Statics - TAM 210 & TAM 211

Lecture 10

February 7, 2018

Announcements

- Upcoming deadlines:
- Quiz 2 (2/7-9)
  - Reserve testing time at CBTF
  - Lectures 5-9
- Friday (2/9)
  - Mastering Engineering Tutorial 5
- Tuesday (2/13)
  - PL Homework 4

Chapter 4: Force System Results

Goals and Objectives

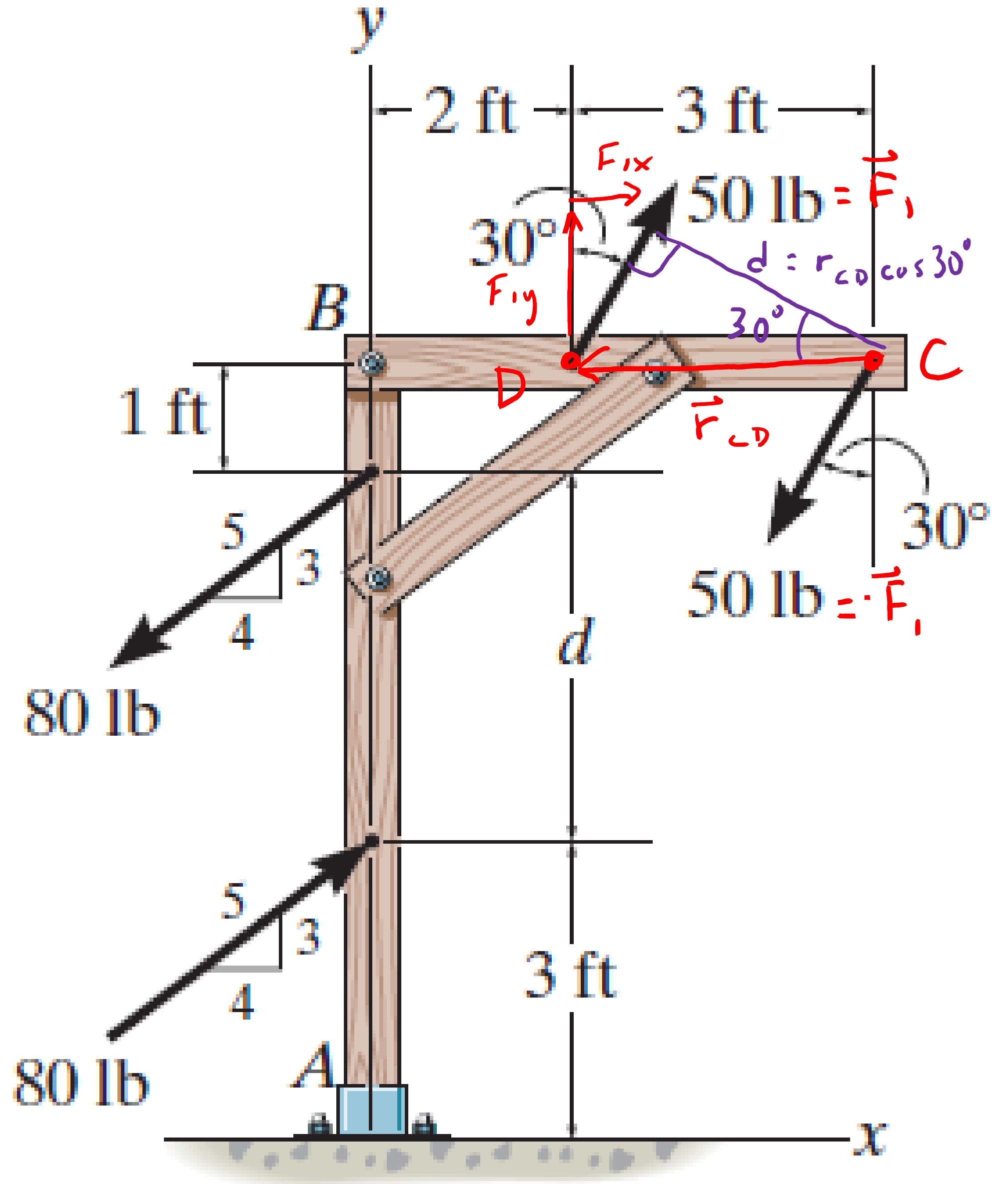
- Discuss the concept of the moment of a force and show how to calculate it in two and three dimensions
- How to find the moment about a specified axis
- Define the moment of a couple
- Finding equivalence force and moment systems
- Reduction of distributed loading

