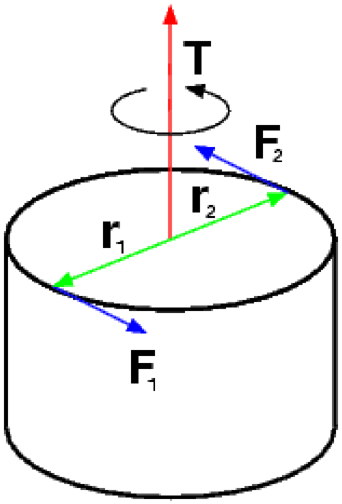
Statics - TAM 210 & TAM 211

Lecture 9

February 5, 2018

Announcements

- Upcoming deadlines:
- Tuesday (2/6)
  - PL Homework 3
- Quiz 2 (2/7-9)
  - Reserve testing time at CBTF
  - Lectures 5-9
- Friday (2/9)
  - Mastering Engineering Tutorial 5



https://fr.wikipedia.org/wiki/Couple_(physique)

Chapter 4: Force System Resultants

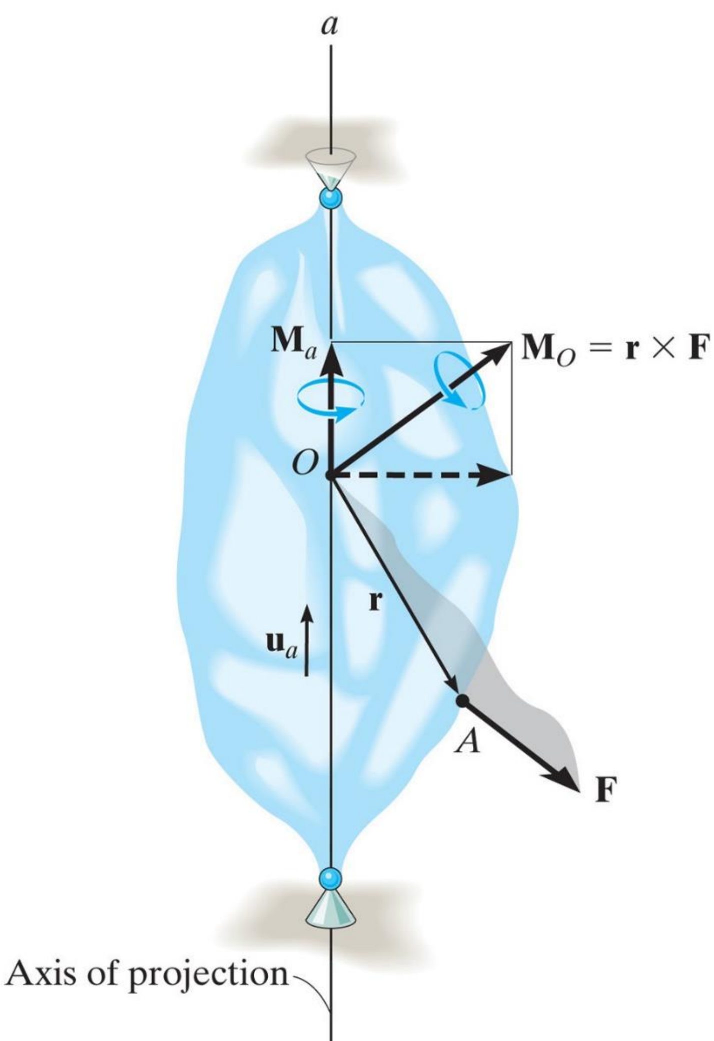
Goals and Objectives

Moment being represented as a vector $\vec{M}_o$

- Discuss the concept of the moment of a force and show how to calculate it in two and three dimensions $\vec{M} = \vec{r} \times \vec{F}$
- How to find the moment about a specified axis $\vec{M}_a$
- Define the moment of a couple
- Finding equivalence force and moment systems
- Reduction of distributed loading

Recap: Moment of a force about a specified axis (Scalar Triple Product)

