Statics - TAM 210 & TAM 211

Lecture 6

January 29, 2018

Announcements

- MATLAB training sessions
  - ~~Wed 24, Thu 25, Fri 26, and~~ Mon 29
  - DCL **L440**, Tutorial: 6:30-7:30 pm, Q&A: 7:30-8:00 pm
- All should have signed up on CATME for discussion section team formation.

- Upcoming deadlines:
- Tuesday (1/30)
  - Prairie Learn HW2
- Quiz 1 (1/31-2/2)
  - Reserve testing time at CBTF
  - https://cbtf.engr.illinois.edu/sched/
  - NO MAKE-UP.
  - Lectures 1- 4 material
- Friday (2/1)
  - Mastering Engineering Tutorial4
- Quiz 2 (2/7-9)

Chapter 3: Equilibrium of a particle

Goals and Objectives

- Practice following general procedure for analysis.
- Introduce the concept of a free-body diagram for an object modeled as a particle.
- Solve equilibrium problems using the equations of equilibrium.
  - 3D, 2D planar, idealizations (smooth surfaces, pulleys, springs)

Recap: General procedure for analysis

1. Read the problem carefully; write it down carefully.
2. MODEL THE PROBLEM: Draw given diagrams neatly and construct additional figures as necessary.
3. Apply principles needed.
4. Solve problem symbolically. Make sure equations are dimensionally homogeneous
5. Substitute numbers. Provide proper units *throughout*. Check significant figures. Box the final answer(s).
6. See if answer is reasonable.

Most effective way to learn engineering mechanics is to ***solve problems!***

Recap: Equilibrium of a particle

3-Dimensional forces: equilibrium requires

$$\sum \boldsymbol{F} = \sum F_x\,\boldsymbol{i} + \sum F_y\,\boldsymbol{j} + \sum F_z\,\boldsymbol{k} = \boldsymbol{0} \quad \Longrightarrow \quad \begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum F_z &= 0 \end{aligned}$$

Coplanar forces: if all forces are acting in a single plane, such as the "xy" plane, then the equilibrium condition becomes

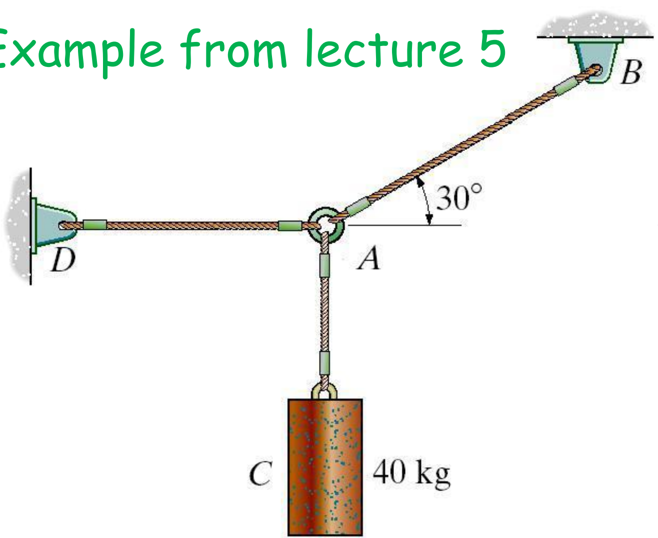
$$\sum \boldsymbol{F} = \sum F_x\,\boldsymbol{i} + \sum F_y\,\boldsymbol{j} = \boldsymbol{0} \quad \Longrightarrow \quad \begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \end{aligned}$$