Recap from lecture 9:

- **Moment of a force couple** ($\vec{F}$ and $-\vec{F}$)
  - $\overrightarrow{M_O} = \vec{r} \times \vec{F}$, $|\overrightarrow{M_O}| = Fd$ (where d ≈ ⊥ dist btw $\vec{F}$ and $-\vec{F}$)
  - Couple moment is a **free vector**, i.e. it is **independent** of the choice of location of O!
  - Spin your i>clicker: apply equal & opposite (not co-linear) forces by each index finger, with **<u>same</u>** force magnitude & gap between fingers, change locations along i>clicker. Does it have the same rotation?

- **Equivalent couple**
  - Spin your i>clicker: ↑ (or ↓) force magnitude and ↓ (or ↑) gap to get the same rotation. $M_O = F\,d$

- **Resultant couple moment**
  - $\overrightarrow{M_R} = \Sigma \overrightarrow{M_i}$



Two couples act on the beam with the geometry shown and d = 4 ft. Find the resultant couple

In response to student question about couple moment when $\vec{r}$ is not ⊥ to $\vec{F}$:

For upper beam, what is the Moment due to the 50 lb force couple? Find: $\vec{M}_{upper}$

$$\vec{M}_{upper} = \vec{r} \times \vec{F}$$
$$= \vec{r}_{CD} \times \vec{F}_1$$
$$= (-3\text{ ft}\,\hat{i}) \times 50(\sin 30\,\hat{i} + \cos 30\,\hat{j})\text{ lb}$$
$$= -130\text{ ft·lb}\,\hat{k}$$

Alternatively,

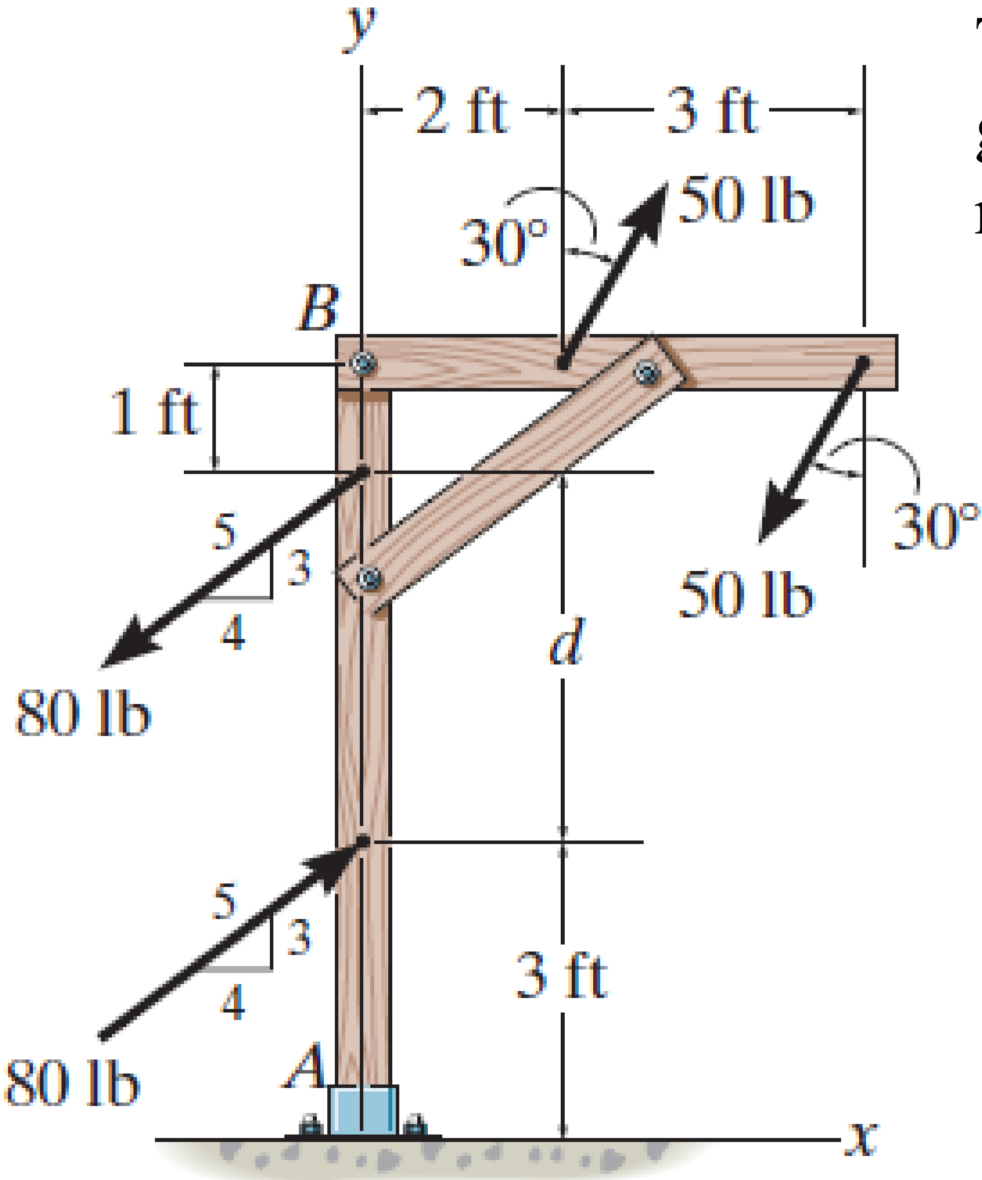
$|M_{upper}| = dF$ where d is ⊥ distance