The magnitude of the projected moment about any generic axis *a* can be computed using the scalar triple product:



$$\left|\overrightarrow{\boldsymbol{M}_a}\right| = \overrightarrow{\boldsymbol{u}_a} \cdot \left(\vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}}\right)$$

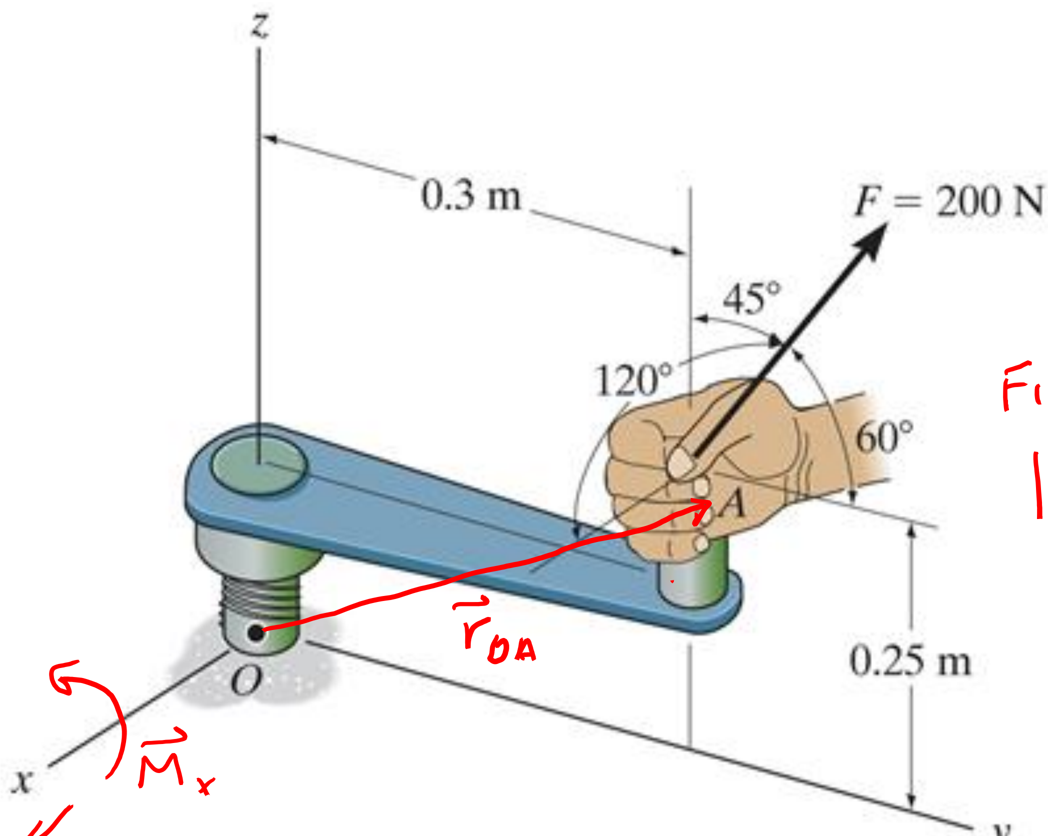
$$= \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

The direction of the projected moment about any generic axis *a* can be defined using :

$$\overrightarrow{\boldsymbol{M}_a} = \left|\overrightarrow{\boldsymbol{M}_a}\right| \overrightarrow{\boldsymbol{u}_a}$$

where $\overrightarrow{\boldsymbol{u}_a}$ is the unit vector along axis *a*

A force is applied to the tool as shown. Find the magnitude of the moment of this force about the x axis.