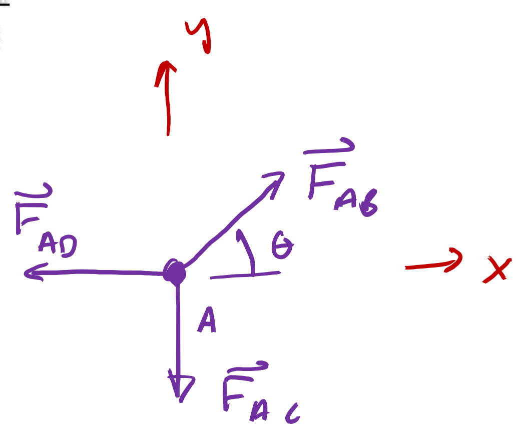
Recap: Free body diagram

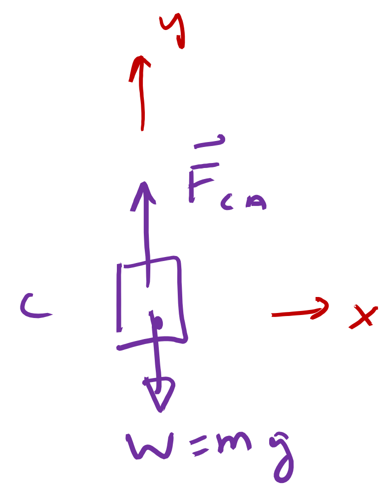
Drawing of a body, or part of a body, on which all forces acting on the body are shown.

- ❑ Draw Outlined Shape: image object free of its surroundings
- ❑ Establish x, y, z axes in any suitable orientation
  - ❑ Show positive directions for translation and rotation
- ❑ Show all forces acting on the object at points of application
- ❑ Label all known and unknown forces
- ❑ Sense ("direction") of unknown force can be assumed. If solution is negative, then the sense is reverse of that shown on FBD

Example from lecture 5

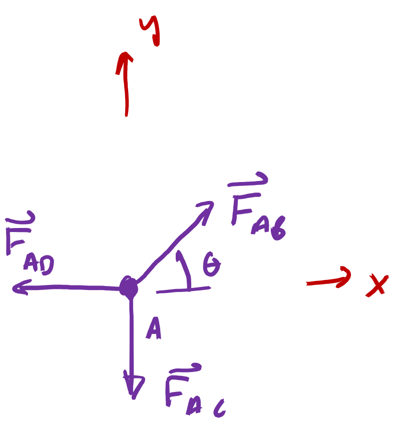






Equations of equilibrium

- Use FBD to write equilibrium equations in x, y, z directions
  - $\sum \overrightarrow{F_x} = 0, \sum \overrightarrow{F_y} = 0,$ and if 3D $\sum \overrightarrow{F_z} = 0,$
  - If # equations ≥ # unknown forces, **statically determinate** (can solve for unknowns)
  - If # equations < # unknown forces, **indeterminate** (can **NOT** solve for unknowns), need more equations
- Get more equations from FBD of other bodies in the problem



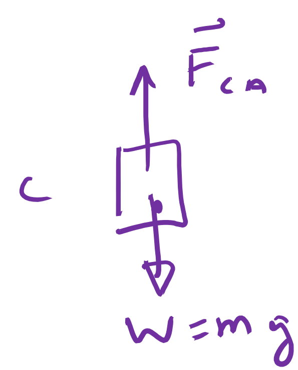
① $\sum F_x = 0 \rightarrow F_{AB}\cos\theta - F_{AD} = 0$

