Statics - TAM 210 & TAM 211

Lecture 7

January 31, 2018

Announcements

- CBTF has physical calculators!!! Casio fx-300MS (see CBTF website)
- Upcoming deadlines:
- Quiz 1 (1/31-2/2)
  - Reserve testing time at CBTF
  - https://cbtf.engr.illinois.edu/sched/
  - NO MAKE-UP.
  - Lectures 1- 4 material
- Friday (2/1)
  - Mastering Engineering Tutorial 4
- Tuesday (2/6)
  - PL Homework 3
- Quiz 2 (2/7-9)
  - Reserve testing time at CBTF

Recap: Equations of equilibrium

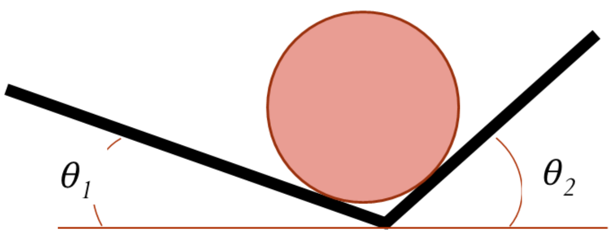
- Use FBD to write equilibrium equations in x, y, z directions
  - $\sum \overrightarrow{\boldsymbol{F_x}} = 0$, $\sum \overrightarrow{\boldsymbol{F_y}} = 0$, and if 3D $\sum \overrightarrow{\boldsymbol{F_z}} = 0$,
  - If # equations ≥ # unknown forces, **statically determinate** (can solve for unknowns)
  - If # equations < # unknown forces, **indeterminate** (can **NOT** solve for unknowns), need more equations
- Get more equations from FBD of other bodies in the problem

Recap: Idealizations

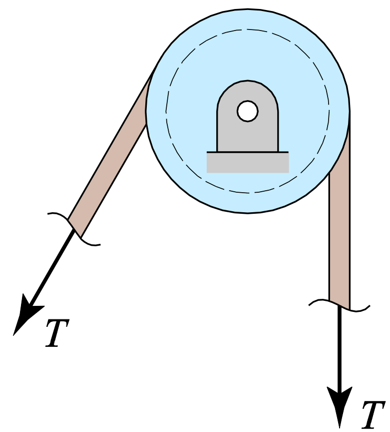
Smooth surfaces: regarded as frictionless; force is perpendicular to surface

Pulleys: (usually) regarded as frictionless; tension around pulley is same on either side.

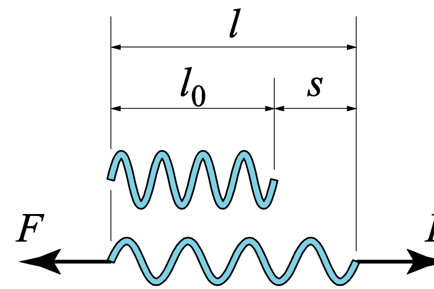
Springs: (usually) regarded as linearly elastic; tension is proportional to *change* in length *s*.



