Statics - TAM 210 & TAM 211

Lecture 3

January 22, 2018

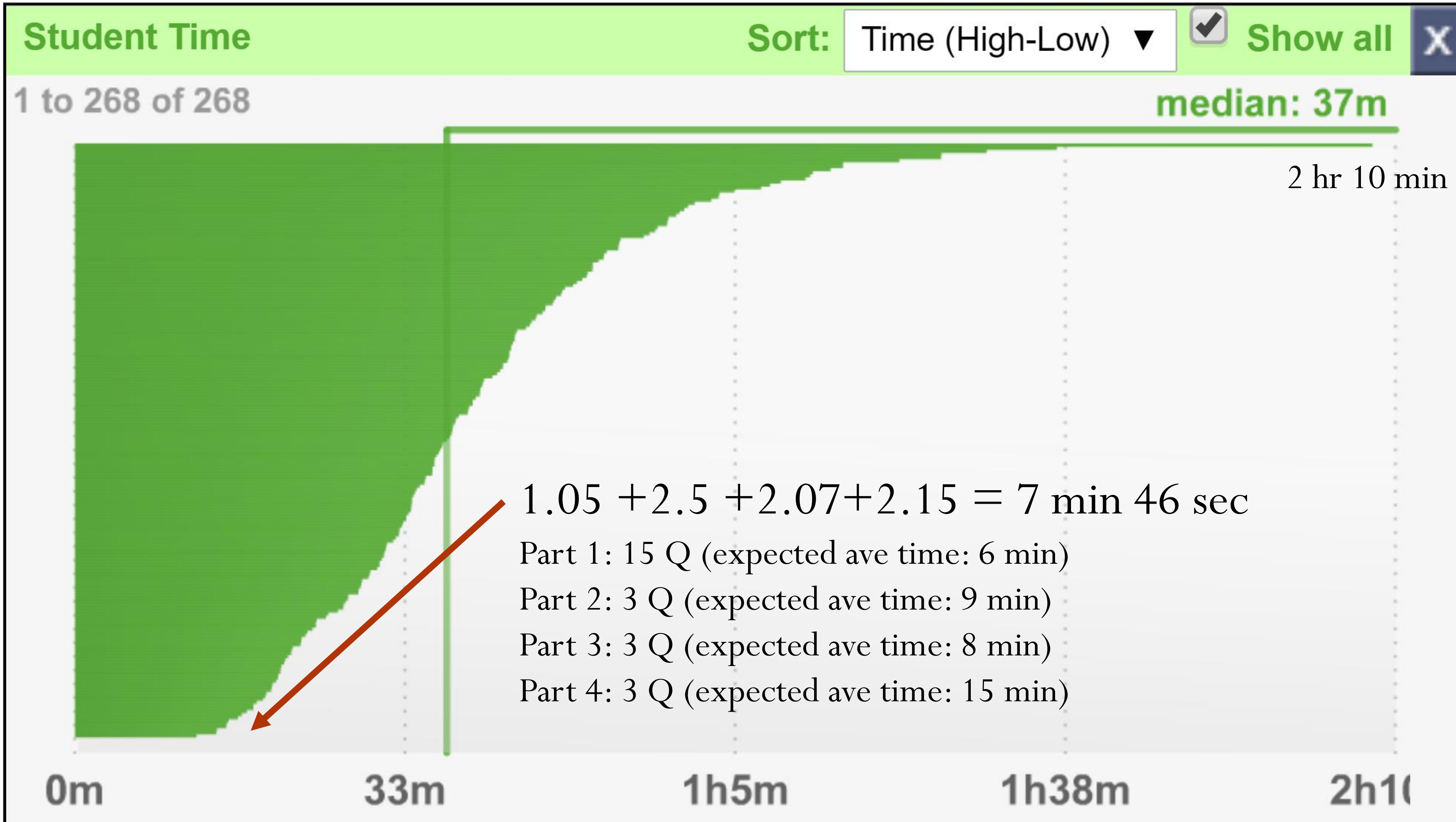
Announcements

- Take practice Quiz 0 on PrairieLearn (not graded)
- MATLAB training sessions
  - Wed 24, Thu 25, Fri 26, and Mon 29
  - DCL 1440, Tutorial: 6:30-7:30 pm, Q&A: 7:30-8:00 pm

