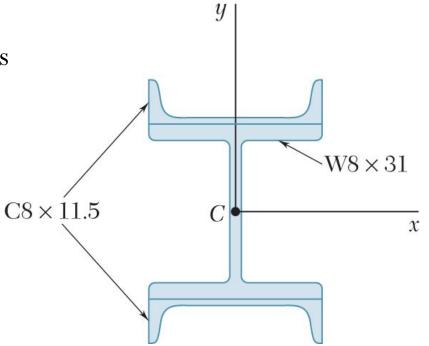
Announcements

• CBTF Quiz 5 continues.

- ☐ Upcoming deadlines:
- Friday (11/30)
 - Written Assignment
- Tuesday (12/4)
 - PL HW



Two channels are welded to a rolled W section as shown. Determine the moments of inertia of the combined section with respect to the centroidal x-axis.



	Area Designation in ²	Arma	Donih	Width	Axis X-X			Axis Y-Y		
		Depth in.	in.	\overline{I}_{x} , in ⁴	\overline{k}_{x} , in.	y , in.	\overline{I}_y , in4	$\overline{k}_{y},$ in.	\overline{x} , in.	
W Shapes (Wide-Flange Shapes) Y	W18 × 76† W16 × 57 W14 × 38 W8 × 31	22.3 16.8 11.2 9.12	18.2 16.4 14.1 8.00	11.0 7.12 6.77 8.00	1330 758 385 110	7.73 6.72 5.87 3.47		152 43.1 26.7 37.1	2.61 1.60 1.55 2.02	
S Shapes (American Standard Shapes)	S18 × 54.7† S12 × 31.8 S10 × 25.4 S6 × 12.5	16.0 9.31 7.45 3.66	18.0 12.0 10.0 6.00	6.00 5.00 4.66 3.33	801 217 123 22.0	7.07 4.83 4.07 2.45		20.7 9.33 6.73 1.80	1.14 1.00 0.960 0.702	
C Shapes (American Standard Channels)	C12×20.7† C10×15.3 C8×11.5 C6×8.2	6.08 4.48 3.37 2.39	12.0 10.0 8.00 6.00	2.94 2.60 2.26 1.92	129 67.3 32.5 13.1	4.61 3.87 3.11 2.34		3.86 2.27 1.31 0.687	0.797 0.711 0.623 0.536	0.698 0.634 0.572 0.512
Angles $X \longrightarrow \overline{x} \longrightarrow \overline{y} \longrightarrow X$	L6×6×1‡ L4×4×½ L3×3×¼ L6×4×½ L5×3×½ L3×2×¼	11.0 3.78 1.44 4.78 3.78 1.19			35.4 5.52 1.23 17.3 9.43 1.09	1.79 1.21 0.926 1.91 1.58 0.963	1.86 1.18 0.836 1.98 1.74 0.980	35.4 5.52 1.23 6.22 2.58 0.390	1.79 1.21 0.926 1.14 0.824 0.569	1.86 1.18 0.836 0.981 0.746 0.487

					Axis X-X			Axis Y-Y		
	Designation	Area mm²	Depth mm	Width mm	105 mm ⁴	\overline{k}_x mm	\overline{y} mm	\overline{I}_y 106 mm4	\overline{k}_y	\overline{x} mm
W Shapes (Wide-Flange Shapes)	W460 × 113† W410 × 85 W360 × 57.8 W200 × 46.1	14400 10800 7230 5880	462 417 358 203	279 181 172 203	554 316 160 45.8	196 171 149 88.1		63.3 17.9 11.1 15.4	66.3 40.6 39.4 51.3	
S Shapes (American Standard Shapes)	S460 × 81.4† S310 × 47.3 S250 × 37.8 S150 × 18.6	10300 6010 4810 2360	457 305 254 152	152 127 118 84.6	333 90.3 51.2 9.16	180 123 103 62.2		8.62 3.88 2.80 0.749	29.0 25.4 24.1 17.8	
C Shapes (American Standard Channels)	C310 × 30.8† C250 × 22.8 C200 × 17.1 C150 × 12.2	3920 2890 2170 1540	305 254 203 152	74.7 66.0 57.4 48.8	53.7 28.0 13.5 5.45	117 98.3 79.0 59.4		1.61 0.945 0.545 0.286	20.2 18.1 15.8 13.6	17.7 16.1 14.5 13.0
Angles $X \longrightarrow \overline{y} \longrightarrow \overline{y} \longrightarrow X$	L152 × 152 × 25.4‡ L102 × 102 × 12.7 L76 × 76 × 6.4 L152 × 102 × 12.7 L127 × 76 × 12.7 L76 × 51 × 6.4	7100 2420 929 3060 2420 768			14.7 2.30 0.512 7.20 3.93 0.454	45.5 30.7 23.5 48.5 40.1 24.2	47.2 30.0 21.2 50.3 44.2 24.9	14.7 2.30 0.512 2.59 1.06 0.162	45.5 30.7 23.5 29.0 20.9 14.5	47.2 30.0 21.2 24.9 18.9 12.4