$\vec{u}_x = \hat{i}$

Find: $\vec{M}_x$

$$|\vec{M}_x| = \underbrace{\vec{u}_x}_{\hat{i}} \cdot \underbrace{(\vec{r}_{OA} \times \vec{F})}_{\vec{M}_O}$$

$$\vec{r}_{OA} = 0\hat{i} + 0.3m\,\hat{j} + 0.25m\,\hat{k}$$

$$\vec{F} = 200(\cos 120\,\hat{i} + \cos 60\,\hat{j} + \cos 45\,\hat{k})$$

$$= -100\hat{i} + 100\hat{j} + 141.4\hat{k} \text{ N}$$

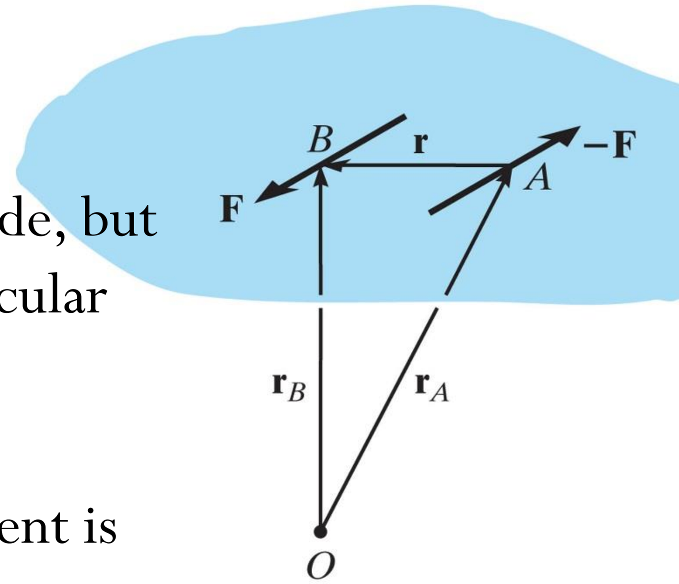
$$M_x = \hat{i} \cdot (\vec{r}_{OA} \times \vec{F})$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.3 & 0.25 \\ -100 & 100 & 141.4 \end{vmatrix}$$

$$= 1[(0.3m \cdot 141.4N) - (0.25m \cdot 100N)] = 17.4 \text{ Nm}$$

$$\therefore \boxed{\vec{M}_x = 17.4 \text{ Nm}\,\hat{i}}$$

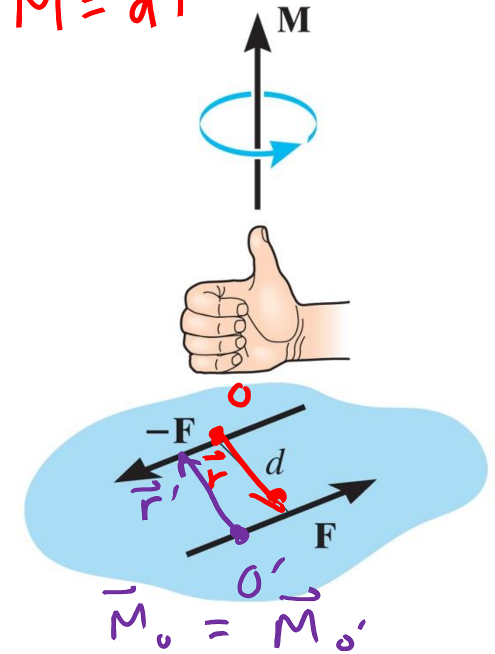
Moment of a couple



Couple: two parallel forces that have same magnitude, but opposite directions, and are separated by a perpendicular distance *d*.

- Resultant force is zero. $\vec{F}_R = \vec{F} + (-\vec{F}) = 0$
- Couple produces actual rotation, or if no movement is possible, tendency of rotation in a specified direction.

Moment produced by a couple is called **couple moment**. $M = dF$



Sum of moments of both couple forces about **any** arbitrary point:

$$\vec{M} = \left(\vec{r_B} \times \vec{F}\right) + \left(\vec{r_A} \times \left(-\vec{F}\right)\right)$$
$$= (\vec{r_B} - \vec{r_A}) \times \vec{F}$$
$$= \vec{r} \times \vec{F}$$

Couple moment is a **free vector**, i.e. is **independent** of the choice of O!