- Upcoming deadlines:
- Tuesday (1/23)
  - Prairie Learn HW1
- Thursday (1/25)
  - Written Assignment 1
- Friday (1/26)
  - Mastering Engineering Tutorial3

giphy.com

Time for completion of Mastering Engineering Tutorial 2



- Not trying to solve problems on your own and copying other's answers will make taking quizzes ∞ more difficult!

Chapter 2: Force vectors
Main goals and learning objectives

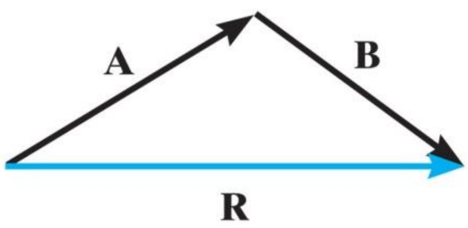
Define scalars, vectors and vector operations and use them to analyze forces acting on objects

- Add forces and resolve them into components
- Express force and position in Cartesian vector form
- Determine a vector’s magnitude and direction
- Introduce the dot product and use it to find the angle between two vectors or the projection of one vector onto another

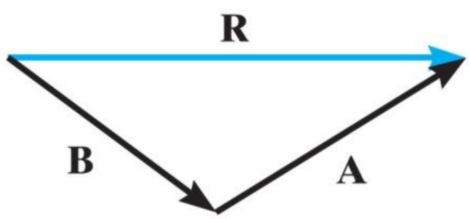
Recap from Lecture 2

- A force can be treated as a vector, since forces obey all the rules that vectors do.

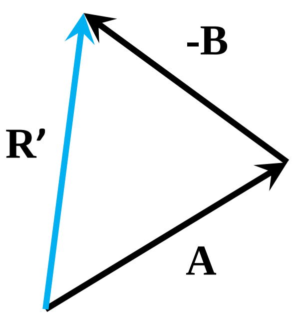
$$\vec{R} = \vec{A} + \vec{B} \qquad \vec{R} = \vec{A} + \vec{B} = \vec{B} + \vec{A} \qquad \vec{R'} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$