Smooth surface

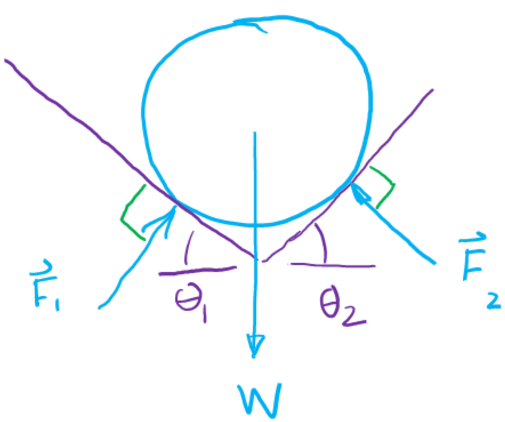


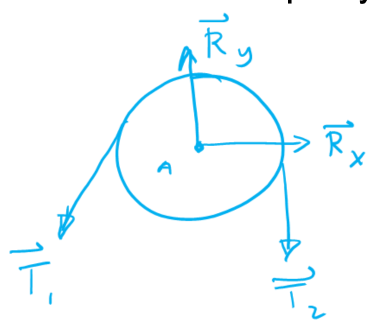
Frictionless pulley



$$F = ks = k(l - l_0)$$

Linearly elastic spring





$|\vec{T}_1| = |\vec{T}_2|$

Assuming cable is massless & rigid

Magnitudes are same

Directions do not need to be the same

$s = l_f - l_0$

if $s > 0 \rightarrow$ elongation

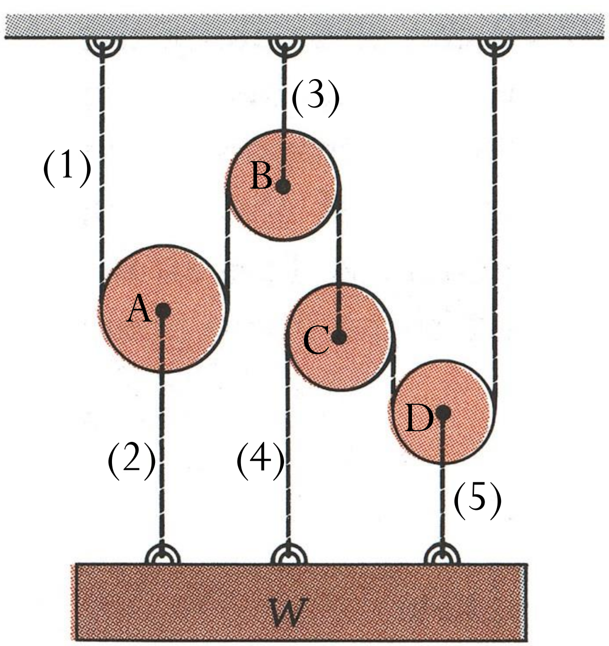
if $s < 0 \rightarrow$ compression

Equilibrium of a system of particles

Some practical engineering problems involve the statics of interacting or interconnected particles. To solve them, we use Newton's first law

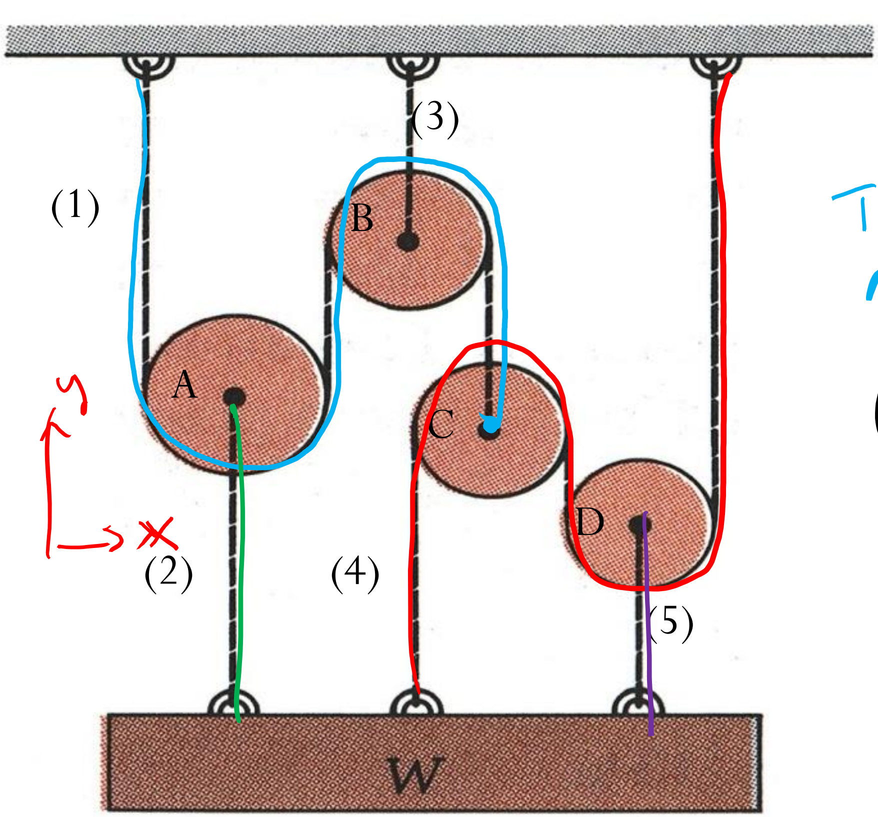
$$\Sigma \mathbf{F} = \mathbf{0}$$

on selected multiple free-body diagrams of particles or groups of particles.