Equivalent couples

Define coordinate frame

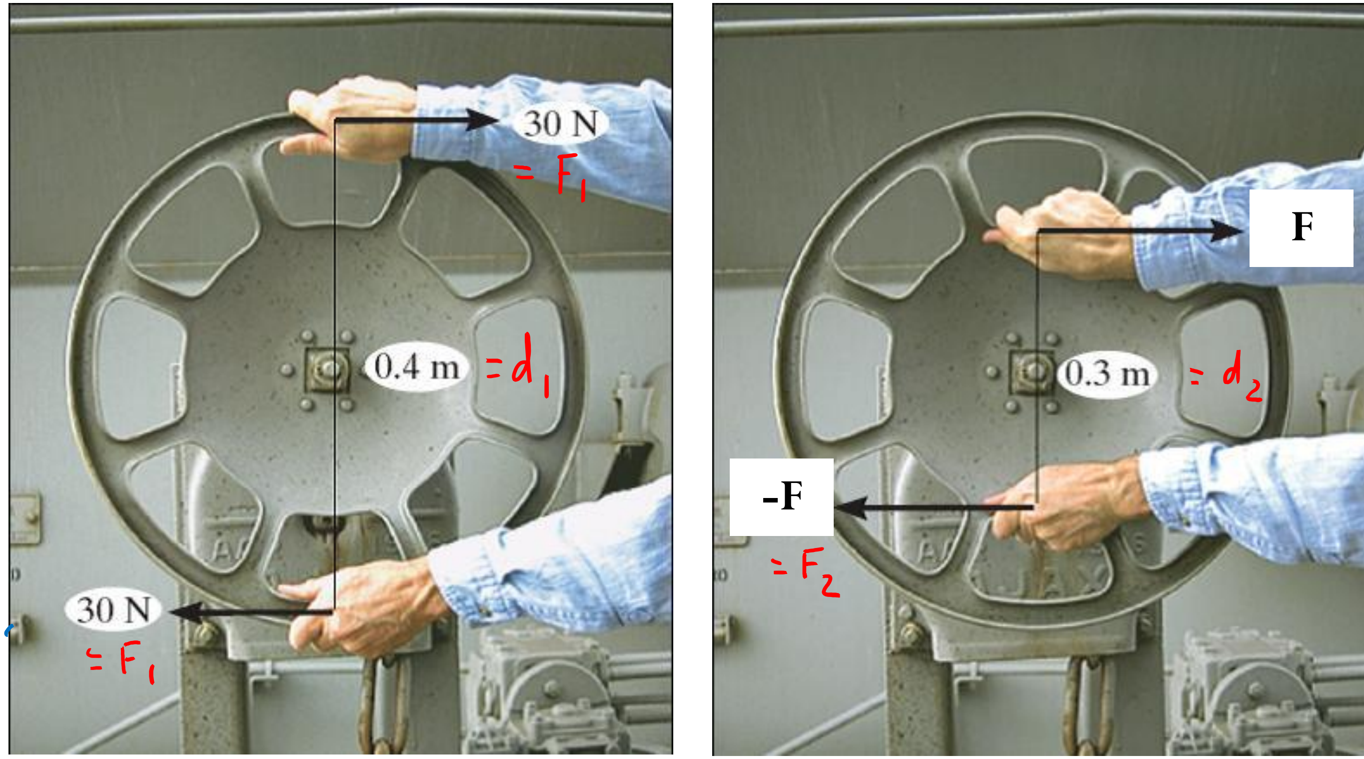
+y, +x, +z (out of screen)

Notation for representing a vector pointing perpendicular (in or out of screen)

⊙ OUT — tip

⊗ IN — tail

← Like arrow



A torque or moment of 12 N·m is required to rotate the wheel.

Would F be greater or less than 30 N?