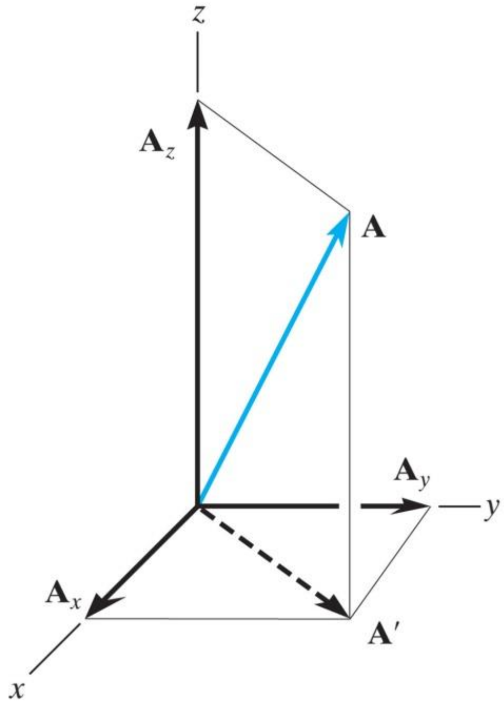
R = A + B

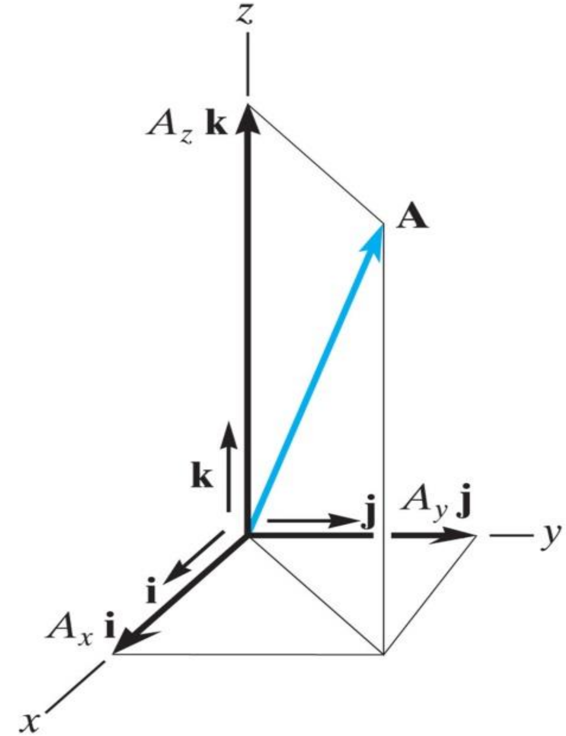


R = B + A



Recap





- Vector representations
  - Rectangular components
  - Cartesian vectors
  - Unit vector

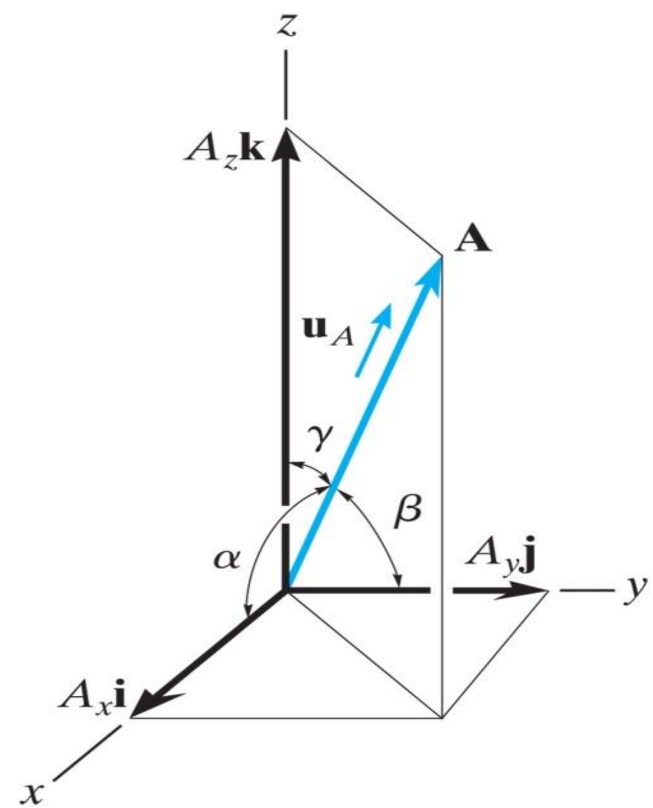
$$\vec{\boldsymbol{A}} = \overrightarrow{\boldsymbol{A_x}} + \overrightarrow{\boldsymbol{A_y}} + \overrightarrow{\boldsymbol{A_z}}$$

$$\vec{\boldsymbol{A}} = A_x\hat{\boldsymbol{\imath}} + A_y\hat{\boldsymbol{\jmath}} + A_z\widehat{\boldsymbol{k}}$$

$$\overrightarrow{\boldsymbol{u_A}} = \frac{\vec{\boldsymbol{A}}}{|\vec{\boldsymbol{A}}|} = \frac{A_x}{|\vec{\boldsymbol{A}}|}\hat{\boldsymbol{\imath}} + \frac{A_y}{|\vec{\boldsymbol{A}}|}\hat{\boldsymbol{\jmath}} + \frac{A_z}{|\vec{\boldsymbol{A}}|}\widehat{\boldsymbol{k}}$$

Recall: Magnitude of a vector (which is a scalar quantity) can be shown as a term with no font modification (A) or vector with norm bars ($|\vec{\boldsymbol{A}}|$), such that $A = |\vec{\boldsymbol{A}}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$$\vec{A} = \vec{A_x} + \vec{A_y} + \vec{A_z} \qquad \vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$