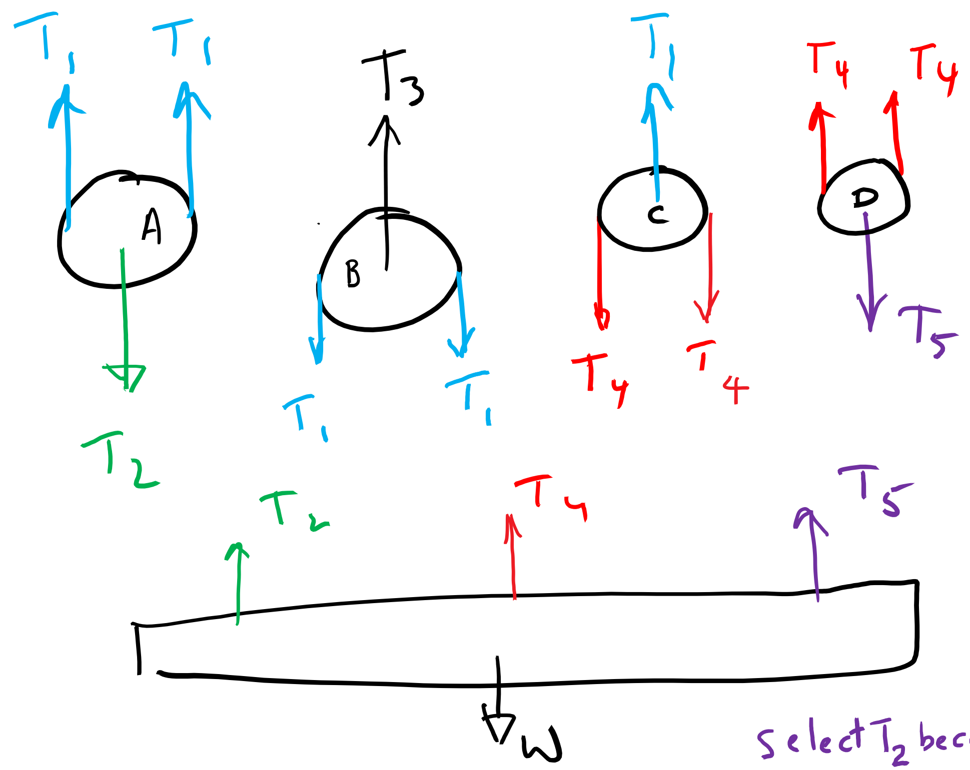


The five ropes can each take 1500 N without breaking. How heavy can *W* be without breaking any?

The five ropes can each take 1500 N without breaking. How heavy can W be without breaking any?



Note: No pin jt reaction forces at center of pulleys because these pulleys are not secured to a fixed (or grounded) pin jt.