② $\sum F_y: F_{AB}\sin\theta - F_{AC} = 0$

3 unk: F_{AB}, F_{AD}, F_{AC}

2 eqn

→ Indeterminate



$\sum F_y: F_{CA} - mg = 0$

$F_{CA} = mg$ ③

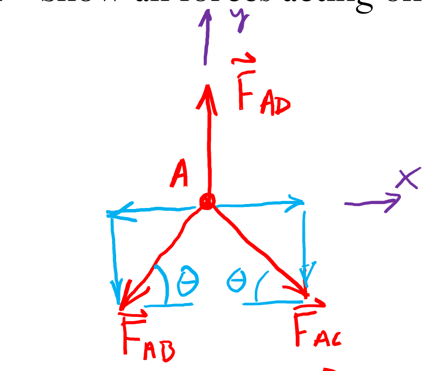
$F_{CA} = -F_{AC}$ ④

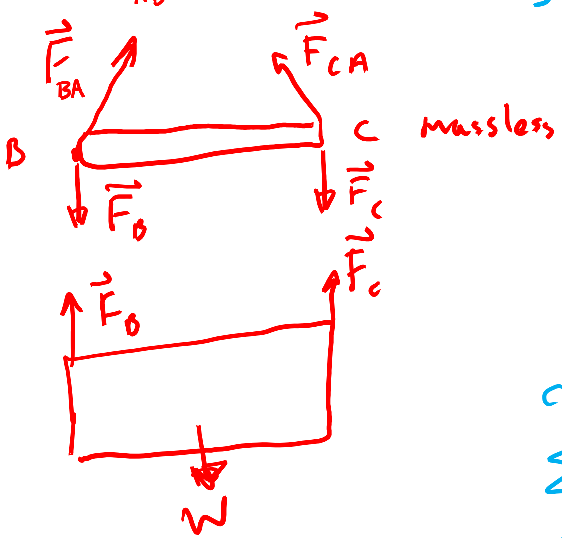
+1 unk: F_{CA}

4 eqn ✓

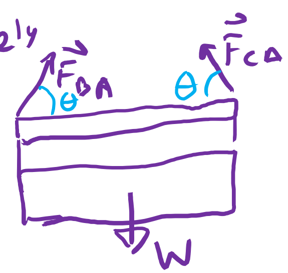
Find the forces in cables AB and AC?

- ❑ Draw Outlined Shape
- ❑ Establish x, y, z axes
- ❑ Show all forces acting on object
- ❑ Label known and unknown forces
- ❑ Assume sense of unknown force





Alternatively



Eqns of Equilibrium

On A:

$\Sigma F_x : |\vec{F}_{AC}|\cos\theta - |\vec{F}_{AB}|\cos\theta = 0$ (1)

$\Sigma F_y : \vec{F}_{AD} - |\vec{F}_{AB}|\sin\theta - F_{AC}\sin\theta = 0$ (2)

3 unk: $\vec{F}_{AB}, \vec{F}_{AC}, \vec{F}_{AD}$

2 eqns → Need more eqns

combined object:

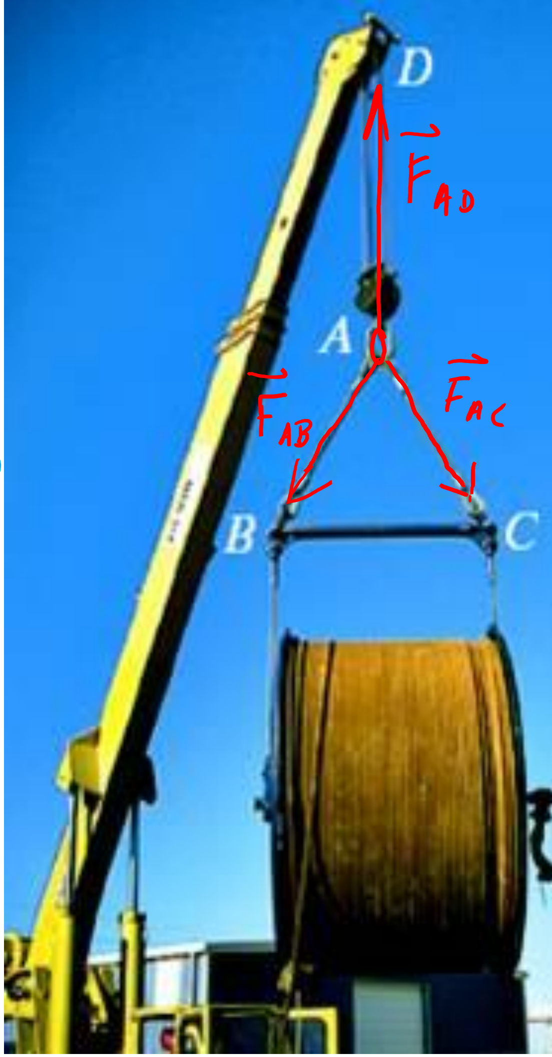
$\Sigma F_x : F_{BA}\cos\theta - F_{CA}\cos\theta = 0$ (3)

$\Sigma F_y : F_{BA}\sin\theta + F_{CA}\sin\theta - W = 0$ (4)

+ 2 unk: F_{BA}, F_{CA}

$F_{BA} = -F_{AB}$ (5) $F_{CA} = -F_{AC}$ (6)

