Statics - TAM 210 & TAM 211

Lecture 4

January 24, 2018

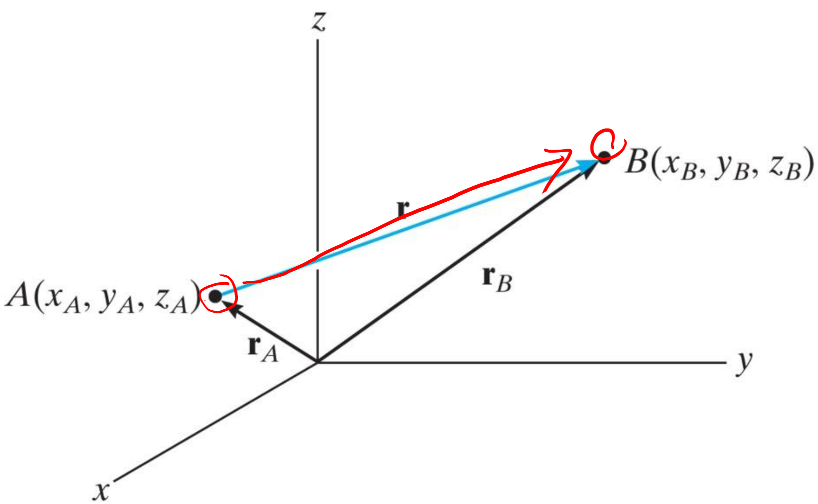
Announcements

- MATLAB training sessions
  - Wed 24, Thu 25, Fri 26, and Mon 29
  - DCL ~~1440~~, Tutorial: 6:30-7:30 pm, Q&A: 7:30-8:00 pm
    L 440 (correction)
- DRES accommodations
  - Send a private post with DRES letter on Piazza to Instructors, if you have not already directly contacted Prof. H-W

- Upcoming deadlines:
- Thursday (1/25)
  - Written Assignment 1
- Friday (1/26)
  - Mastering Engineering Tutorial3
- Tuesday (1/30)
  - Prairie Learn HW2

Recap from Lecture 3

- Position vector

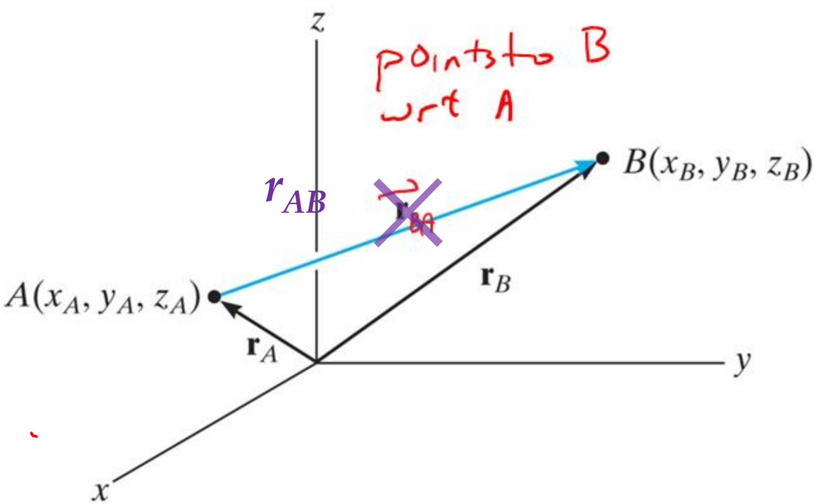


Head   Tail

$$\boldsymbol{r} = \boldsymbol{r}_B - \boldsymbol{r}_A$$
$$= (x_B\,\boldsymbol{i} + y_B\,\boldsymbol{j} + z_B\,\boldsymbol{k}) - (x_A\,\boldsymbol{i} + y_A\,\boldsymbol{j} + z_A\,\boldsymbol{k})$$