$\Sigma F_y = 0$

write eqns of equilibrium from each FBD

A: $2T_1 - T_2 = 0$

B: $-2T_1 + T_3 = 0$

C: $T_1 - 2T_4 = 0$

D: $2T_4 - T_5 = 0$

W: $T_2 + T_4 + T_5 - W = 0$

$T_2 = 2T_1$

$T_3 = 2T_1 = T_2$

$T_4 = \frac{1}{2}T_1$

$T_5 = 2T_4 = T_1$

$W = 2T_1 + \frac{1}{2}T_1 + T_1 \Rightarrow W = 3.5T_1 = \frac{3.5}{2}T_2$

If $T_2 = 1500\text{N}$

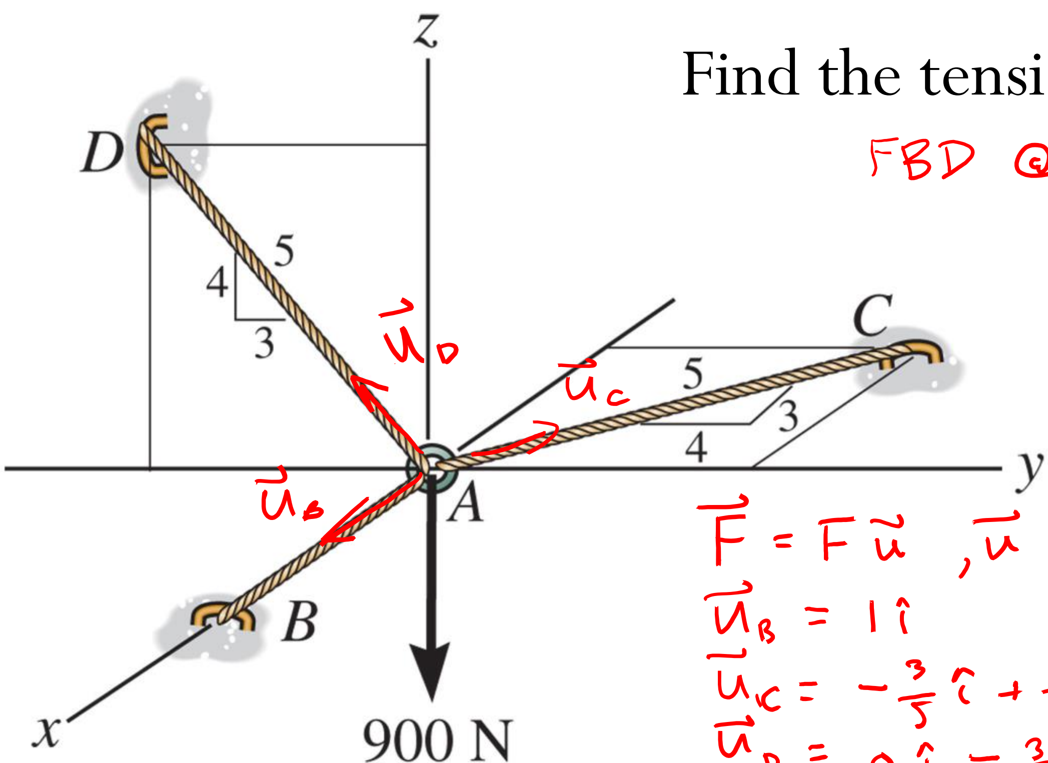
$W = 2625\text{N}$

Select T_2 because want a rope that will support the largest load. Then set it's load to the breakage limit. T_3 could also be possible choice

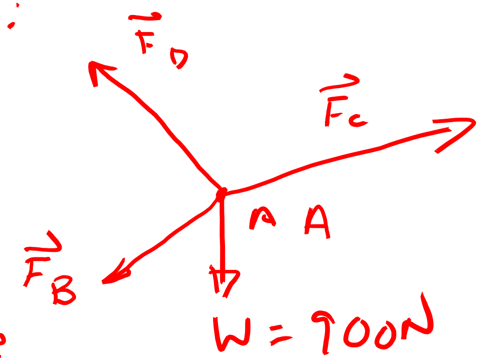
3D force systems

Use $\Sigma \overrightarrow{F_x} = 0, \Sigma \overrightarrow{F_y} = 0, \Sigma \overrightarrow{F_z} = 0$

Find the tension developed in each cable



FBD @ A:



