Statics - TAM 210 & TAM 211

Lecture 5

January 26, 2018

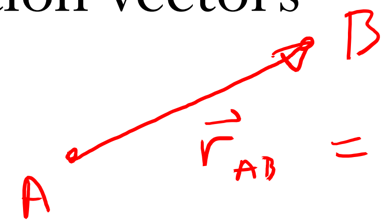
Announcements

- MATLAB training sessions
  - ~~Wed 24, Thu 25,~~ Fri 26, and Mon 29
  - DCL **L440**, Tutorial: 6:30-7:30 pm, Q&A: 7:30-8:00 pm
- Discussion section team formation using CATME. Sign up by Sunday night. Look for email from Matt Milner with info.

- Upcoming deadlines:
- Friday (1/26)
  - Mastering Engineering Tutorial3
- Tuesday (1/30)
  - Prairie Learn HW2
- Quiz 1 (1/31-2/2)
  - Reserve testing time at CBTF
  - https://cbtf.engr.illinois.edu/sched/
  - DO NOT MISS TEST TIME.
  - NO MAKE-UP.

Recap of Lecture 4

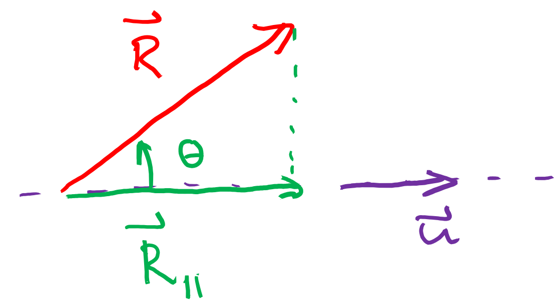
- Position vectors



$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A = \vec{r}_{HEAD} - \vec{r}_{TAIL}$$

- Force vector directed along a line

$$\vec{F} = F\,\vec{u}\,, \quad \vec{u} = \frac{\vec{r}}{|\vec{r}|}$$



- Dot (scalar) product

$$\vec{A}\cdot\vec{B} = C = |\vec{A}|\,|\vec{B}|\cos\theta = \sum_{i=x,y,z} A_i B_i$$

$$\vec{R}_{\parallel} = (\vec{R}\cdot\vec{u})\,\vec{u}$$

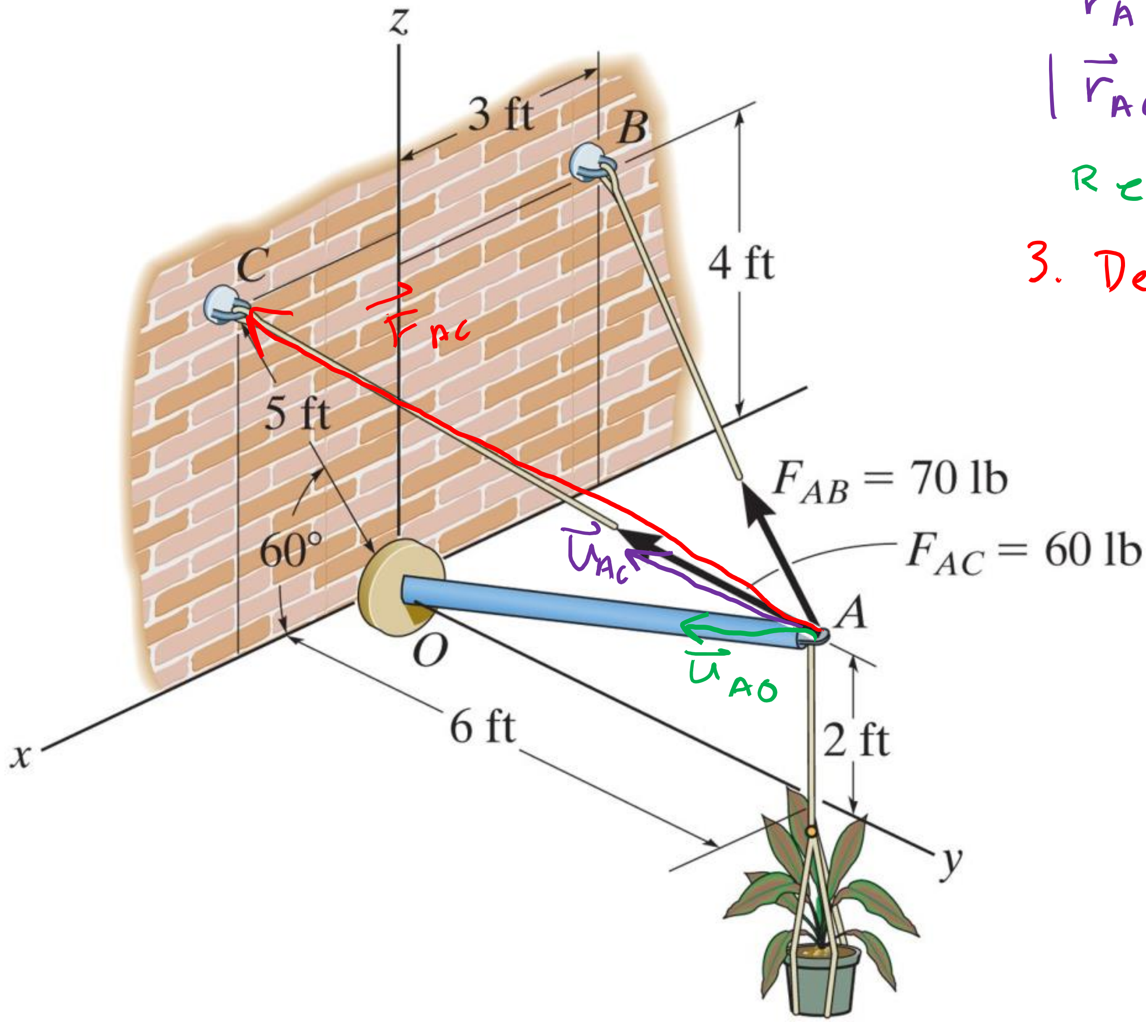
$$|\vec{R}_{\parallel}| = |\vec{R}|\cos\theta$$