- How to define A_x , A_y, A_z?
  - Direction cosines

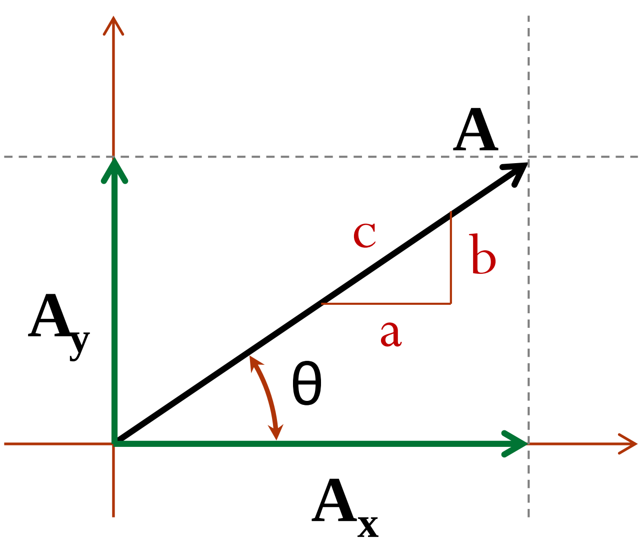
$$cos(\alpha) = \frac{A_x}{A},\ cos(\beta) = \frac{A_y}{A}, cos(\gamma) = \frac{A_z}{A}$$

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$
$$= A\,cos(\alpha)\hat{\imath} + A\,cos(\beta)\hat{\jmath} + Acos(\gamma)\hat{k}$$

  - Rectangular components

$$A_x = A\,cos(\theta),\ A_y = A\,sin(\theta)$$

$$A_x = A\left(\frac{a}{c}\right),\ A_y = A\left(\frac{b}{c}\right)$$



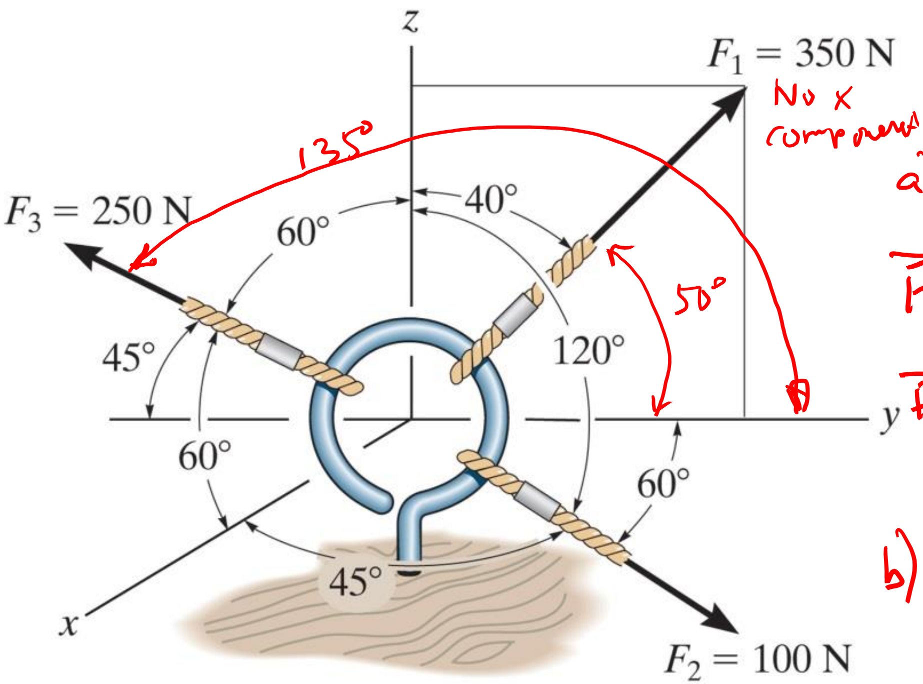
The cables attached to the screw eye are subjected to three forces shown.