$\vec{F} = F\vec{u}$, $\vec{u} = \frac{\vec{r}}{|\vec{r}|}$

$\vec{u}_B = 1\hat{i}$

$\vec{u}_C = -\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}$

$\vec{u}_D = 0\hat{i} - \frac{3}{5}\hat{j} + \frac{4}{5}\hat{k}$

$\Sigma F_x: \; F_B - F_C\left(\frac{3}{5}\right) = 0$

$\Sigma F_y: \; -F_D\left(\frac{3}{5}\right) + F_C\left(\frac{4}{5}\right) = 0$

$\Sigma F_z: \; F_D\left(\frac{4}{5}\right) - 900 = 0$

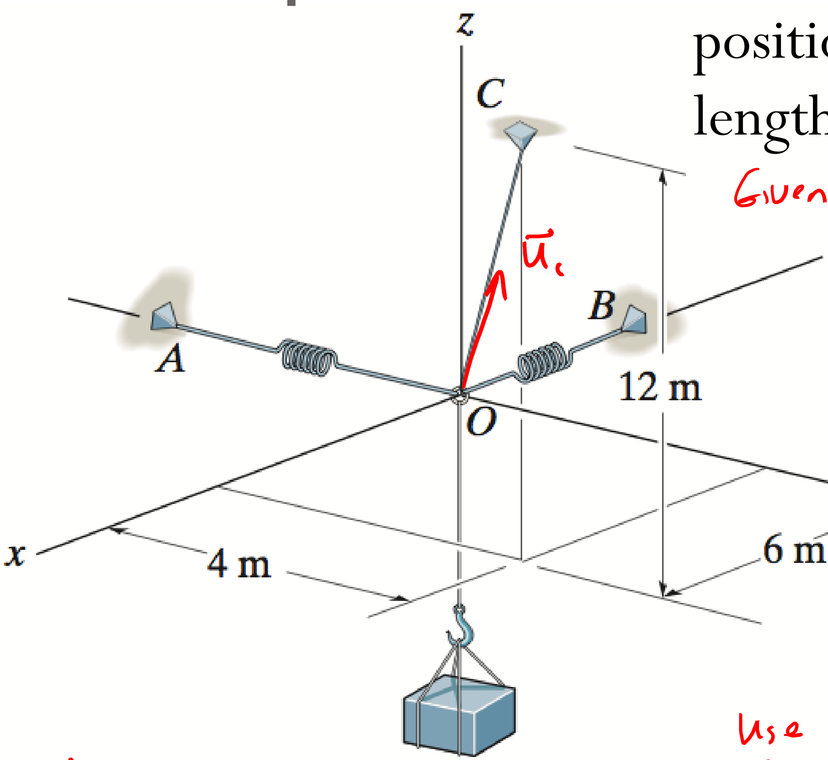
$\Rightarrow F_B, F_C, F_D$

Solve for the magnitudes (tensions) of the 3 cables

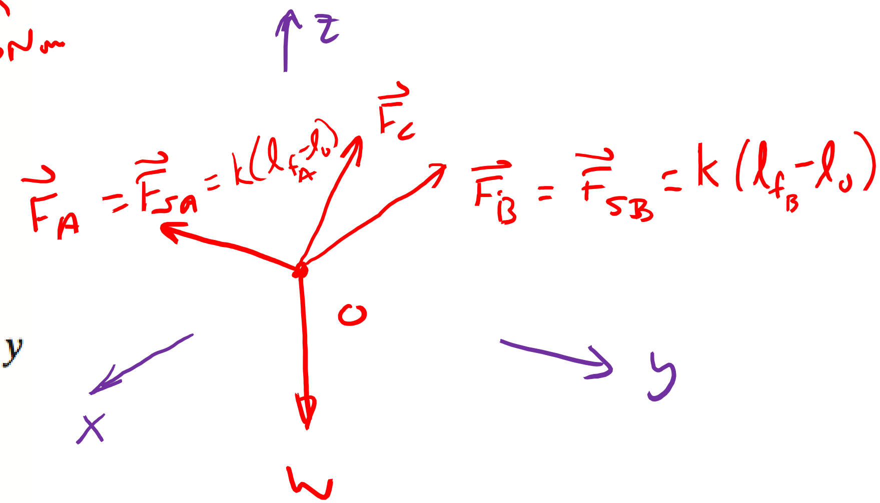
If wanted the forces, then compute the vectors. $\vec{F}_B = F_B\vec{u}_B$, etc.

Example – 3D

Determine the stretch in each of the two springs required to hold the 20-kg crate in the equilibrium position shown. Each spring has an unstretched length of 2 m and a stiffness of $k = 360$ N-m.



Given: $l_0 = 2\,m$
$k = 360\,N/m$



$\vec{F}_A = -F_A\hat{j} = -F_{sA}$

$\vec{F}_B = -F_B\hat{i} = -F_{sB}$

$\vec{F}_c = F_c\vec{u}_c$

$\vec{u}_c = \dfrac{\vec{r}}{|\vec{r}|} = \dfrac{6\hat{i} + 4\hat{j} + 12\hat{k}}{\sqrt{6^2 + 4^2 + 12^2}}$

$\vec{W} = -mg\hat{k}$

Use

$\Sigma F_x:\ -F_B + F_{cx} = 0 \rightarrow F_B = F_{cx},\ F_{cx} = F_c u_{cx} \Rightarrow F_B = F_c u_{cx}$

$\Sigma F_y:\ -F_A + F_{cy} = 0$

$\Sigma F_z:\ F_{cz} - mg = 0 \Rightarrow F_{cz} = mg,\ F_{cz} = F_c u_{cz} \Rightarrow F_c = \dfrac{mg}{u_{cz}}$

$y:\ F_A = F_{cy} \rightarrow F_A = F_c u_{cy},\ F_A = k s_A \Rightarrow \boxed{s_A = \dfrac{F_c u_{cy}}{k}}$

$x:\ F_B = F_{cx} \rightarrow F_B = F_c u_{cx},\ F_B = k s_B \Rightarrow \boxed{s_B = \dfrac{F_c u_{cx}}{k}}$

Chapter 4: Force System Resultants

Goals and Objectives

- Discuss the concept of the moment of a force and show how to calculate it in two and three dimensions