5 unknowns, 6 eqns → Statically Determinate ✓

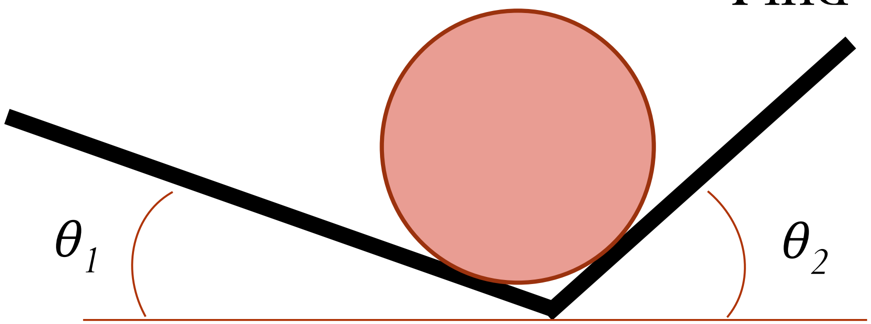


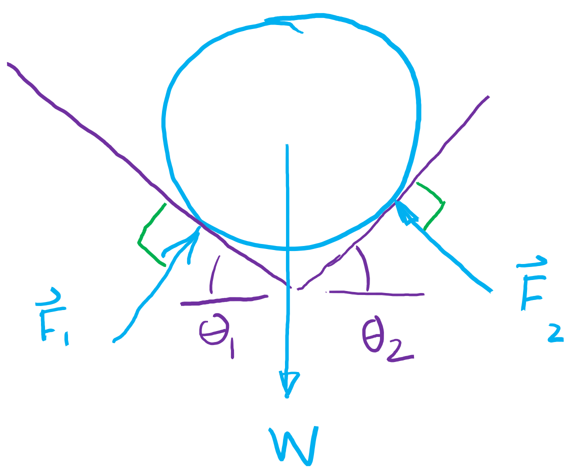
Idealizations

Find contact forces on smooth surface

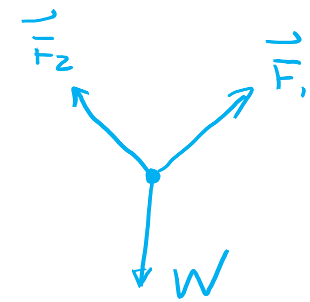
No Friction

Only $\perp$ forces (Normal force)





$\Rightarrow$



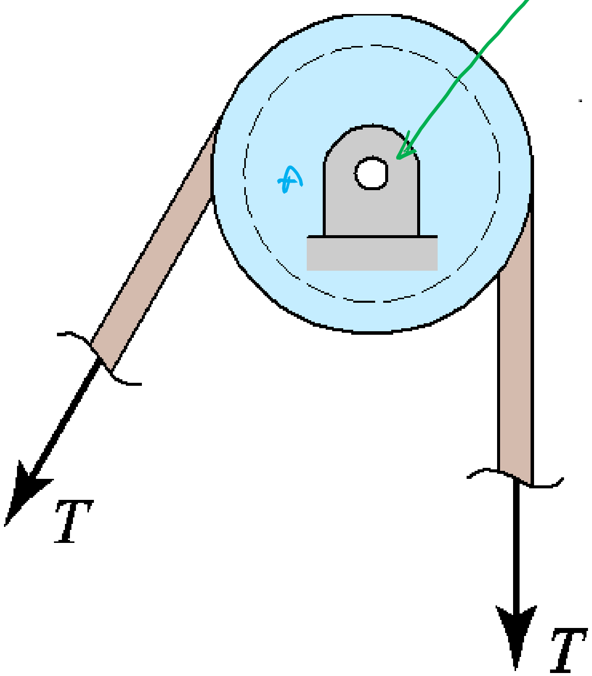
Can collapse to point mass

Idealizations

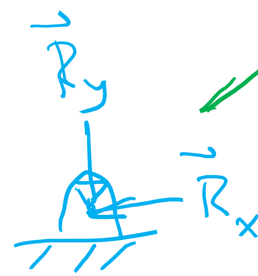
massless &

Pulleys are (usually) regarded as frictionless, then the tension in a rope or cord around the pulley is the same on either side.