$M_1 = d_1 F_1 = (0.4\,m)(30\,N)$ cw moment $(-\hat{k})$

$\vec{M}_1 = -12\,Nm\,\hat{k}$

$\vec{M}_2 = -12\,Nm\,\hat{k}$

$M_2 = d_2 F$

$12\,Nm = 0.3\,m\,F$

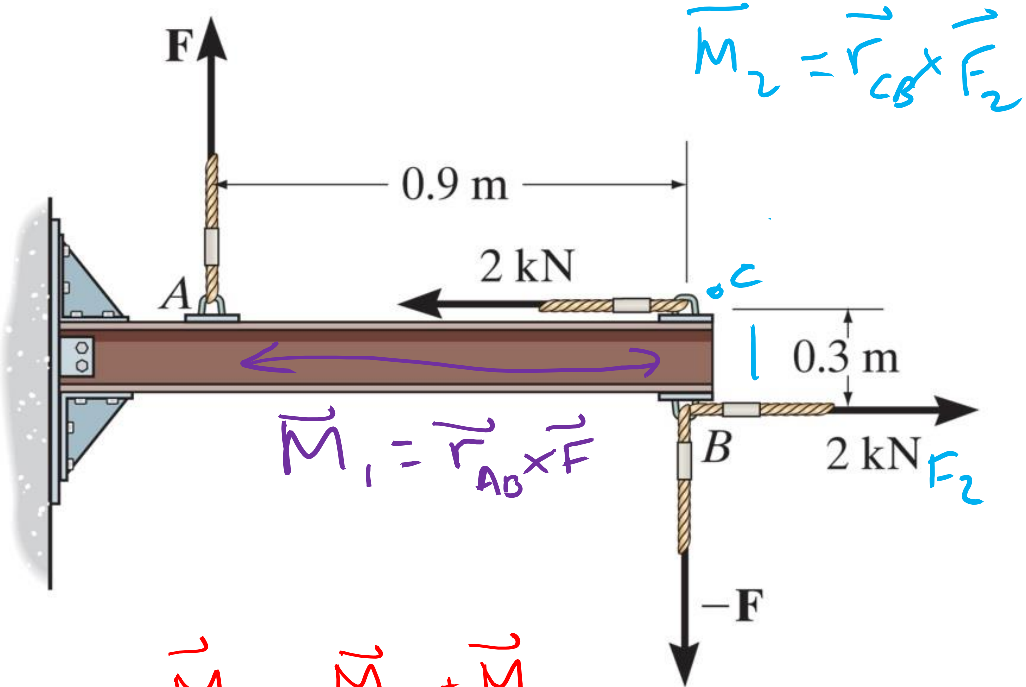
$F = 40\,N$

$\vec{F} = 40\,N\,\hat{i}$

Resultant Couple Moment

Since couple moments are vectors, their resultant is due to vector addition:

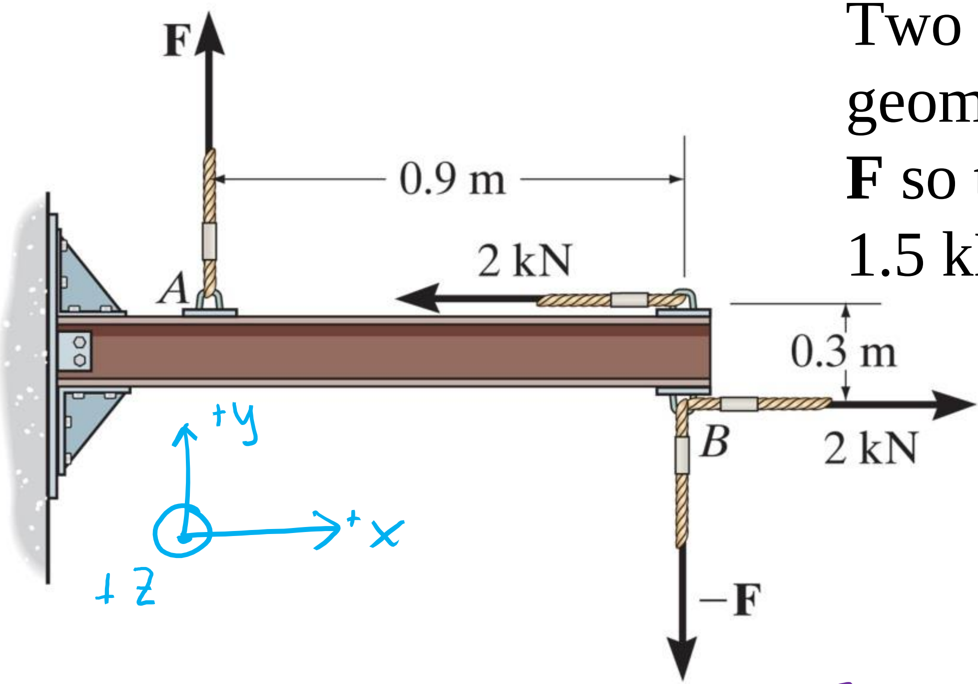
$$\overrightarrow{M_R} = \overrightarrow{M_1} + \overrightarrow{M_2} + \cdots$$
$$= \Sigma\overrightarrow{M_i}$$
$$= \Sigma\left(\vec{r} \times \vec{F}\right)$$