- How to find the moment about a specified axis

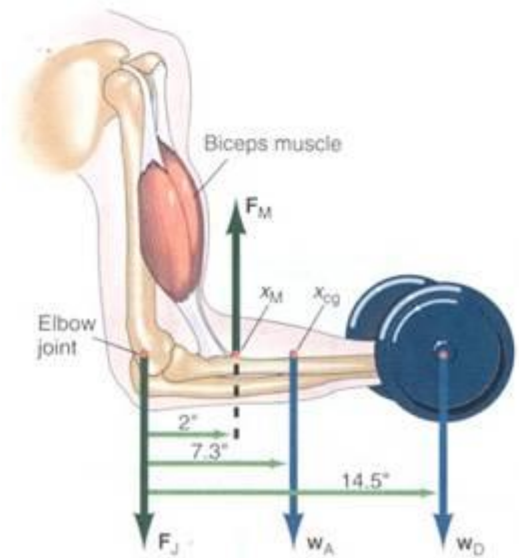
- Define the moment of a couple

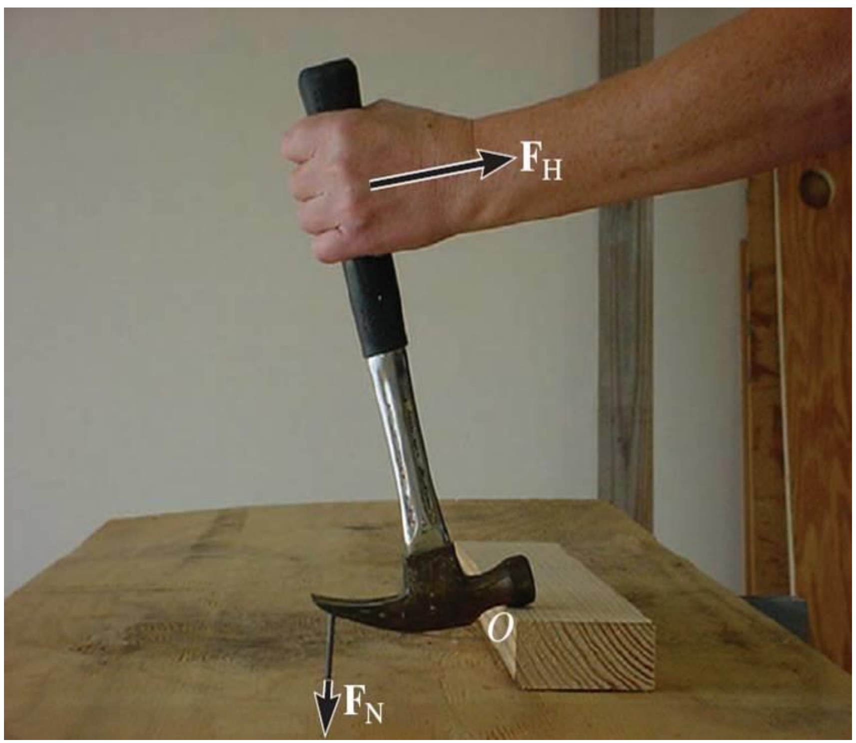
- Finding equivalence force and moment systems

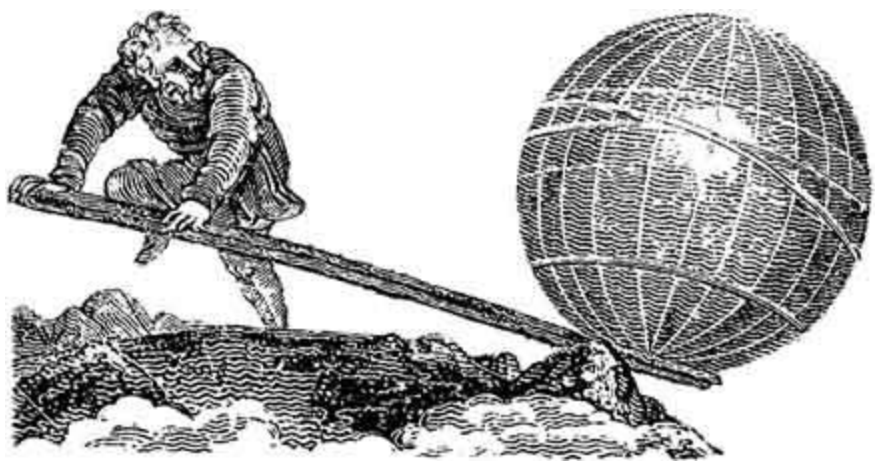
- Reduction of distributed loading

Moment of a force

The **moment of a force about a point** provides a measure of the **tendency for rotation** (sometimes called a torque).