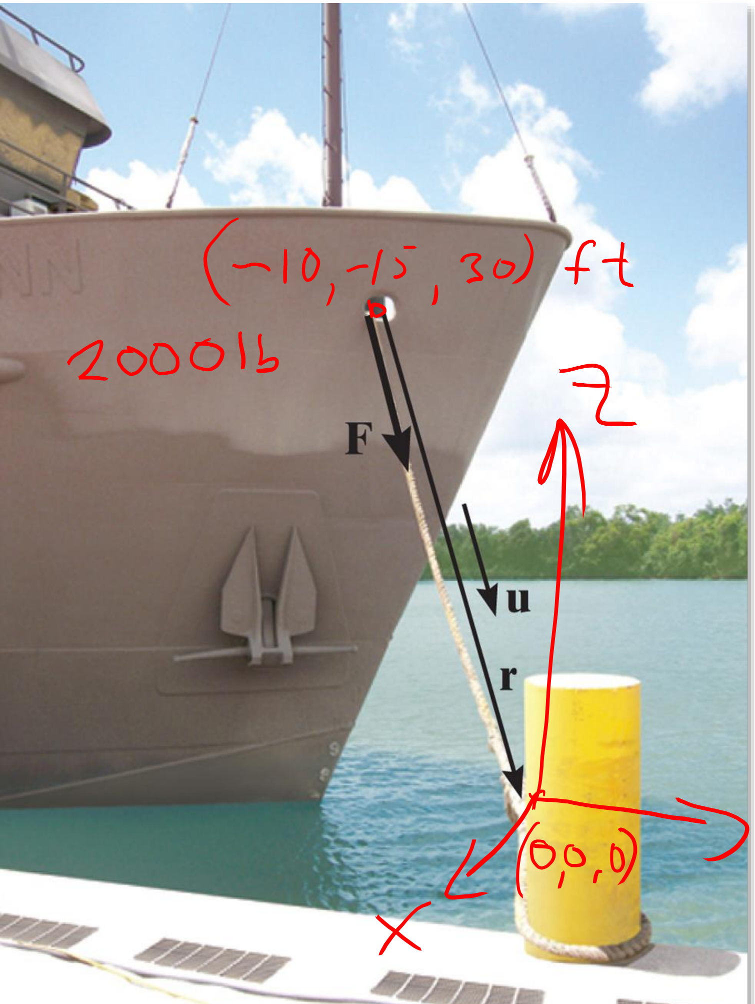
$$\boldsymbol{r} = (x_B - x_A)\,\boldsymbol{i} + (y_B - y_A)\,\boldsymbol{j} + (z_B - z_A)\,\boldsymbol{k}$$

Thus, the $(\boldsymbol{i},\ \boldsymbol{j},\ \boldsymbol{k})$ components of the positon vector $\boldsymbol{r}$ may be formed by taking the coordinates of the tail (point A) and subtracting them from the corresponding coordinates of the head (point B).



Modification to notation introduced in lecture 3: Use r_{AB} (not r_{BA}) to be consistent with PL and book

Force vector directed along a line



The force vector ***F*** acting a long the rope can be defined by the unit vector ***u*** (defined the <u>direction</u> of the rope) and the <u>magnitude</u> F of the force.

$$\vec{\boldsymbol{F}} = F\vec{\boldsymbol{u}}$$

The unit vector ***u*** is specified by the position vector ***r***:

Such that $\vec{\boldsymbol{u}} = \dfrac{\vec{\boldsymbol{r}}}{|\vec{\boldsymbol{r}}|}$ $\dfrac{ft}{ft}$

Note that $\vec{\boldsymbol{u}}$ is unitless and points in the direction of $\vec{\boldsymbol{r}}$.

where

$$\vec{\boldsymbol{r}} = (x_B - x_A)\hat{\boldsymbol{\imath}} + (y_B - y_A)\hat{\boldsymbol{\jmath}} + (z_B - z_A)\hat{\boldsymbol{k}}$$