- Cross (vector) product

$$\vec{A}\times\vec{B} = \vec{C} = (|\vec{A}|\,|\vec{B}|\sin\theta)\,\vec{u}_c = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

← determinant

Determine the projected component of the force vector $\overrightarrow{F_{AC}}$ along the axis of strut AO. Express your result as a Cartesian vector



Plan: use $(\vec{F}_{AC})_{AO} = (\vec{F}_{AC} \cdot \vec{u}_{AO})\vec{u}_{AO}$

1. Find $\vec{u}_{AC}$ & $\vec{u}_{AO}$

2. $\vec{u}_{AC} = \dfrac{\vec{r}_{AC}}{|\vec{r}_{AC}|}$

$\vec{r}_{AC} = (x_C - x_A)\hat{i} + (y_C - y_A)\hat{j} + (z_C - z_A)\hat{k}$

$|\vec{r}_{AC}| = \sqrt{(r_{AC})_x^2 + (r_{AC})_y^2 + (r_{AC})_z^2}$

Repeat for $\vec{u}_{AO}$

3. Derive $\vec{F}_{AC} = |\vec{F}_{AC}|\,\vec{u}_{AC}$

↑ 60 lb, given

4. $(\vec{F}_{AC})_{AO} = (\vec{F}_{AC} \cdot \vec{u}_{AO})\,\vec{u}_{AO}$

Example

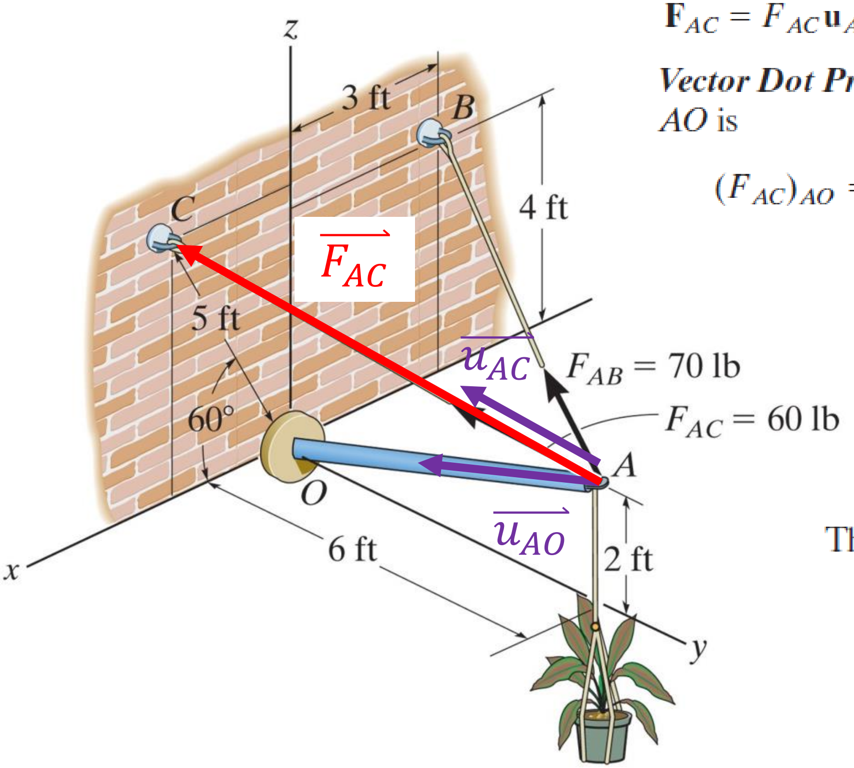
Determine the projected component of the force vector $\boldsymbol{F}_{AC}$ along the axis of strut AO. Express your result as a Cartesian vector

