Statics - TAM 210 & TAM 211

Lecture 8

February 2, 2018

Announcements

- CBTF has physical calculators!!! Casio fx-300MS (see CBTF website)
- Upcoming deadlines:
- Quiz 1 (1/31-2/2)
  - Reserve testing time at CBTF
  - https://cbtf.engr.illinois.edu/sched/
  - NO MAKE-UP.
  - Lectures 1- 4 material
- Friday (2/~~1~~) 2
  - Mastering Engineering Tutorial 4
- Tuesday (2/6)
  - PL Homework 3
- Quiz 2 (2/7-9)
  - Reserve testing time at CBTF
  - Lectures 5-9

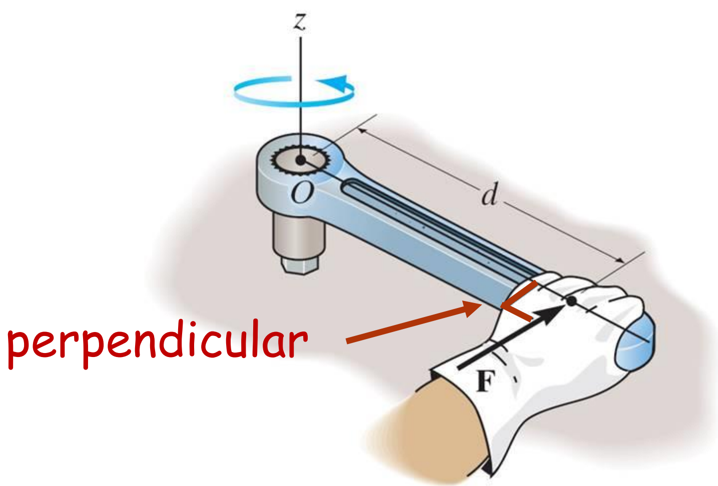
http://www.groundhogday2018shadow.info/2016/11/happy-groundhog-day-images.html

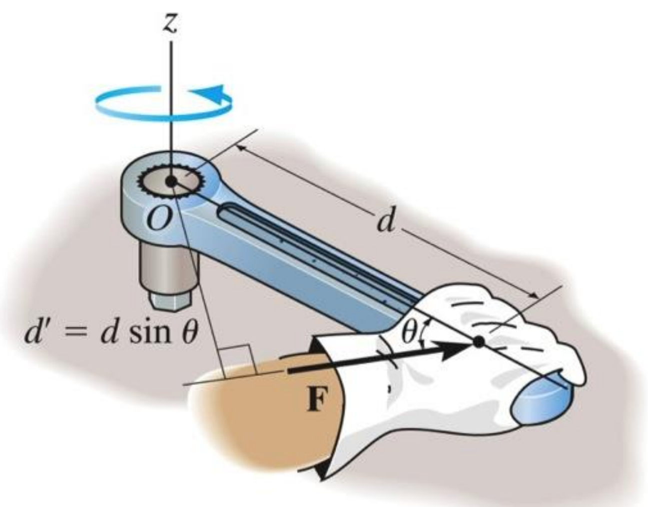
Chapter 4: Force System Resultants

Goals and Objectives

- Discuss the concept of the moment of a force and show how to calculate it in two and three dimensions
- How to find the moment about a specified axis
- Define the moment of a couple
- Finding equivalence force and moment systems
- Reduction of distributed loading