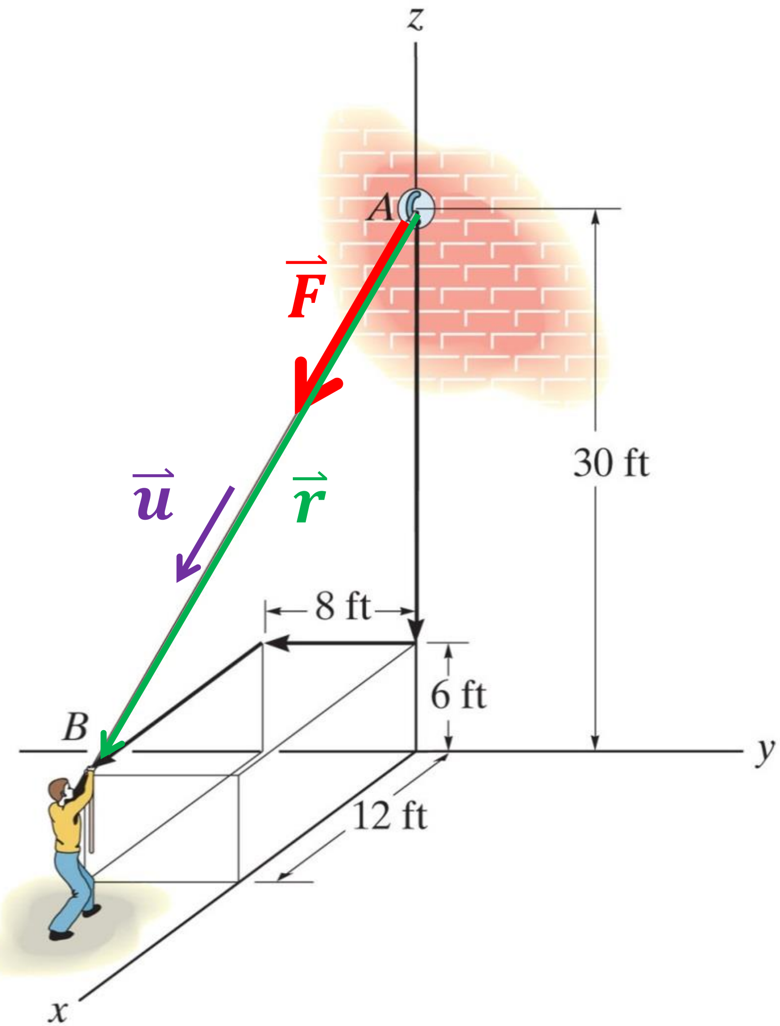
$\vec{r} = (10\hat{i} + 15\hat{j} + (-30)\hat{k})$ ft

wrong units

lb

$$\vec{F} = 2000\,(lb)\left[\frac{10\hat{i} - 15\hat{j} - 30\hat{k}}{\sqrt{10^2 + 15^2 + 30^2}}\right] \overset{?}{=} \frac{lb \cdot \cancel{ft}}{\cancel{ft}} = lb$$

Force vector directed along a line



The man pulls on the cord with a force of 70 lb. Represent the force **F** as a Cartesian vector.

i>clicker time

What is the "correct" (as per PrairieLearn and textbook) notation for the position vector **r** shown in green?

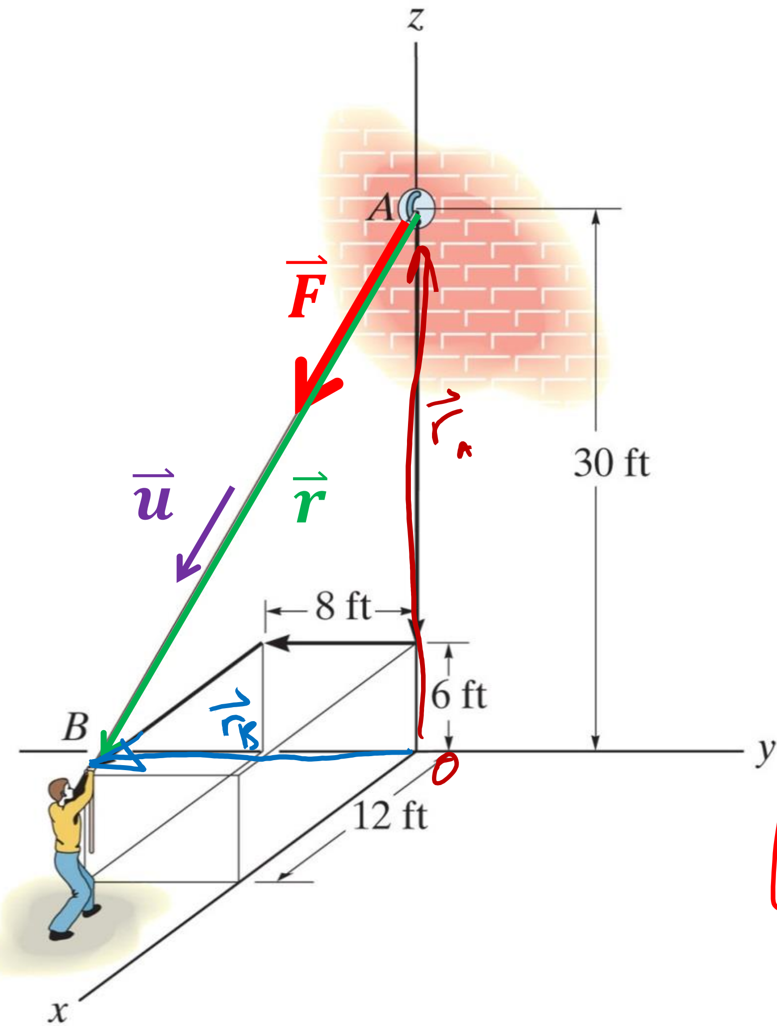
A) $\mathbf{r}_{\mathbf{AB}}$ $= \vec{r}_B - \vec{r}_A$