$$= (|r_{CD}|\cos 30°\text{ ft})(50\text{ lb})$$
$$= (3\cos 30°)\,50\text{ ft·lb}$$
$$= 130\text{ ft·lb ccw } (-\hat{k})$$

$\vec{M}_{upper} = -130\text{ ft·lb}\,\hat{k}$ ✓ same

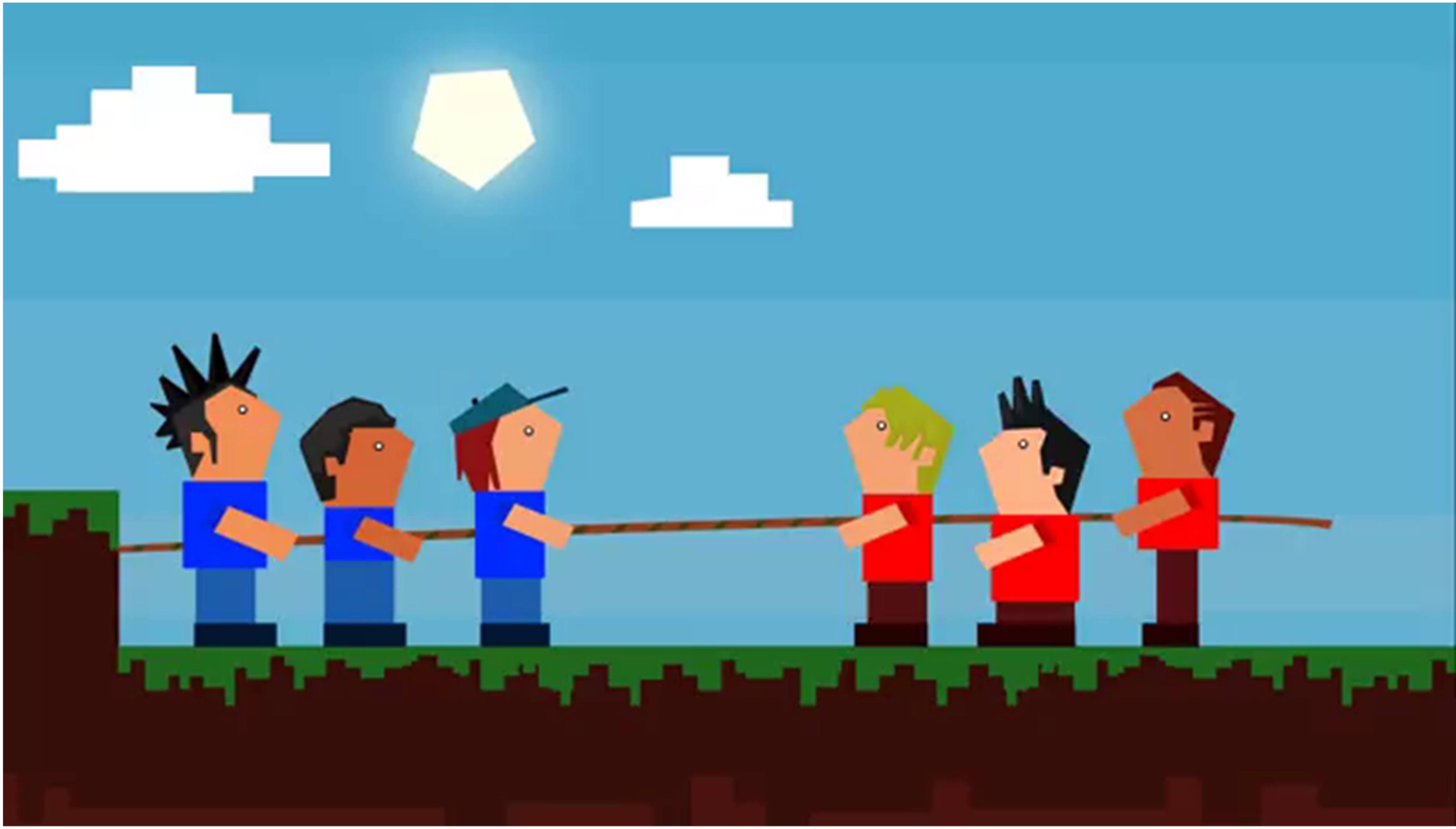
Due to running out to time, we'll address the typed