Recap: Moment of a force





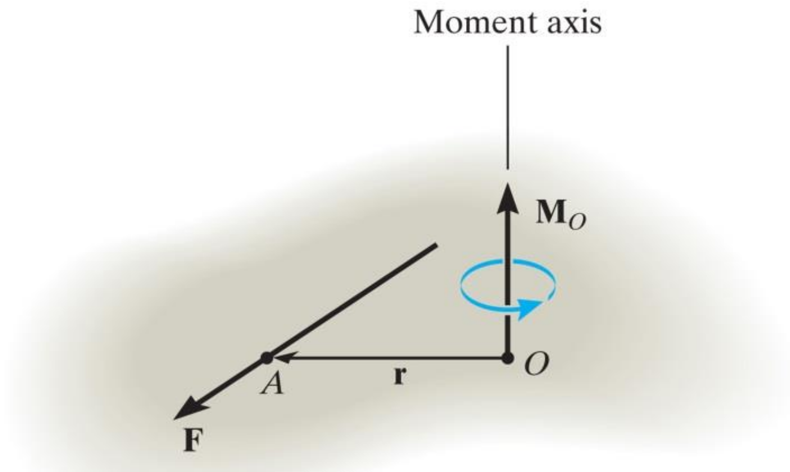
Scalar Formulation: $M_O = F\,d$

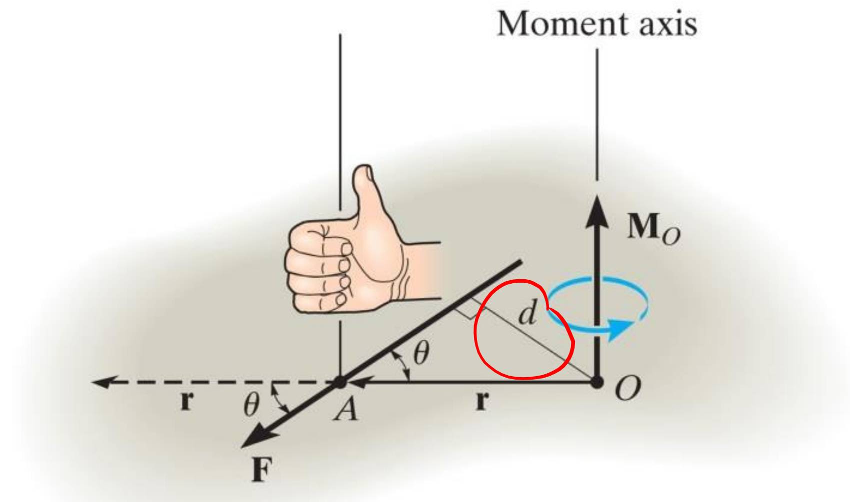
Scalar Formulation: $M_O = F\,d'$

Direction: Moment about point O $\overrightarrow{M_O}$ is perpendicular to the plane that contains the force $\vec{F}$ and its moment arm $\vec{d}$. The right-hand rule is used to define the sense.

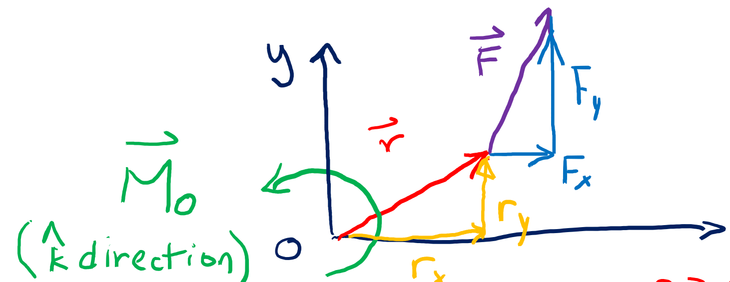
Magnitude: In a 2D case (where $\vec{F}$ is perpendicular to $\vec{d}$), the magnitude of the moment about point O is $M_O = F\,d$

Recap: Moment of a force





Vector Formulation



Use cross product: $\overrightarrow{M_O} = \vec{r} \times \vec{F}$

Direction: Defined by right hand rule.

$$\overrightarrow{M_O} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = (r_yF_z - r_zF_y)\hat{\imath} - (r_xF_z - r_zF_x)\hat{\jmath} + \overbrace{(r_xF_y - r_yF_x)\hat{k}}^{\text{2D (x,y) case}}$$

$= \vec{M}_o$ (x,y case)

Magnitude:

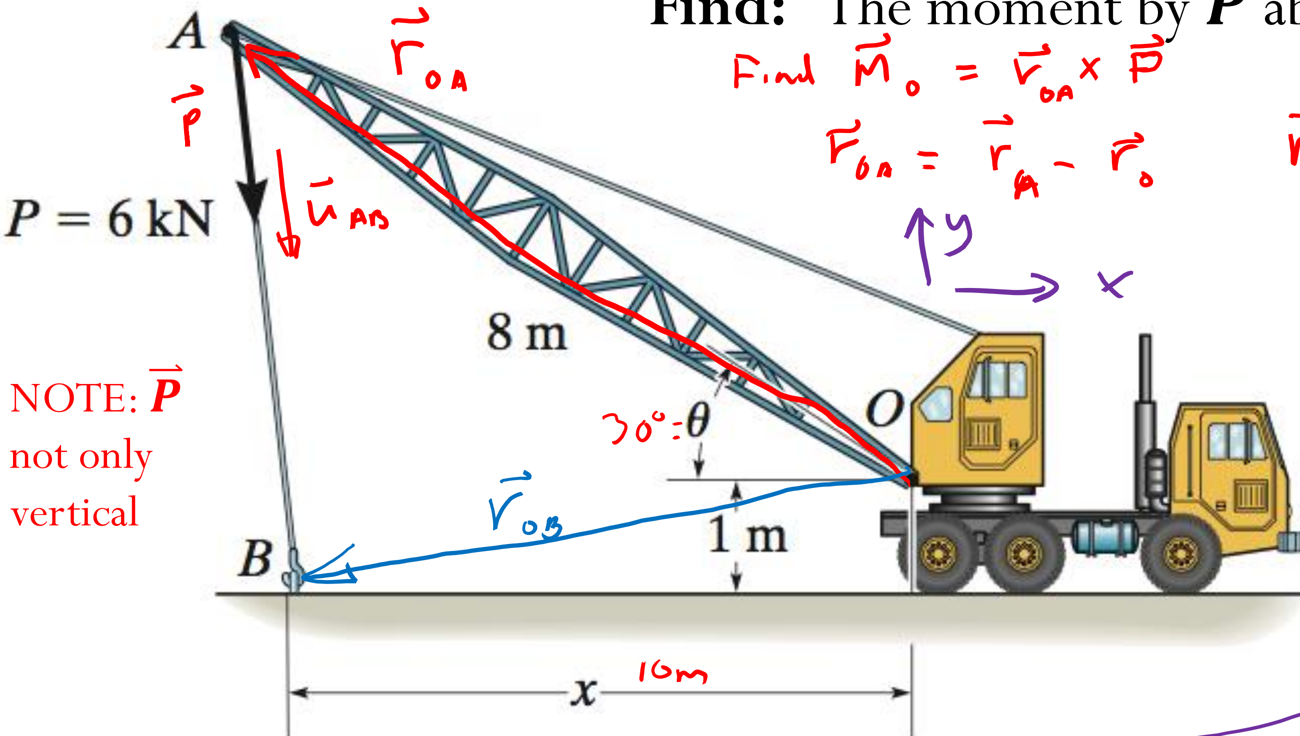
$$M_O = \left|\overrightarrow{M_O}\right| = |\vec{r}|\left|\vec{F}\right| \sin\theta = F(r\sin\theta) = Fd$$

d is the perpendicular distance from O to $\vec{F}$

Example

Given: The angle $\theta = 30°$ and $x = 10$ m.

Find: The moment by $\vec{\boldsymbol{P}}$ about point O.



Find $\vec{M}_O = \vec{r}_{OA} \times \vec{P}$

$\vec{r}_{OA} = \vec{r}_A - \vec{r}_O$

$$\vec{M}_O = \vec{r}_{OA} \times \vec{P} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_{OA_x} & r_{OA_y} & r_{OA_z} \\ P_x & P_y & P_z \end{vmatrix} = (r_{OA_x}P_y - r_{OA_y}P_x)\hat{k}$$