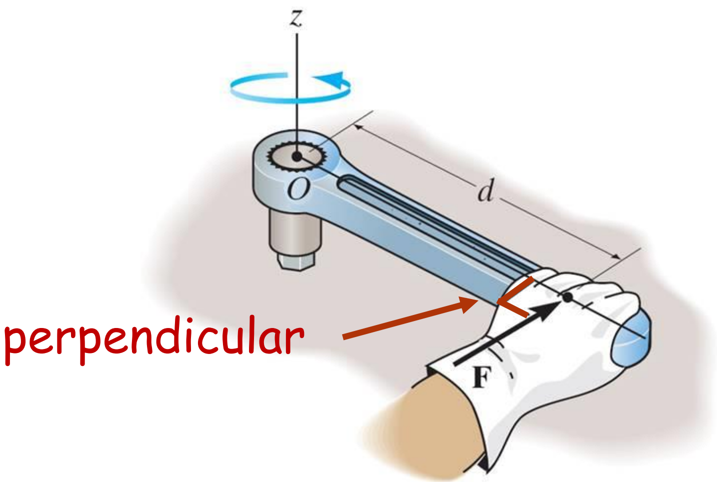


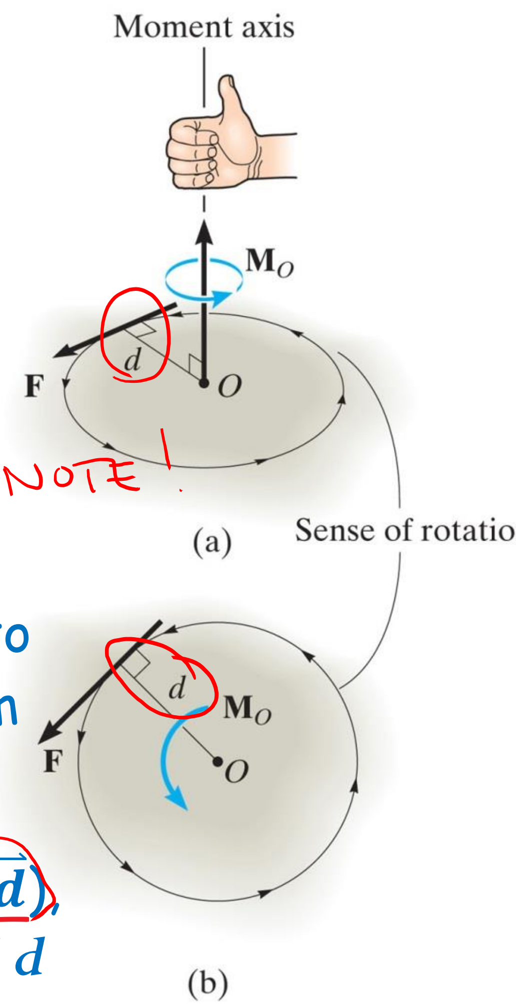


Moment 1.A very brief period of time. An exact point in time. 2. Importance. **3. A turning effect produced by a force acting at a distance on an object.** Oxford Dictionary

Moment of a force – scalar formulation

The **moment of a force about a point** provides a measure of the **tendency for rotation** (sometimes called a torque).





IMPORTANT POINT TO NOTE!

Direction: Moment about point O $\overrightarrow{M_O}$ is perpendicular to the plane that contains the force $\vec{F}$ and its moment arm $\vec{d}$. The right-hand rule is used to define the sense.

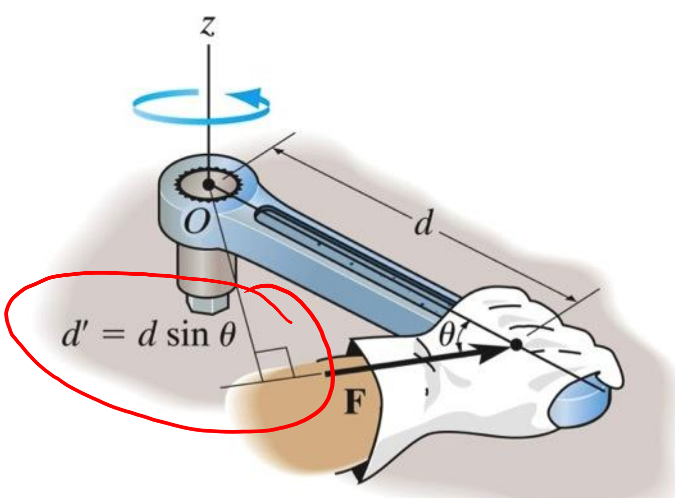
Magnitude: In a 2D case (where $\vec{F}$ is perpendicular to $\vec{d}$), the magnitude of the moment about point O is $M_O = F\,d$