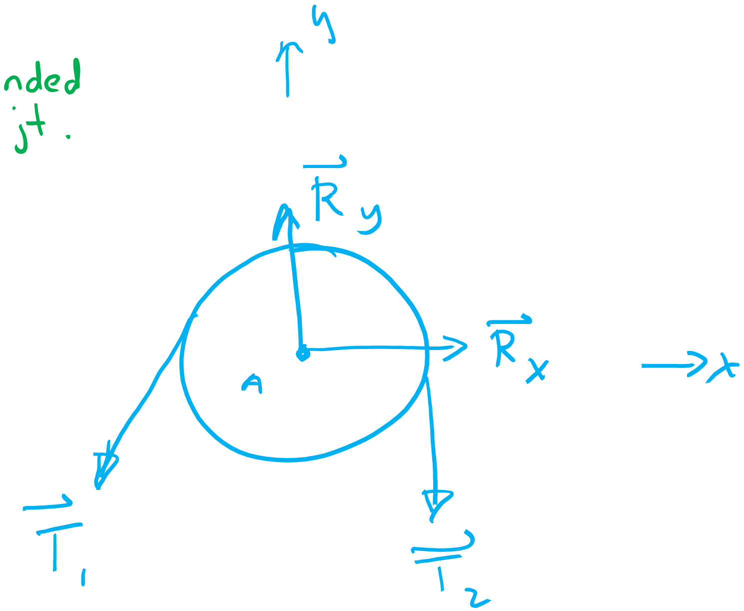
This pulley is secured in position by grounded pin jt.



Frictionless pulley



Reaction Forces on pin jt



$$|\vec{T}_1| = |\vec{T}_2|$$

Assuming cable is massless & rigid

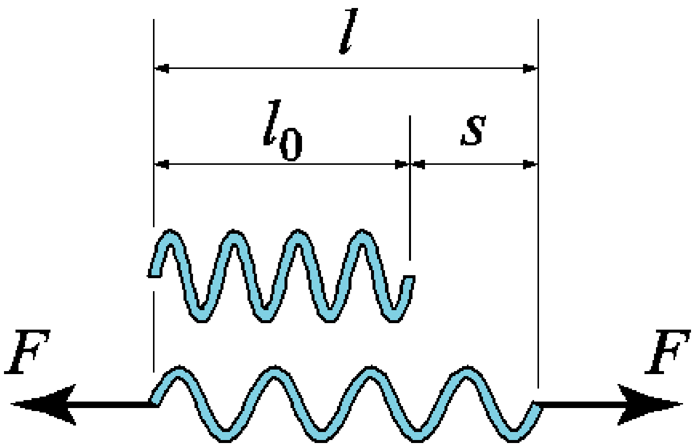
Magnitudes are same

Directions do not need to be the same

Idealizations

massless

Springs are (usually) regarded as linearly elastic; then the tension is proportional to the *change* in length *s*.