problem statement in our next lecture. Hint to find M_R, need to also determine M_{lower}, such that

$M_R = M_{upper} + M_{lower}$



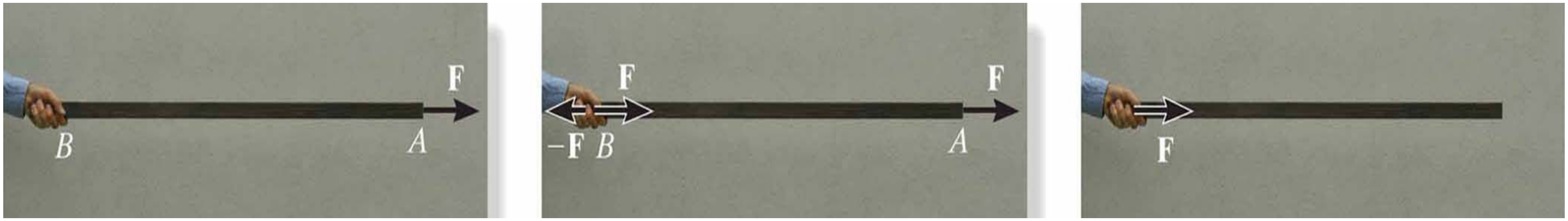
Two couples act on the beam with the geometry shown and d = 4 ft. Find the resultant couple