(a) Express each force vector using the Cartesian vector form (components form).

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} = A\left[\cos(\alpha)\hat{\imath} + \cos(\beta)\hat{\jmath} + \cos(\gamma)\hat{k}\right]$$

(b) Determine the magnitude of the resultant force vector

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$



a) $\vec{F}_1 = 0\hat{\imath} + 350(\cos 50)\hat{\jmath} + 350(\cos 40)\hat{k}$ N

$\vec{F}_2 = 100(\cos 45\hat{\imath} + \cos 60\hat{\jmath} + \cos 120\hat{k})$ N

$\vec{F}_3 = 250(\cos 60\hat{\imath} + \cos 135\hat{\jmath} + \cos 60\hat{k})$ N

($\cos 135\hat{\jmath}$ ↑ $-\cos 45\hat{\jmath}$)

b) $\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$

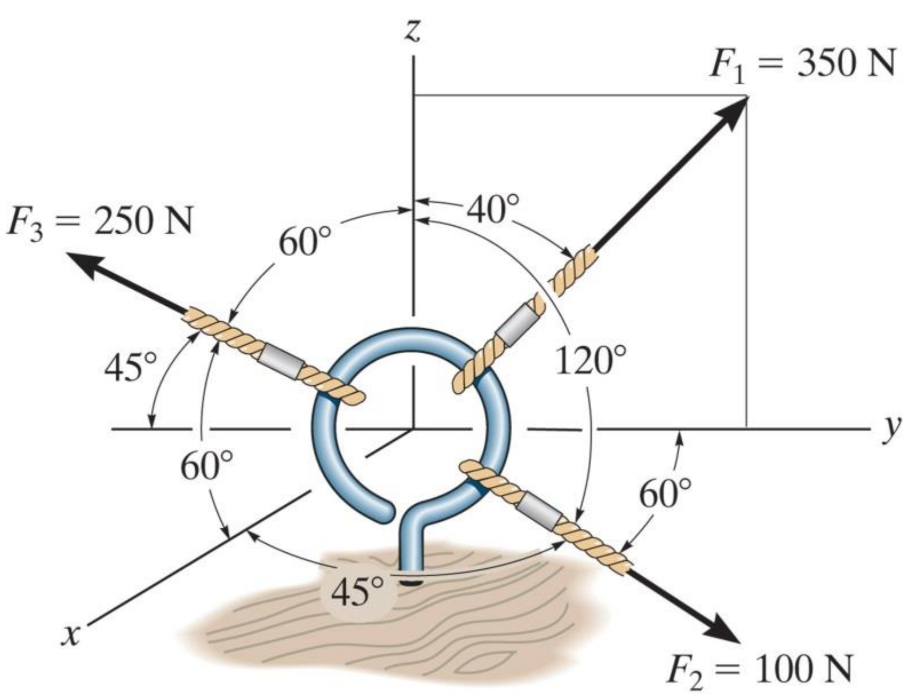
$$\vec{F}_R = (0 + 100\cos 45 + 250\cos(60))\hat{\imath} + (350\cos 50 + 100\cos 60 + 250\cos 135)\hat{\jmath} + (350\cos 40 + 100\cos 120 + 250\cos 60)\hat{k} \text{ N}$$

$$= 195.71\hat{\imath} + 98.20\hat{\jmath} + 343.12\hat{k} = F_{Rx}\hat{\imath} + F_{Ry}\hat{\jmath} + F_{Rz}\hat{k}$$

$$\Rightarrow |\vec{F}_R| = \sqrt{F_{Rx}^2 + F_{Ry}^2 + F_{Rz}^2} = 407.03 \text{ N} = \boxed{407 \text{ N}}$$

The cables attached to the screw eye are subjected to three forces shown.