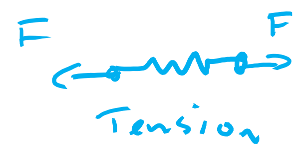


$$F = ks = k(l - l_0)$$

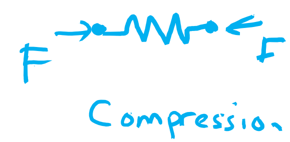
Linearly elastic spring

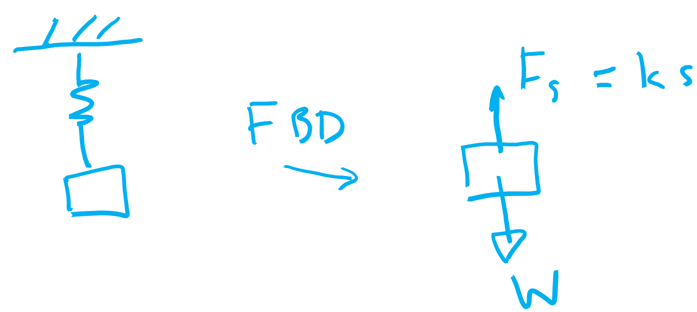
$s = l_f - l_0$

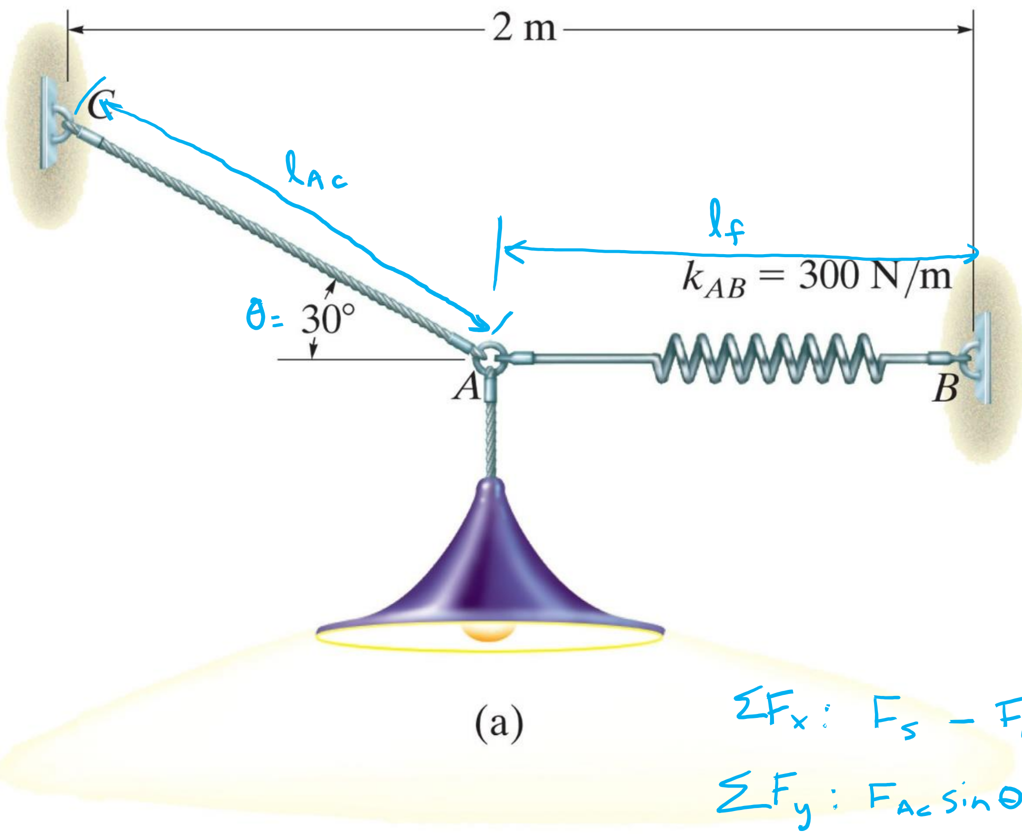
if $s > 0 \rightarrow$ elongation



if $s < 0 \rightarrow$ compression







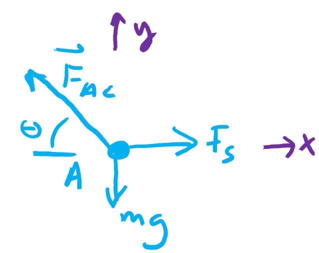
(a)

Determine the required length of cord AC so that the 8-kg lamp can be suspended in the position shown. The undeformed spring length is 0.4 m and has a stiffness of 300 N/m.

Given: $m = 8\,kg$, $l_0 = 0.4\,m$, $k_{AB} = 300\,N/m$
$\theta = 30°$

Find: l_{AC}

Soln: FBD of A



$\Sigma F_x: \; F_s - F_{AC}\cos\theta = 0$ ①

$\Sigma F_y: \; F_{AC}\sin\theta - mg = 0$ ②

$F_s = k_{AB}\, s = k_{AB}(l_f - l_0)$ ③

③ into ①: $k_{AB}(l_f - l_0) - F_{AC}\cos\theta = 0$

insert ②: $k_{AB}(l_f - l_0) - \left(\frac{mg}{\sin\theta}\right)\cos\theta = 0 \Rightarrow l_f = \left(\frac{mg}{k_{AB}}\right)\frac{\cos\theta}{\sin\theta} + l_0 = 0.853\,m$

