Announcements

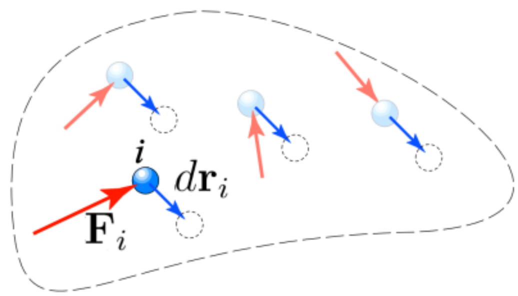
- Last day of class: Monday, Dec. 9
- No discussion sections next week
- Last day of office hours and Piazza help: Wednesday, Dec. 12
- CBTF (last) Quiz 6 starts Thursday, Dec. ~~14~~ 13

Written Exam: MEB 224

- Upcoming deadlines:
- Tuesday (12/4)
  - PL HW
- Friday (12/7)
  - Written assignment 9

Definition of Work

Work of a force



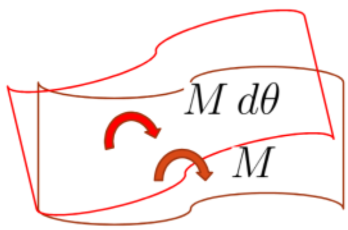
A force does work when it undergoes a displacement in the direction of the line of action.

The work dU produced by the force $\boldsymbol{F}$ when it undergoes a differential displacement $d\boldsymbol{r}$ is given by

$$dU = \mathbf{F} \cdot d\mathbf{r}$$

Work of a couple

$$dU = M\mathbf{k} \cdot d\theta\,\mathbf{k} = M\,d\theta$$



Virtual Displacements

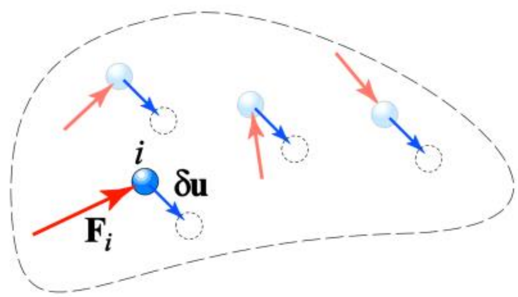
A *virtual displacement* is a conceptually possible displacement *or* rotation of all *or* part of a system of particles. The movement is assumed to be possible, but actually does not exist. These "movements" are first order differential quantities:

$\delta x, \delta y, \delta \theta$

Virtual Work

The principle of virtual work states that if a body is in equilibrium, then the algebraic sum of the virtual work done by all the forces and couple moments acting on the body is zero for any virtual displacement of the body. Thus,

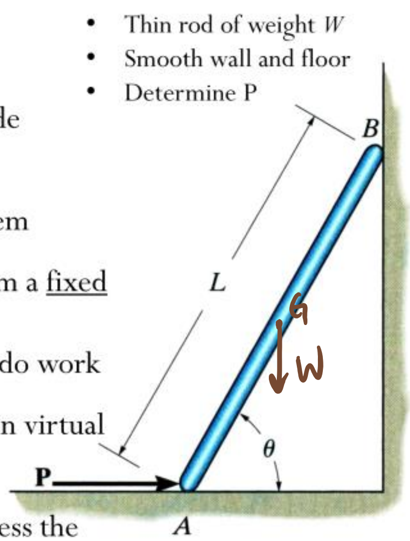
$dU = 0 = \Sigma dU_i$



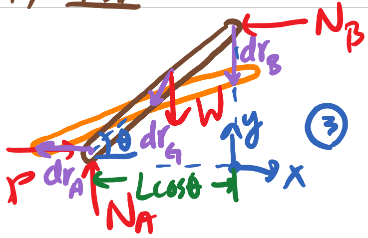
Procedure for Analysis

1. Draw FBD of the entire system and provide coordinate system
2. Sketch the "deflected position" of the system
3. Define position coordinates measured from a <u>fixed</u> point and select the parallel line of action component and remove forces that do no do work
4. <u>Differentiate</u> position coordinates to obtain virtual displacement
5. Write the virtual work equation and express the virtual work of each force/couple moment
6. Factor out the comment virtual displacement term and solve

- Thin rod of weight W
- Smooth wall and floor
- Determine P



1) <u>FBD</u>



2.) Pretend P is absent (pick 1 force/moment)

$dU = \vec{F} \cdot d\vec{r}$

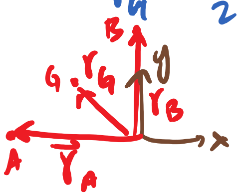
3.) Define key positions (where F is applied)

4) differentiate

$\vec{r}_A = -L\cos\theta\,\hat{i} \xrightarrow{\text{differentiate}} \dfrac{d\vec{r}_A}{d\theta} = -L(-\sin\theta)\hat{i} \rightarrow d\vec{r}_A = L\sin\theta\, d\theta\,\hat{i}$

$\vec{r}_B = L\sin\theta\,\hat{j} \longrightarrow d\vec{r}_B = L\cos\theta\, d\theta\,\hat{j}$

$\vec{r}_G = -\frac{L}{2}\cos\theta\,\hat{i} + \frac{L}{2}\sin\theta\,\hat{j} \longrightarrow d\vec{r}_G = \left(\frac{L}{2}\sin\theta\, d\theta\,\hat{i} + \frac{L}{2}\cos\theta\, d\theta\,\hat{j}\right)$



5.) N_B & N_A don't do work because $\vec{F} \cdot d\vec{r} = 0$.