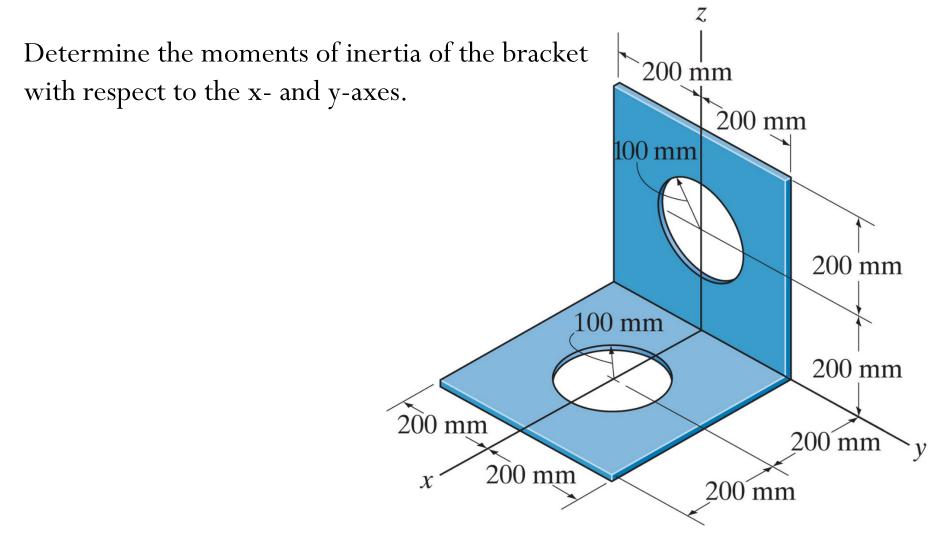
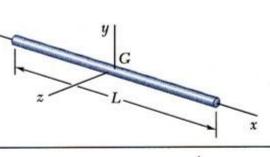


Figure: 10_P106-107

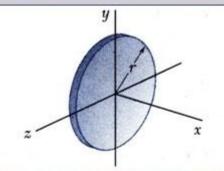
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Vector Mechanics for Engineers: Statics

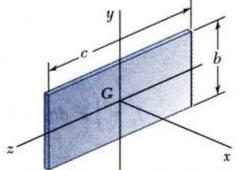
Moments of Inertia of Common Geometric Shapes



$$I_y=I_z=\frac{1}{12}\,mL^2$$



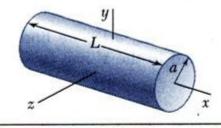
$$\begin{split} I_x &= \frac{1}{2} m r^2 \\ I_y &= I_z = \frac{1}{4} m r^2 \end{split}$$



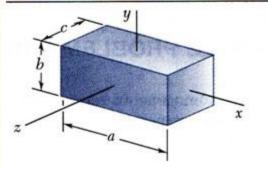
$$I_x = \frac{1}{12} \, m (b^2 + c^2)$$

$$I_y = \frac{1}{12} mc^2$$

$$I_z = \frac{1}{12} mb^2$$



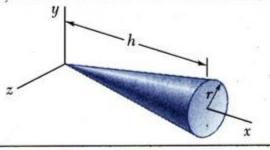
$$\begin{split} I_x &= \frac{1}{2} \, m a^2 \\ I_y &= I_z = \frac{1}{12} \, m (3 a^2 + L^2) \end{split}$$