B) $\mathbf{r}_{\mathbf{BA}}$

$$\overrightarrow{r_{AB}} = (x_B - x_A)\hat{\imath} + (y_B - y_A)\hat{\jmath} + (z_B - z_A)\hat{k}$$

Thus, the $(\boldsymbol{i},\ \boldsymbol{j},\ \boldsymbol{k})$ components of the positon vector $\boldsymbol{r}$ may be formed by taking the coordinates of the tail (point A) and subtracting them from the corresponding coordinates of the head (point B).

Force vector directed along a line



The man pulls on the cord with a force of 70 lb. Represent the force **F** as a Cartesian vector.

$$\vec{F} = F\,\vec{u}_{AB}$$

$$F = 70\text{ lb} \qquad \vec{u} = \frac{\vec{r}}{|\vec{r}|}$$

$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$$

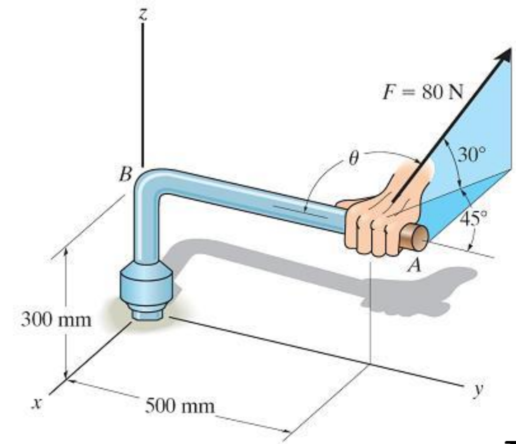
$$\vec{r}_A = 30\,\hat{k} \qquad \vec{r}_B = 12\,\hat{i} - 8\,\hat{j} + 6\,\hat{k}$$

$$\vec{r}_{AB} = 12\,\hat{i} - 8\,\hat{j} + (6-30)\,\hat{k}\ \text{ft}$$

$$|\vec{r}| = \sqrt{12^2 + 8^2 + 24^2} = 28\text{ ft}$$

$$\boxed{\vec{F} = (70\text{ lb})\left(\frac{12}{28}\,\hat{i} - \frac{8}{28}\,\hat{j} - \frac{24}{28}\,\hat{k}\right)}$$

Dot (or scalar) product

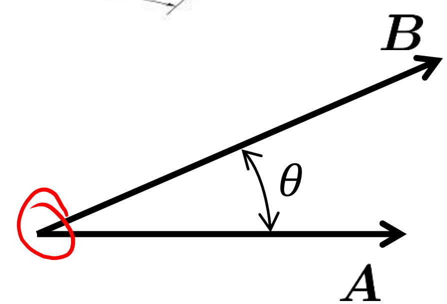


The dot product of vectors **A** and **B** is defined as such

$$\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = |\vec{A}||\vec{B}| \cos\theta$$

Use of dot product:

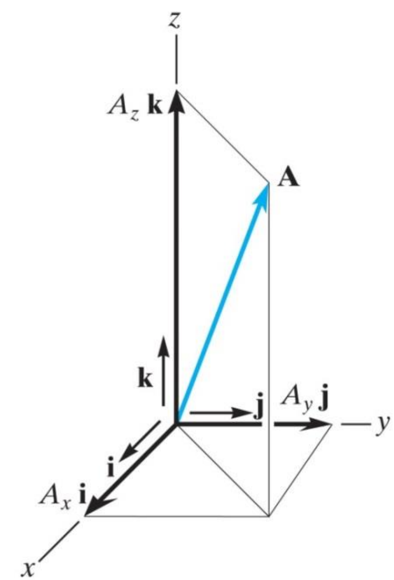
- Find angle btw 2 vectors $\quad \cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}|\,|\vec{B}|}$
- Find projections $\parallel$ or $\perp$ to a line



Laws of operation:

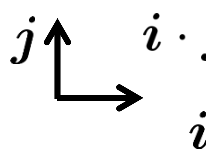
$$\boldsymbol{A}\cdot\boldsymbol{B} = \boldsymbol{B}\cdot\boldsymbol{A}$$
$$\alpha(\boldsymbol{A}\cdot\boldsymbol{B}) = \alpha\boldsymbol{A}\cdot\boldsymbol{B} = \boldsymbol{A}\cdot\alpha\boldsymbol{B}$$
$$\boldsymbol{A}\cdot(\boldsymbol{B}+\boldsymbol{C}) = \boldsymbol{A}\cdot\boldsymbol{B} + \boldsymbol{A}\cdot\boldsymbol{C}$$

Cartesian vector formulation:



$$\begin{aligned}\boldsymbol{A}\cdot\boldsymbol{B} &= (A_x\,\boldsymbol{i} + A_y\,\boldsymbol{j} + A_z\,\boldsymbol{k})\cdot(B_x\,\boldsymbol{i} + B_y\,\boldsymbol{j} + B_z\,\boldsymbol{k}) \\ &= A_x\,B_x + A_y\,B_y + A_z\,B_z\end{aligned}$$

Note that:

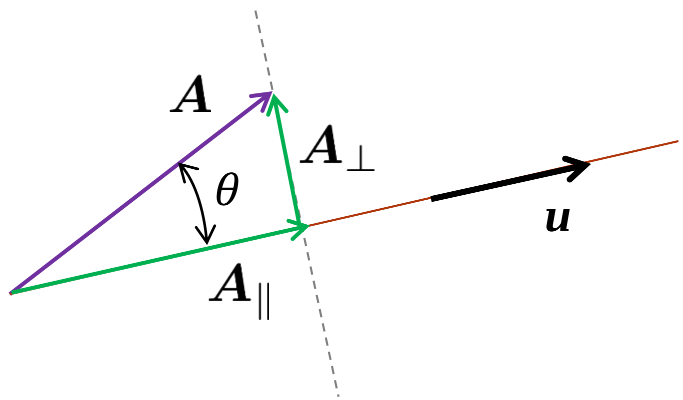


$$\boldsymbol{i}\cdot\boldsymbol{j} = 0$$

$$\boldsymbol{i}\cdot\boldsymbol{i} = 1$$

Projection of vector onto parallel and perpendicular lines

The scalar component $A_{||}$ of a vector $\boldsymbol{A}$ along (parallel to) a line with unit vector $\boldsymbol{u}$ is given by:



$$A_{||} = \boldsymbol{A} \cdot \boldsymbol{u} = |\boldsymbol{A}| \cos(\theta)$$

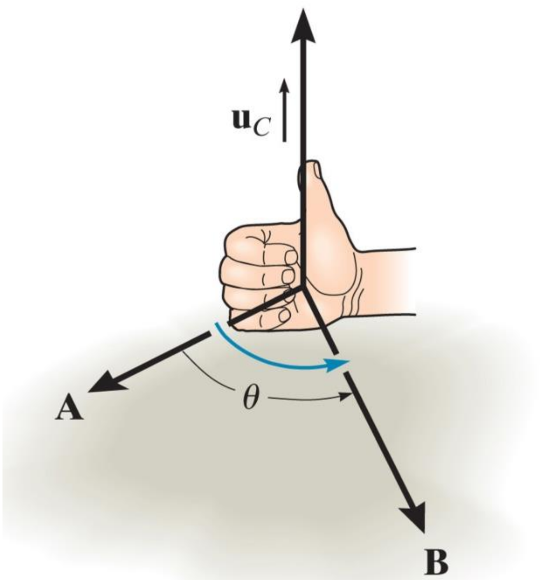
And thus the <u>vector</u> components $\boldsymbol{A}_{||}$ and $\boldsymbol{A}_{\perp}$ are given by:

$$\vec{\boldsymbol{A}}_{||} = A_{||}\, \vec{\boldsymbol{u}} = (\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{u}})\vec{\boldsymbol{u}}$$

$$\vec{\boldsymbol{A}}_{\perp} = \vec{\boldsymbol{A}} - \vec{\boldsymbol{A}}_{||}$$

Cross (or vector) product

The cross product of vectors **A** and **B** yields the vector **C**, which is written



$$\boxed{\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}}$$

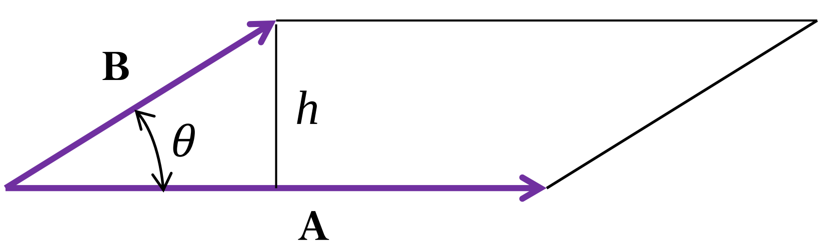
The magnitude of vector **C** is given by:

$$C = |\boldsymbol{C}| = |\boldsymbol{A}|\,|\boldsymbol{B}|\sin(\theta)$$

The vector **C** is perpendicular to the plane containing **A** and **B** (specified by the **right-hand rule**). Hence,

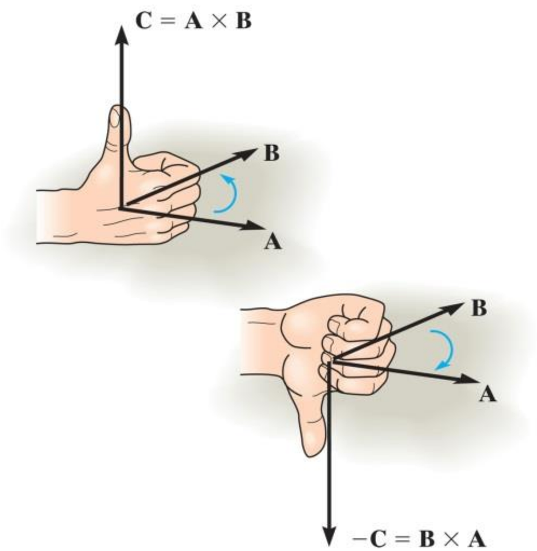
$$\vec{\boldsymbol{C}} = A\,B\,\sin(\theta)\,\vec{\boldsymbol{u}}_c$$

Geometric definition of the cross product: the magnitude of the cross product is given by the area of a parallelogram



$$Area = |\boldsymbol{A}|\,h = |\boldsymbol{A}||\boldsymbol{B}|\sin(\theta)$$
$$= |\boldsymbol{A} \times \boldsymbol{B}|$$

Cross (or vector) product



Laws of operation:

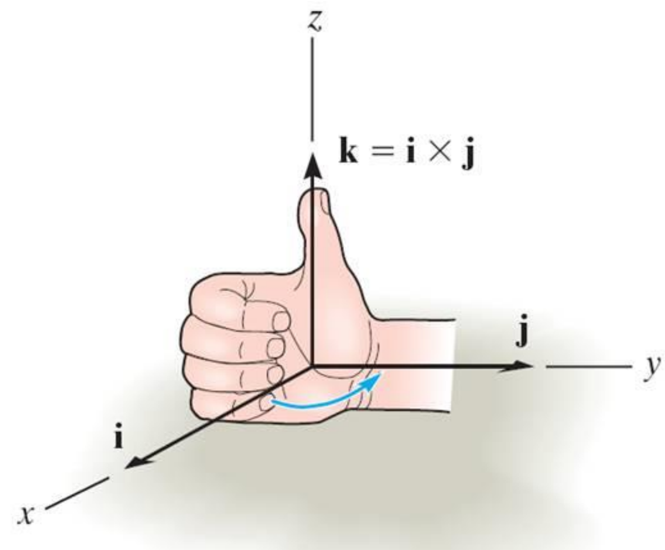
$$\boldsymbol{A} \times \boldsymbol{B} = -\boldsymbol{B} \times \boldsymbol{A}$$