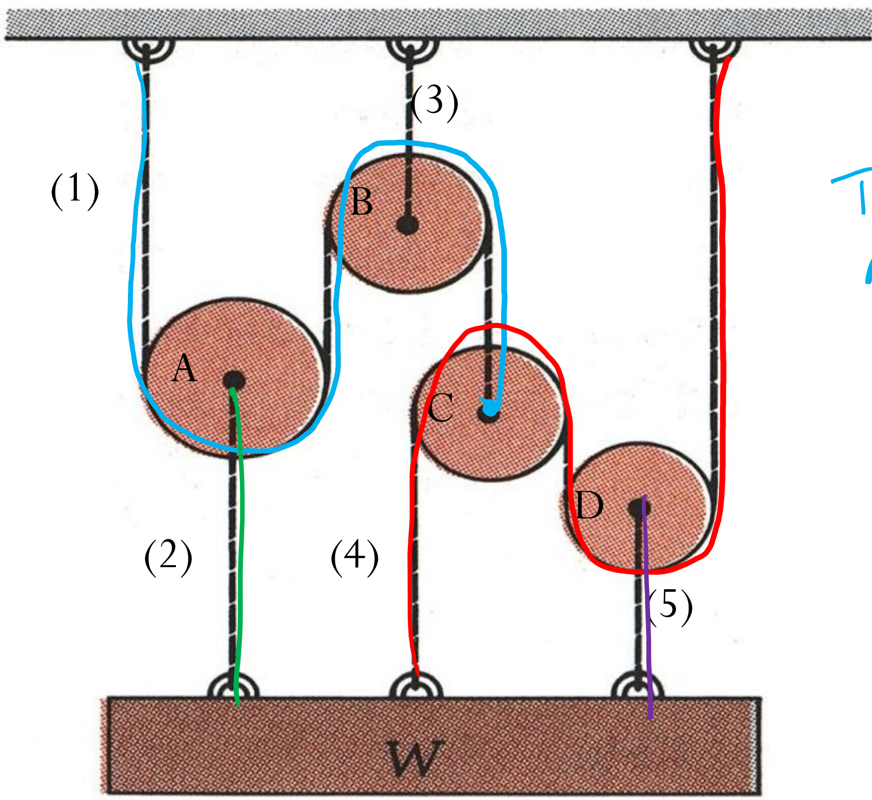
Use geometrical constraint:

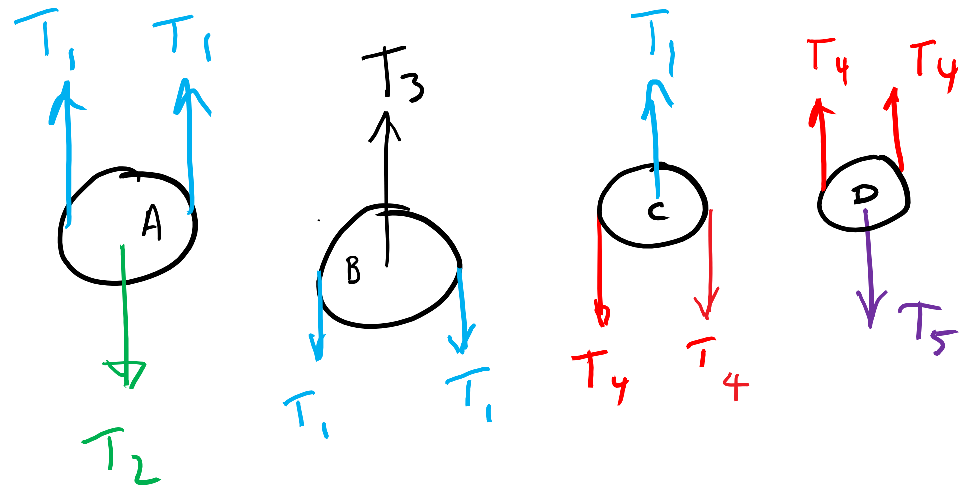
$2m = l_f + l_{AC}\cos\theta$

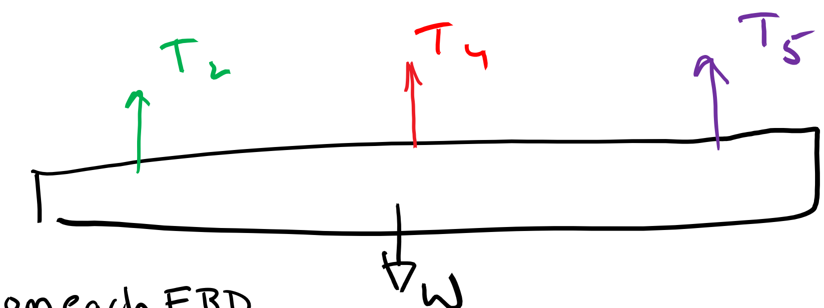
$l_{AC} = \frac{2m - l_f}{\cos\theta} = \boxed{1.32\,m = l_{AC}}$

The five ropes can each take 1500 N without breaking. How heavy can *W* be without breaking any?

Note: No pin jt reaction forces at center of pulleys because these pulleys are not secured to a fixed (or grounded) pin jt.







$$\Sigma F_y = 0$$

write eqns of equilibrium from each FBD