Unit Vectors: The unit vectors $\mathbf{u}_{AC}$ and $\mathbf{u}_{AO}$ must be determined first.

$$\mathbf{u}_{AC} = \frac{(5\cos 60^\circ - 0)\mathbf{i} + (0-6)\mathbf{j} + (5\sin 60^\circ - 2)\mathbf{k}}{\sqrt{(5\cos 60^\circ - 0)^2 + (0-6)^2 + (\cancel{0} - 2)^2}} = 0.3621\mathbf{i} - 0.8689\mathbf{j} + 0.3375\mathbf{k}$$

$5\sin 60^o$

$$\mathbf{u}_{AO} = \frac{(0-0)\mathbf{i} + (0-6)\mathbf{j} + (0-2)\mathbf{k}}{\sqrt{(0-0)^2 + (0-6)^2 + (0-2)^2}} = -0.9487\mathbf{j} - 0.3162\,\mathbf{k}$$



Thus, the force vectors $\mathbf{F}_{AC}$ is given by

$$\mathbf{F}_{AC} = F_{AC}\mathbf{u}_{AC} = 60(0.3621\mathbf{i} - 0.8689\mathbf{j} + 0.3375\mathbf{k}) = \{21.72\mathbf{i} - 52.14\mathbf{j} + 20.25\mathbf{k}\}\text{ lb}$$

Vector Dot Product: The magnitude of the projected component of $\mathbf{F}_{AC}$ along strut AO is

$$\begin{aligned}(F_{AC})_{AO} = \mathbf{F}_{AC}\cdot\mathbf{u}_{AO} &= (21.72\mathbf{i} - 52.14\mathbf{j} + 20.25\mathbf{k})\cdot(-0.9487\mathbf{j} - 0.3162\mathbf{k}) \\ &= (21.72)(0) + (-52.14)(-0.9487) + (20.25)(-0.3162) \\ &= 43.057\text{ lb}\end{aligned}$$

Thus, $(\mathbf{F}_{AC})_{AO}$ expressed in Cartesian vector form can be written as

$$(\mathbf{F}_{AC})_{AO} = (F_{AC})_{AO}\mathbf{u}_{AO} = 43.057(-0.9487\mathbf{j} - 0.3162\mathbf{k})$$

$$\boxed{= \{-40.8\mathbf{j} - 13.6\mathbf{k}\}\text{ lb}}$$

Chapter 3: Equilibrium of a particle

Goals and Objectives

- Practice following general procedure for analysis.

- Introduce the concept of a free-body diagram for an object modeled as a particle.

- Solve particle equilibrium problems using the equations of equilibrium.

General procedure for analysis

1. Read the problem carefully; write it down carefully.
2. MODEL THE PROBLEM: Draw given diagrams neatly and construct additional figures as necessary.
3. Apply principles needed.
4. Solve problem symbolically. Make sure equations are dimensionally homogeneous
5. Substitute numbers. Provide proper units *throughout*. Check significant figures. Box the final answer(s).
6. See if answer is reasonable.

Most effective way to learn engineering mechanics is to ***solve problems!***

Equilibrium of a particle

According to Newton's first law of motion, a particle will be in **equilibrium** (that is, it will remain at rest or continue to move with constant velocity) if and only if

$$\sum \vec{F} = 0$$

where $\vec{\boldsymbol{F}}$ is the resultant force vector of all forces acting on a particle.