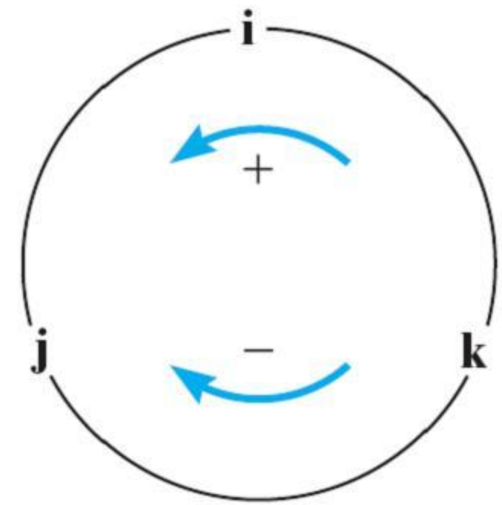
$$\alpha(\boldsymbol{A} \times \boldsymbol{B}) = (\alpha\boldsymbol{A}) \times \boldsymbol{B} = \boldsymbol{A} \times (\alpha\boldsymbol{B}) = (\boldsymbol{A} \times \boldsymbol{B})\alpha$$

$$\boldsymbol{A} \times (\boldsymbol{B} + \boldsymbol{D}) = \boldsymbol{A} \times \boldsymbol{B} + \boldsymbol{A} \times \boldsymbol{D}$$

Cross (or vector) product

The right-hand rule is a useful tool for determining the direction of the vector resulting from a cross product. Note that a vector crossed into itself is zero, e.g., $\boldsymbol{i} \times \boldsymbol{i} = 0$





Considering the cross product in Cartesian coordinates

$$\begin{aligned}
\boldsymbol{A} \times \boldsymbol{B} &= (A_x\,\boldsymbol{i} + A_y\,\boldsymbol{j} + A_z\,\boldsymbol{k}) \times (B_x\,\boldsymbol{i} + B_y\,\boldsymbol{j} + B_z\,\boldsymbol{k}) \\
&= +A_xB_x(\boldsymbol{i} \times \boldsymbol{i}) + A_xB_y(\boldsymbol{i} \times \boldsymbol{j}) + A_xB_z(\boldsymbol{i} \times \boldsymbol{k}) \\
&\quad +A_yB_x(\boldsymbol{j} \times \boldsymbol{i}) + A_yB_y(\boldsymbol{j} \times \boldsymbol{j}) + A_yB_z(\boldsymbol{j} \times \boldsymbol{k}) \\
&\quad +A_zB_x(\boldsymbol{k} \times \boldsymbol{i}) + A_zB_y(\boldsymbol{k} \times \boldsymbol{j}) + A_zB_z(\boldsymbol{k} \times \boldsymbol{k}) \\
&= (A_yB_z - A_zB_y)\boldsymbol{i} - (A_xB_z - A_zB_x)\boldsymbol{j} + (A_xB_y - A_yB_x)\boldsymbol{k}
\end{aligned}$$

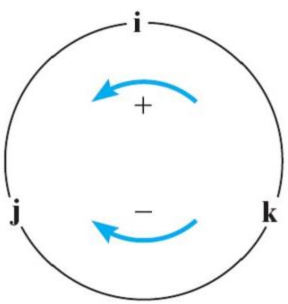
Cross (or vector) product

Also, the cross product can be written as a determinant.

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Each component can be determined using 2 × 2 determinants.



For element **i**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \mathbf{i}(A_yB_z - A_zB_y)$

Remember the negative sign

For element **j**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = -\mathbf{j}(A_xB_z - A_zB_x)$

For element **k**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \mathbf{k}(A_xB_y - A_yB_x)$