(c) Determine the direction cosines of the resultant force vector



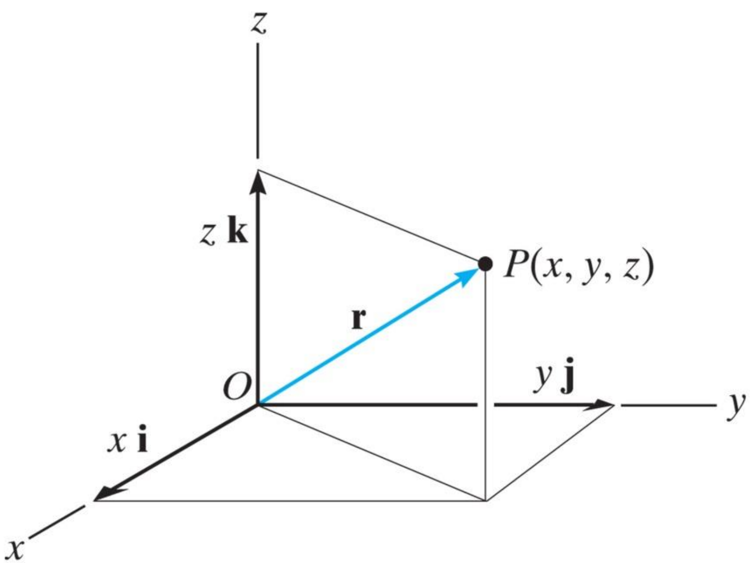
$$\cos(\alpha) = \frac{A_x}{A},\ \cos(\beta) = \frac{A_y}{A},\ \cos(\gamma) = \frac{A_z}{A}$$

$$\cos\alpha = \frac{F_{Rx}}{F_R} = \frac{195.71}{407.03}$$

$$\cos\beta = \frac{F_{Ry}}{F_R} =$$

$$\cos\gamma = \frac{F_{Rz}}{F_R} =$$

Position vectors



A position vector $\boldsymbol{r}$ is defined as a fixed vector which locates a point in space relative to another point. For example,

$$\boldsymbol{r} = x\,\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$$

expresses the position of point $P(x,y,z)$ with respect to the origin O.

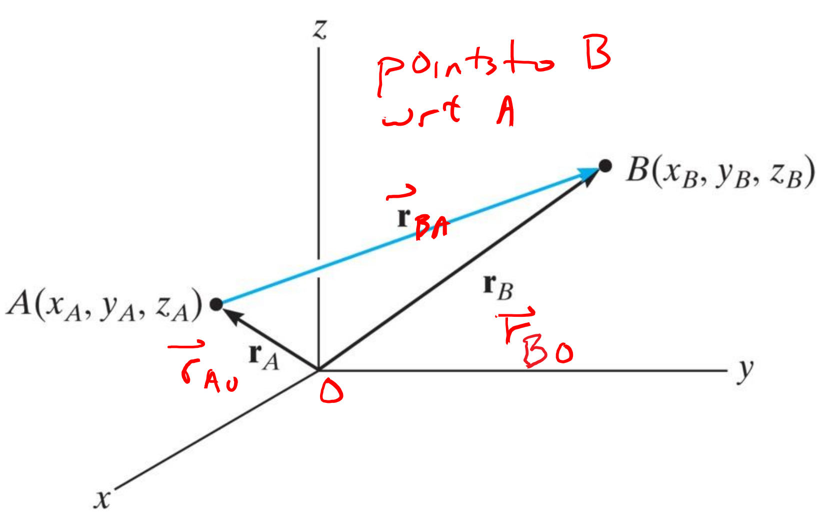
The position vector $\boldsymbol{r}$ of point $\boldsymbol{B}$ with respect to point $\boldsymbol{A}$ is obtained from

$$\boldsymbol{r}_A + \boldsymbol{r} = \boldsymbol{r}_B$$

Hence,

$$\begin{aligned} \boldsymbol{r} &= \boldsymbol{r}_B - \boldsymbol{r}_A \\ &= (x_B\,\boldsymbol{i} + y_B\,\boldsymbol{j} + z_B\,\boldsymbol{k}) - (x_A\,\boldsymbol{i} + y_A\,\boldsymbol{j} + z_A\,\boldsymbol{k}) \end{aligned}$$