Moving a force on its line of action



https://www.wikihow.com/Win-at-Tug-of-War

Moving a force on its line of action

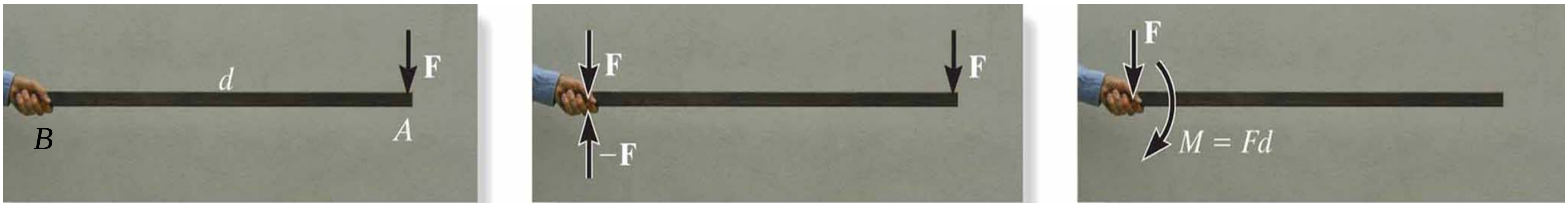


Moving a force from A to B, when both points are on the vector's line of action, does not change the **external effect**.

Hence, a force vector is called a **sliding vector**.

However, the **internal effect** of the force on the body does depend on where the force is applied.

Moving a force off of its line of action



The two force systems are equipollent since the resultant force is the same in both systems, and the resultant moment with respect to any point P is the same in both systems.

So moving a force off its line of action means you have to "add" a new couple. Since this new couple moment is a **free vector**, it can be applied at any point on the body.

Equipollent (or equivalent) force systems

A force **system** is a collection of **forces** and **couples** applied to a body.

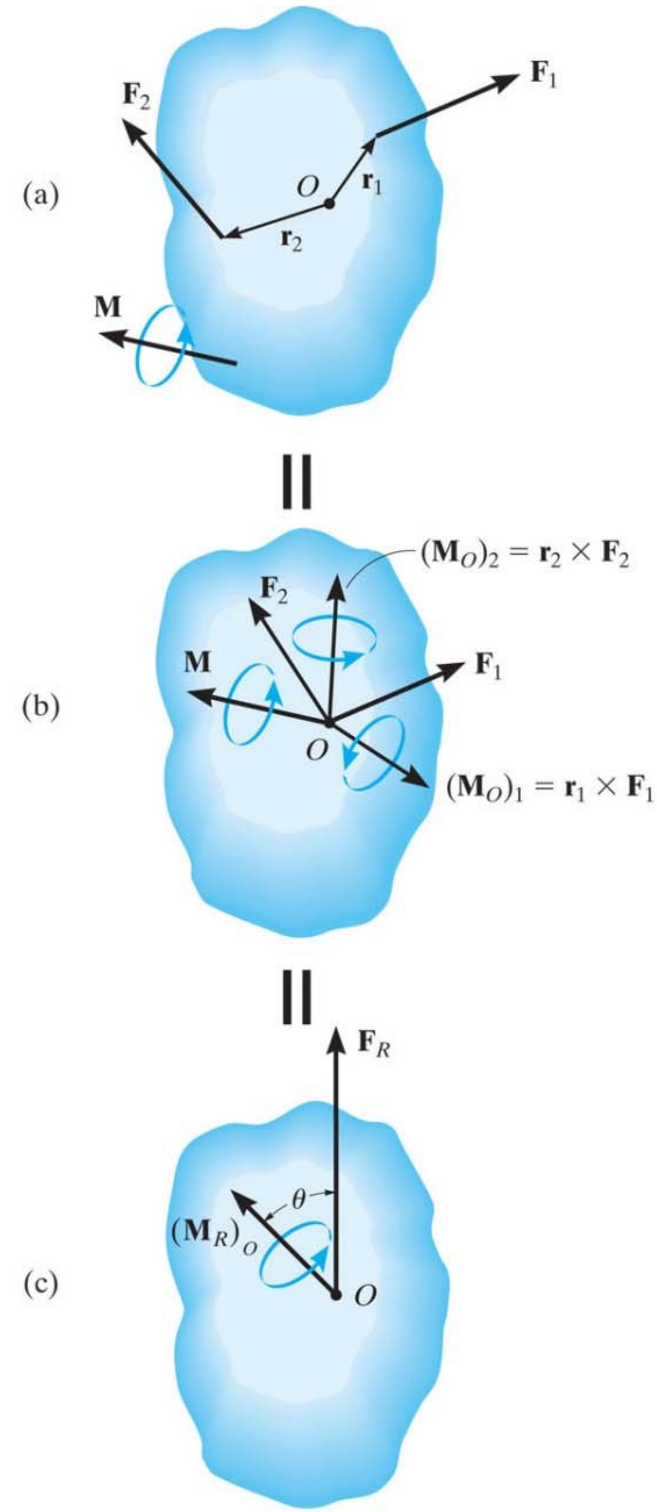
Two force systems are said to be **equipollent** (or equivalent) if they have the **same resultant force** AND the **same resultant moment** with respect to any point O.