3-Dimensional forces: equilibrium requires

$$\sum \vec{F} = \sum F_x \hat{i} + \sum F_y \hat{j} + \sum F_z \hat{k} \Rightarrow \boxed{\begin{array}{l} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{array}}$$

Equilibrium of a particle (cont)

Coplanar forces: if all forces are acting in a single plane, such as the "xy" plane, then the equilibrium condition becomes

$$\sum \vec{F} = 0 = \sum F_x \hat{i} + \sum F_y \hat{j}$$

$$\Rightarrow \boxed{\begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \end{aligned}}$$

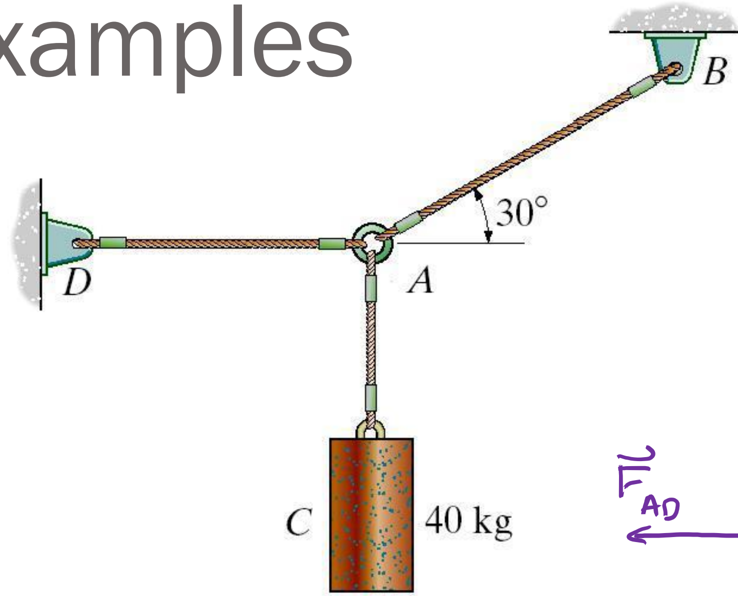
Free body diagram

Drawing of a body, or part of a body, on which all forces acting on the body are shown.

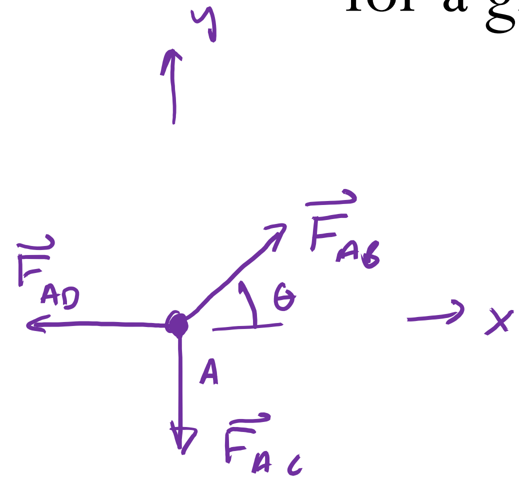
- Key to writing the equations of equilibrium.
- Can draw for any object/subsystem of system. Pick the most appropriate object. (Equal & opposite forces on interacting bodies.)

- ❑ Draw Outlined Shape: image object free of its surroundings
    - ❑ Sometimes may collapse large object into point mass
- ❑ Establish x, y, z axes in any suitable orientation
    - ❑ Show positive directions for translation and rotation
- ❑ Show all forces acting on the object at points of application
- ❑ Label all known and unknown forces
- ❑ Sense ("direction") of unknown force can be assumed. If solution is negative, then the sense is reverse of that shown on FBD

Examples



Find the tension in the cables for a given mass.



① $\Sigma F_x = 0 \Rightarrow F_{AB}\cos\theta - F_{AD} = 0$

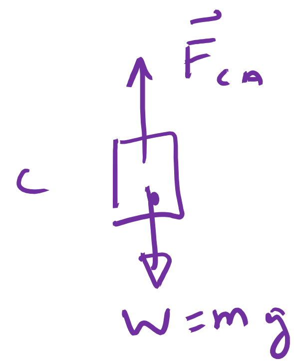
② $\Sigma F_y: F_{AB}\sin\theta - F_{AC} = 0$

3 unk: F_{AB}, F_{AD}, F_{AC}

2 eqn

→ Indeterminate

- ❑ Draw Outlined Shape
- ❑ Establish x, y, z axes
- ❑ Show all forces acting on object
- ❑ Label known and unknown forces
- ❑ Assume sense of unknown force



$\Sigma F_y: F_{CA} - mg = 0$

$F_{CA} = mg$ ③

$F_{CA} = -F_{AC}$ ④

+1 unk: F_{CA}

4 eqn ✓