$\vec{M}_R = \vec{M}_1 + \vec{M}_2$
$= (\vec{r}_{AB} \times \vec{F}) + (\vec{r}_{CB} \times \vec{F}_2)$

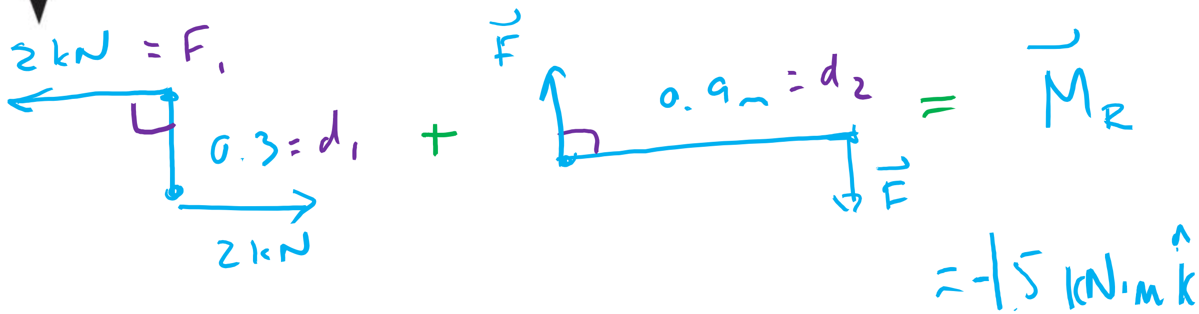
Note that $\vec{M}_R$, since it is a free vector, can be placed anywhere on the beam.

Two couples act on the beam with the geometry shown. Find the magnitude of **F** so that the resultant couple moment is 1.5 kN·m clockwise.