$\Rightarrow \boxed{M_O \approx 48.0 \text{ kN.m}}$

$\hat{k}$ direction

$$\vec{r}_{OA} = (-8\cos 30)\hat{i} + (8\sin 30)\hat{j}$$

$$\vec{P} = P\vec{u}_{AB}$$

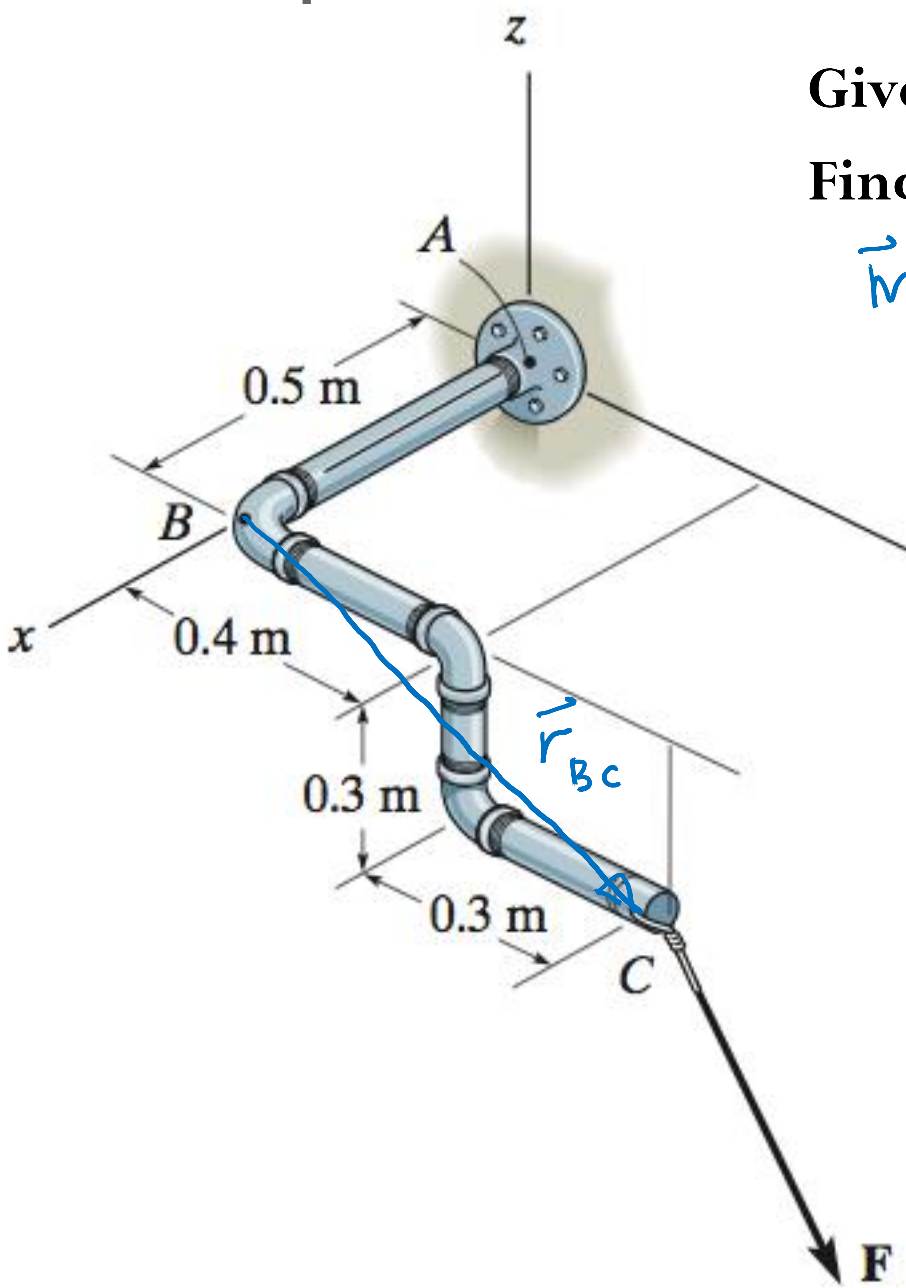
$$\vec{u}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{OB}|} = \frac{\vec{r}_B - \vec{r}_A}{r_{AB}}$$

$$= \frac{(-10 + 8\cos 30)\hat{i} - (-1 - 8\sin 30)\hat{j}}{\sqrt{(x)^2 + (y)^2}} = \frac{(-3.07\hat{i} - 5\hat{j})\text{m}}{5.87 \text{ m}}$$

what if use $\vec{r}_{OB}$

$\vec{M}_O = \vec{r}_{OB} \times \vec{P}$?

Example – Vector Formulation



Given: **F** = {600**i** + 800**j** - 500**k**} N

Find: Moment of the force about point *B*.

$\vec{M}_B = \vec{r}_{BC} \times \vec{F}$

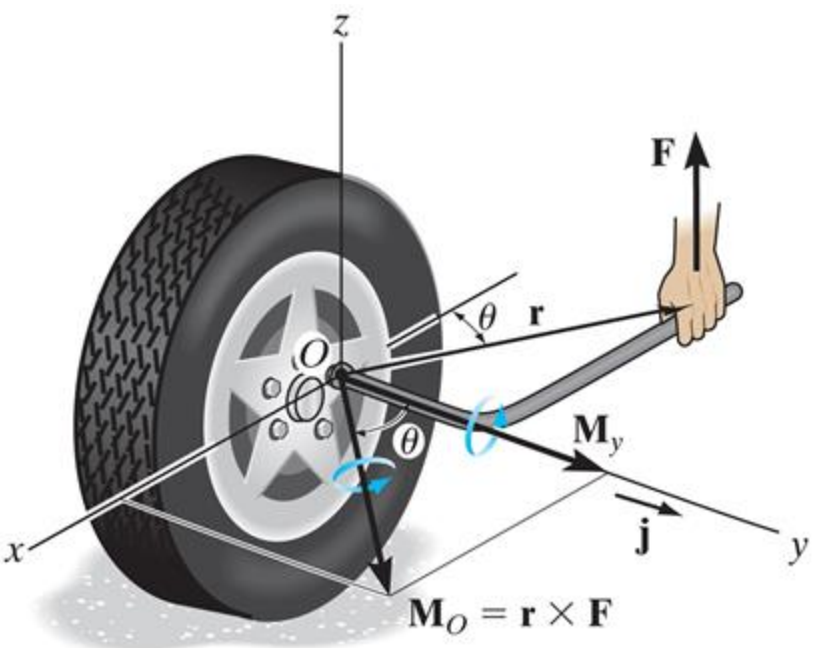
$\vec{r}_{BC} = 0\,\hat{i} + (0.4 + 0.3)\,\hat{j} - 0.3\,\hat{k}$