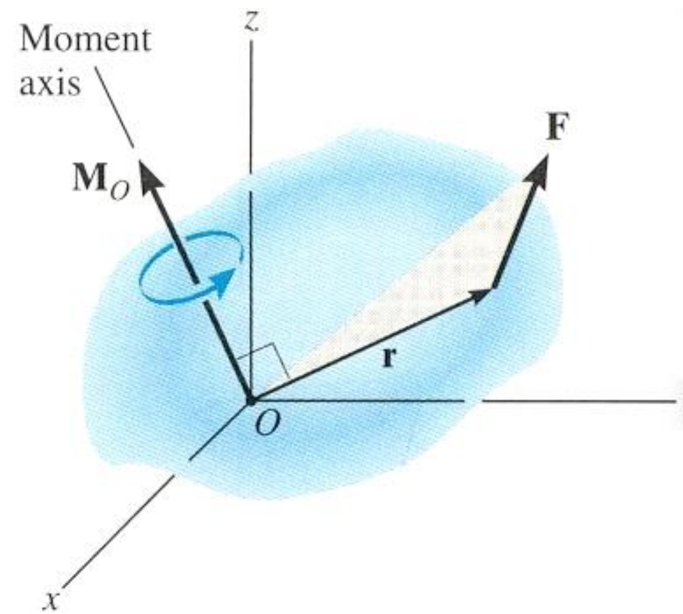
Moment of a force – vector formulation

The moment of a force $\vec{F}$ about point O, or actually about the moment axis passing through O and perpendicular to the plane containing O and $\vec{F}$, can be expressed using the cross (vector) product, namely:

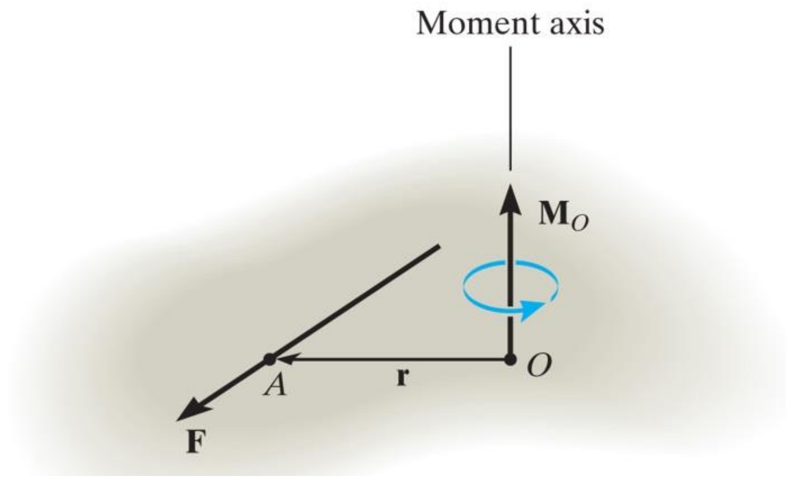
$$\overrightarrow{M_O} = \vec{r} \times \vec{F}$$

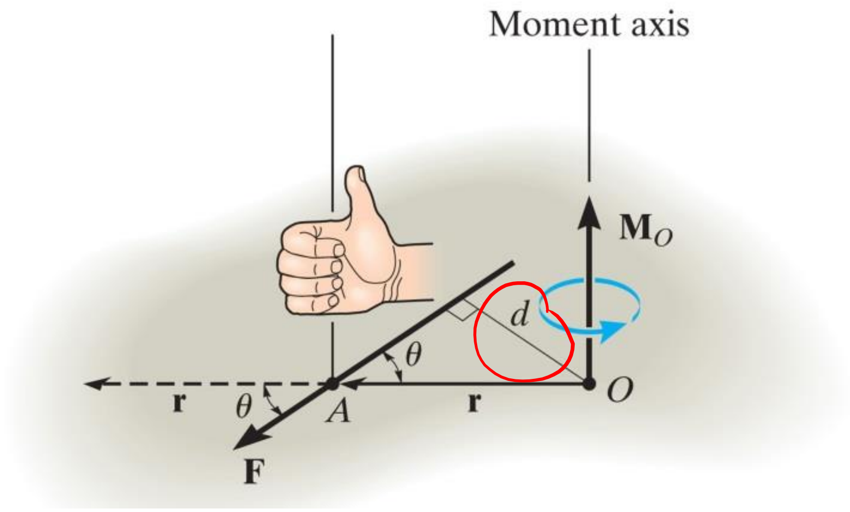
where $\vec{r}$ is the position vector directed from O to any point on the line of action of $\vec{F}$.





Moment of a force – vector formulation





Use cross product: $\overrightarrow{\boldsymbol{M_O}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}}$

Direction: Defined by right hand rule.

$$\overrightarrow{\boldsymbol{M_O}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}} = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = (r_yF_z - r_zF_y)\hat{\boldsymbol{\imath}} - (r_xF_z - r_zF_x)\hat{\boldsymbol{\jmath}} + (r_xF_y - r_yF_x)\widehat{\boldsymbol{k}}$$

Magnitude:

$$M_O = \left|\overrightarrow{\boldsymbol{M_O}}\right| = |\vec{\boldsymbol{r}}|\left|\vec{\boldsymbol{F}}\right| sin\theta = F(rsin\theta) = Fd$$

d is the perpendicular distance from O to $\vec{F}$