Find : $\vec{F}$

Given : $\vec{M}_R = 1.5 \text{ kN}\cdot\text{m} \, (-\hat{k})$ CW

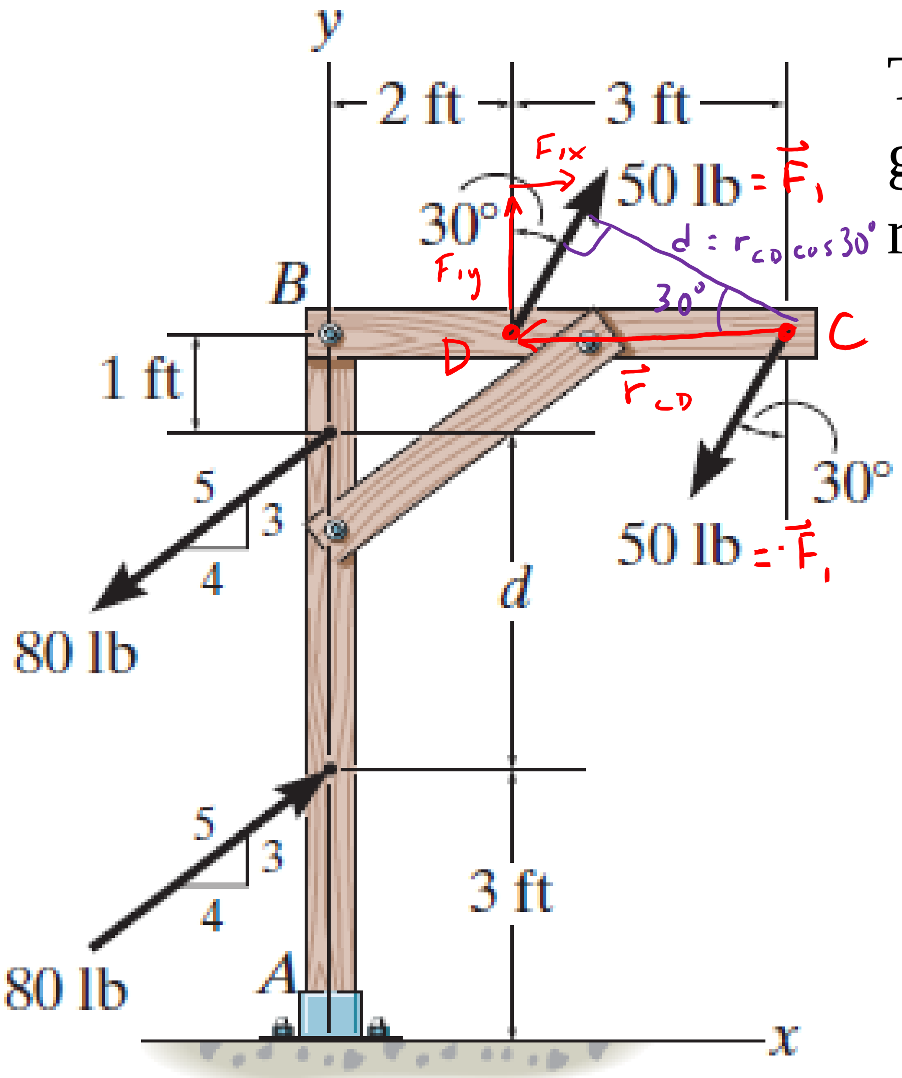


$$\Sigma \vec{M}_R = d_1 F_1 \hat{k} + d_2 F (-\hat{k})$$

$$= (0.3\text{ m})(2\text{ kN})\hat{k} + -(0.9\text{ m } F)\hat{k} = -1.5 \text{ kN}\cdot\text{m}\,\hat{k}$$

$$\Rightarrow \boxed{F = 2.33 \text{ kN}}$$

Two couples act on the beam with the geometry shown and d = 4 ft. Find the resultant couple



In response to student question about couple moment when $\vec{r}$ is not $\perp$ to $\vec{F}$:

For upper beam, what is the Moment due to the 50 lb force couple? Find: $\vec{M}_{upper}$

$$\vec{M}_{upper} = \vec{r} \times \vec{F}$$
$$= \vec{r}_{CD} \times \vec{F}_1$$
$$= (-3\text{ ft}\ \hat{i}) \times 50(\sin 30\hat{i} + \cos 30\hat{j})\text{ lb}$$
$$= -130\text{ ft}\cdot\text{lb}\ \hat{k}$$

Alternatively,

$$|M_{upper}| = dF$$ where d is $\perp$ distance

$$= (|r_{CD}|\cos 30°\text{ ft})(50\text{ lb})$$
$$= (3\cos 30°)\,50\text{ ft}\cdot\text{lb}$$
$$= 130\text{ ft}\cdot\text{lb ccw } (-\hat{k})$$

$$\vec{M}_{upper} = -130\text{ ft}\cdot\text{lb}\ \hat{k}$$ ✓ same

Due to running out to time,we'll address the typed problem statement in our next lecture. Hint to find M_R, need to also determine M_{lower}, such that

$M_R = M_{upper} + M_{lower}$