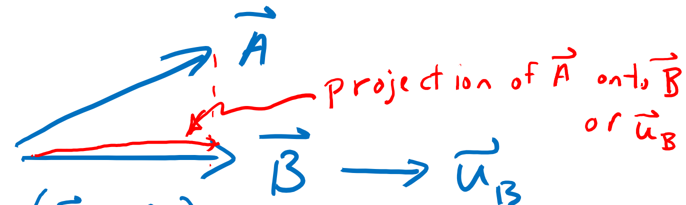
$\vec{M}_B = (-110\,\hat{i} - 180\,\hat{j} - 420\,\hat{k})$ N·m

Moment of a force about a specified axis (use of Scalar Triple Product)

A force is applied to the tool as shown. Find the magnitude of the moment of this force about the *y*-axis.

Remember, the component of a vector, $\vec{A}$, along the direction of another, $\vec{B}$, can be determined using the dot product:





$$\text{proj}(\vec{A}, \vec{B}) = (\vec{A} \cdot \vec{u}_B)\, \vec{u}_B$$

$\updownarrow$ equivalent

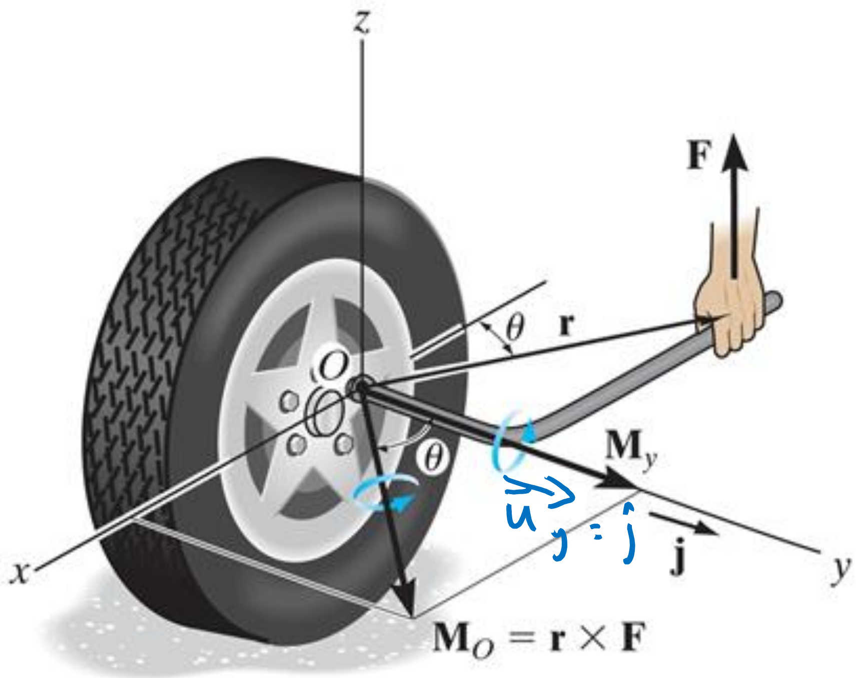
$$= (\vec{u}_B \cdot \vec{A})\, \vec{u}_B$$

Find moment about generic axis a

$$|\vec{M}_a| = \vec{u}_a \cdot \vec{M}_o$$

$$= \underbrace{\vec{u}_a \cdot (\vec{r} \times \vec{F})}_{\text{Scalar Triple Product}}$$

$$\vec{M}_a = \begin{vmatrix} u_{ax} & u_{ay} & u_{az} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$



$$\vec{F} = F\hat{k}$$

$$\vec{r} = -x\hat{i} + y\hat{j}$$

$$\vec{M}_o = \vec{r} \times \vec{F} = yF\hat{i} - (-xF)\hat{j} + 0\hat{k}$$

$$|\vec{M}_y| = \hat{j} \cdot \vec{M}_o = \hat{j} \cdot (\vec{r} \times \vec{F})$$

$$= \boxed{xF \ \ N \cdot m}$$

scalar magnitude of moment projected along y-axis