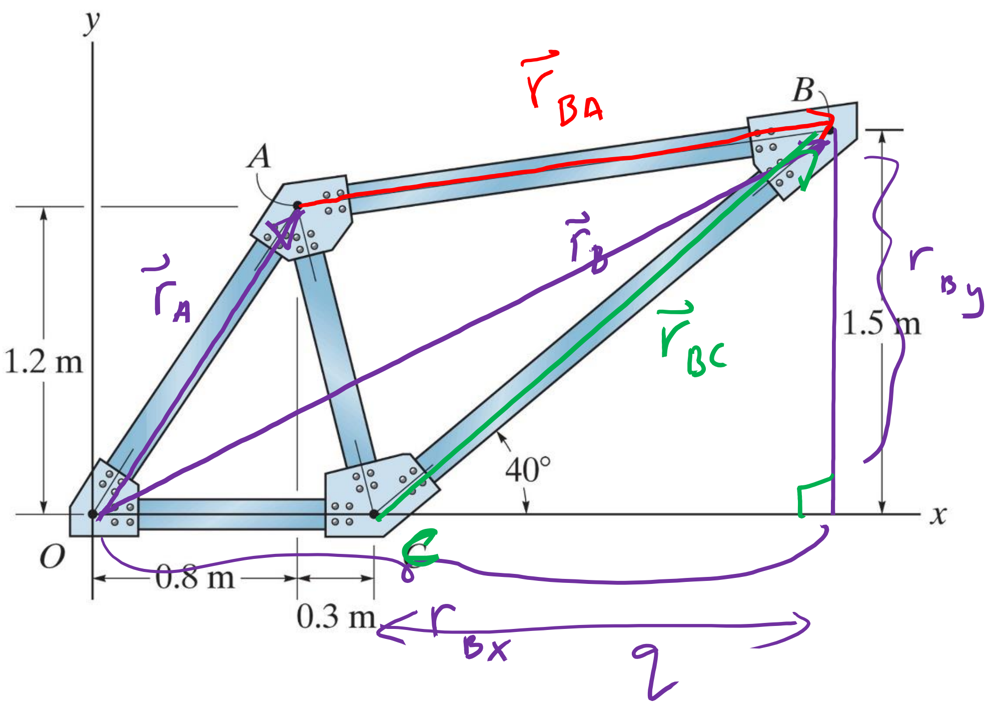
$$\boldsymbol{r} = (x_B - x_A)\,\boldsymbol{i} + (y_B - y_A)\,\boldsymbol{j} + (z_B - z_A)\,\boldsymbol{k}$$



Thus, the $(\boldsymbol{i},\ \boldsymbol{j},\ \boldsymbol{k})$ components of the positon vector $\boldsymbol{r}$ may be formed by taking the coordinates of the tail (point A) and subtracting them from the corresponding coordinates of the head (point B).

Example

Determine the lengths of bars AB, BC and AC.



Length = magnitude

$\overline{AB} = |\vec{r}_{BA}|$

$\vec{r}_{BA} = \vec{r}_B - \vec{r}_A$

$\vec{r}_A = 0.8\hat{i} + 1.2\hat{j}$ m

$\vec{r}_B = ? = \_\_\,\hat{i} + \_\_\,\hat{j}$

$= (0.8 + 0.3 + q)\hat{i} + 1.5\hat{j}$ m

$q = ?$

$\tan 40° = \frac{opp}{adj} = \frac{1.5}{q}$

$q = 1.5/\tan 40° = 1.79 = 1.8$ m

$\Rightarrow \vec{r}_B = 2.9\hat{i} + 1.5\hat{j}$ m

$\therefore \vec{r}_{BA} = \vec{r}_B - \vec{r}_A = (2.9 - 0.8)\hat{i} + (1.5 - 1.2)\hat{j} = 2.1\hat{i} + 0.3\hat{j}$

$\overline{AB} = |\vec{r}_{BA}| = \sqrt{2.1^2 + 0.3^2} = \sqrt{4.5} =$ **2.1 m = $\overline{AB}$**