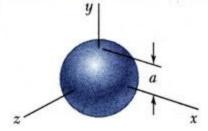
$$I_x = \frac{1}{12} m(b^2 + c^2)$$

$$I_y = \frac{1}{12} m(c^2 + a^2)$$

$$I_z = \frac{1}{12} \, m (a^2 + b^2)$$

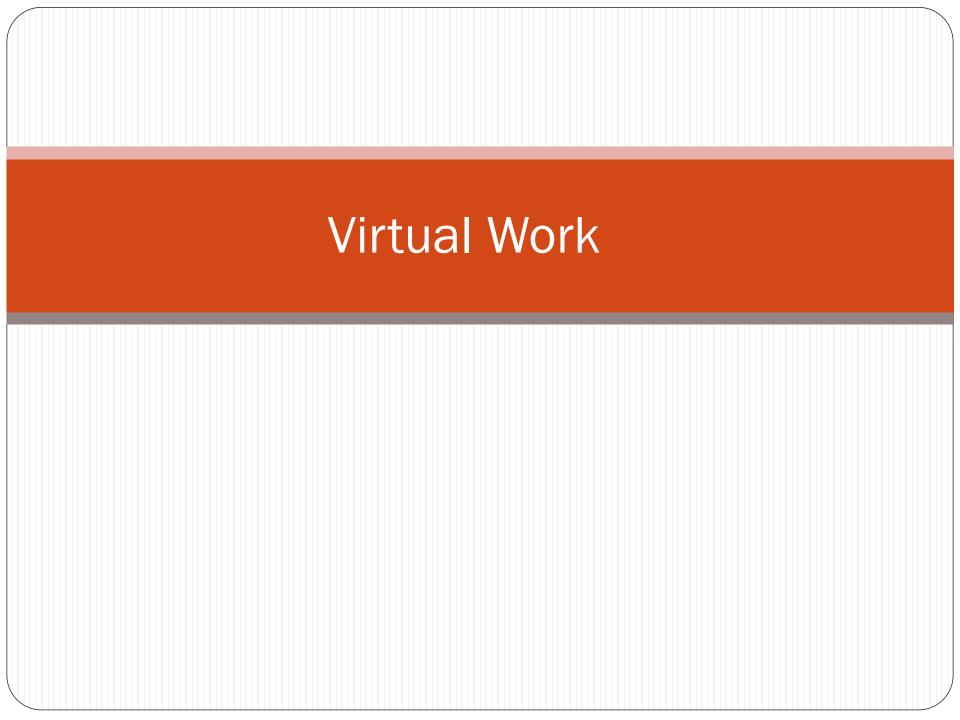


$$\begin{split} I_x &= \frac{3}{10} m a^2 \\ I_y &= I_z = \frac{3}{5} m (\frac{1}{4} a^2 + h^2) \end{split}$$



$$I_x = I_y = I_z = \frac{2}{5} ma^2$$





Main goals and learning objectives

• Introduce the principle of virtual work

• Show how it applies to determining the equilibrium configuration of a series of pin-connected members

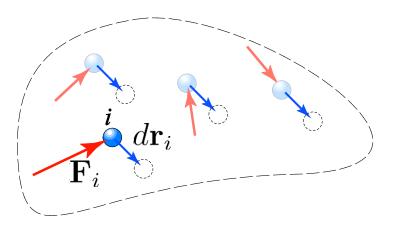
Definition of Work

Work of a force

A force does work when it undergoes a displacement in the direction of the line of action.

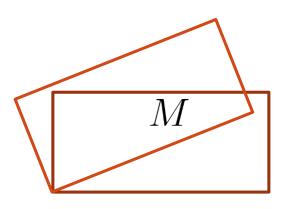
The work dU produced by the force ${\pmb F}$ when it undergoes a differential displacement $d{\pmb r}$ is given by

$$dU = \mathbf{F} \cdot d\mathbf{r}$$



Definition of Work

Work of a couple $dU = M\mathbf{k} \cdot d\theta \,\mathbf{k} = M \,d\theta$



Virtual Displacements

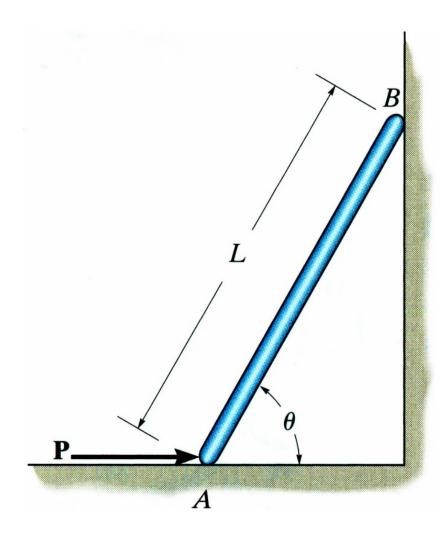
A *virtual displacement* is a conceptually possible displacement *or* rotation of all *or* part of a system of particles. The movement is assumed to be possible, but actually does not exist.

Principle of Virtual Work

The principle of virtual work states that if a body is in equilibrium, then the algebraic sum of the virtual work done by all the forces and couple moments acting on the body is zero for any virtual displacement of the

δu

body. Thus,



The thin rod of weight *W* rests against the smooth wall and floor. Determine the magnitude of force *P* needed to hold it in equilibrium.