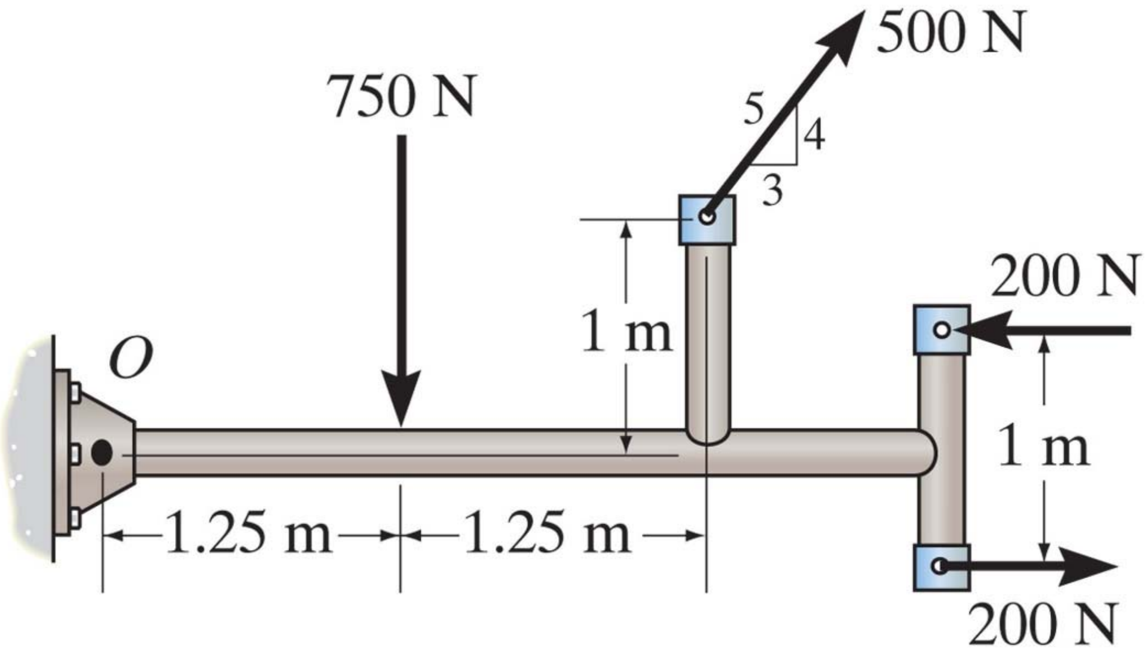
Reducing a force system to a single resultant force $\boldsymbol{F}_R$ and a single resultant couple moment $(\boldsymbol{M}_R)_o$:

$$\overrightarrow{\boldsymbol{F}_R} = \Sigma F_x \hat{\boldsymbol{\imath}} + \Sigma F_y \hat{\boldsymbol{\jmath}} + \Sigma F_z \widehat{\boldsymbol{k}}$$

$$|\overrightarrow{\boldsymbol{F}_R}| = \sqrt{F_x^2 + F_y^2 + F_z^2} \qquad \theta = \tan^{-1} \frac{F_{opp}}{F_{adj}}$$

$$(\boldsymbol{M}_R)_o = \sum \boldsymbol{M}_o + \sum \boldsymbol{M}$$





Replace the forces and couple system acting on the member by an equivalent force and couple moment acting at point O.