- For N_B: $\vec{N}_B = N_B(-\hat{i})$ $\quad d\vec{r}_B = L\cos\theta\, d\theta\,\hat{j}$

$$\vec{N}_B \cdot d\vec{r}_B = (-N_B \cdot 0) + 0 \cdot (L\cos\theta\, d\theta) = 0$$

P will do work: $\vec{P} = P\hat{i}$, $dU_p = \vec{P} \cdot d\vec{r}_A = (+P\hat{i}) \cdot (L\sin\theta\, d\theta\, \hat{i})$

W will do work: $\vec{W} = -W\hat{j}$,

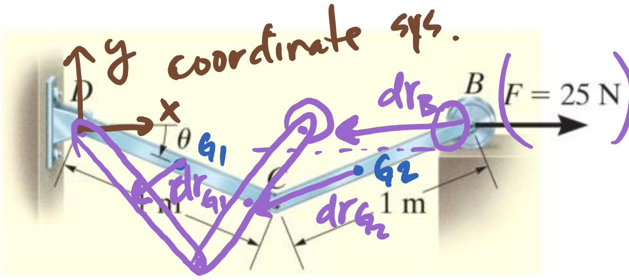
$$dU_W = \vec{W} \cdot d\vec{r}_G = (-W\hat{j}) \cdot \left(\frac{L}{2}\sin\theta\, d\theta\, \hat{i} + \frac{L}{2}\cos\theta\, d\theta\, \hat{j}\right)$$

$$= -W\left(\frac{L}{2}\cos\theta\, d\theta\right)$$

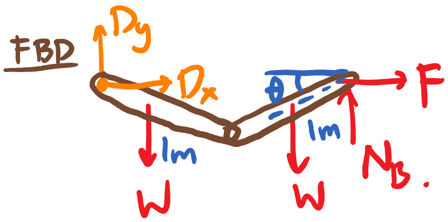
$$dU = dU_p + dU_W = 0 = PL\sin\theta\, d\theta + \left(-\frac{WL}{2}\cos\theta\, d\theta\right)$$

$$0 = L\,d\theta\left(P\sin\theta - \frac{W}{2}\cos\theta\right) \rightarrow P\sin\theta = \frac{W}{2}\cos\theta$$

$$P = \frac{W\cos\theta}{2\sin\theta}$$



Determine the angle for equilibrium of the two-member linkage. Each member has a mass of 10 kg.



Use virtual work:

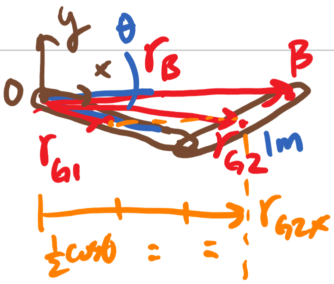
- D_x, D_y, N_B don't do work.
- W & F will do virtual work.

$$dU = dU_F + dU_{W1} + dU_{W2}.$$

$$\vec{F} = 25N\,\hat{i} \qquad d\vec{r}_B = 2(-\sin\theta)\,d\theta\,\hat{i}$$

$$\vec{W}_1 = -(10)(9.81)N\,\hat{j} \qquad d\vec{r}_{G1} = \tfrac{1}{2}(-\sin\theta)\cancel{d\theta\,\hat{i}} - \tfrac{1}{2}(\cos\theta)\,d\theta\,\hat{j}$$

$$\vec{W}_2 = -(10)(9.81)N\,\hat{j} \qquad d\vec{r}_{G2} = \tfrac{3}{2}(-\sin\theta)\cancel{d\theta\,\hat{i}} - \tfrac{1}{2}(\cos\theta)\,d\theta\,\hat{j}$$



$$\vec{r}_B = 2(1m)(\cos\theta)\,\hat{i}$$

$$r_{G1} = \frac{1m}{2}\cdot\cos\theta\,\hat{i} - \frac{1m}{2}\sin\theta\,\hat{j}$$

$$r_{G2} = \frac{3m}{2}\cdot\cos\theta\,\hat{i} - \frac{1m}{2}\sin\theta\,\hat{j}$$

$$dU = \vec{F}\cdot d\vec{r}_B + \vec{W}_1\cdot d\vec{r}_{G1} + \vec{W}_2\cdot d\vec{r}_{G2} = 0$$

$$= 25(-2\sin\theta\,d\theta) + (-98.1)(-\tfrac{1}{2}\cos\theta\,d\theta) + (-98.1)(\tfrac{1}{2}\cos\theta\,d\theta)$$

$$\rightarrow -50\sin\theta + 98.1\cos\theta = 0.$$

$$\tan\theta = \frac{98.1}{50} \rightarrow \theta = \tan^{-1}\left(\frac{98.1}{50}\right)$$