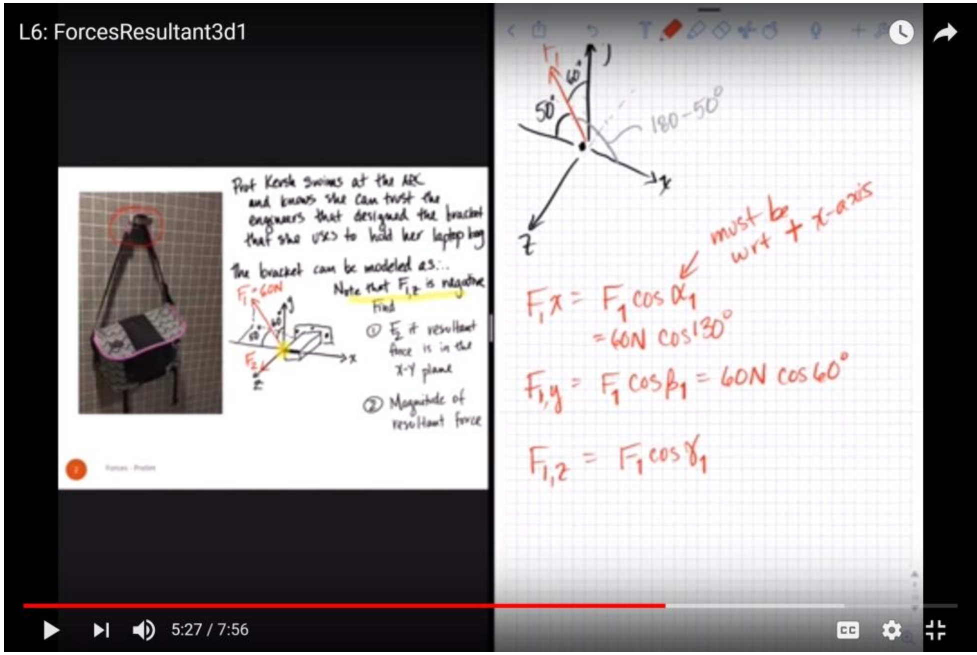
Statics - TAM 210 & TAM 211

Lecture 11

February 9, 2018

Announcements

- Note there are videos of solving more sample problems under Schedule tab of course website

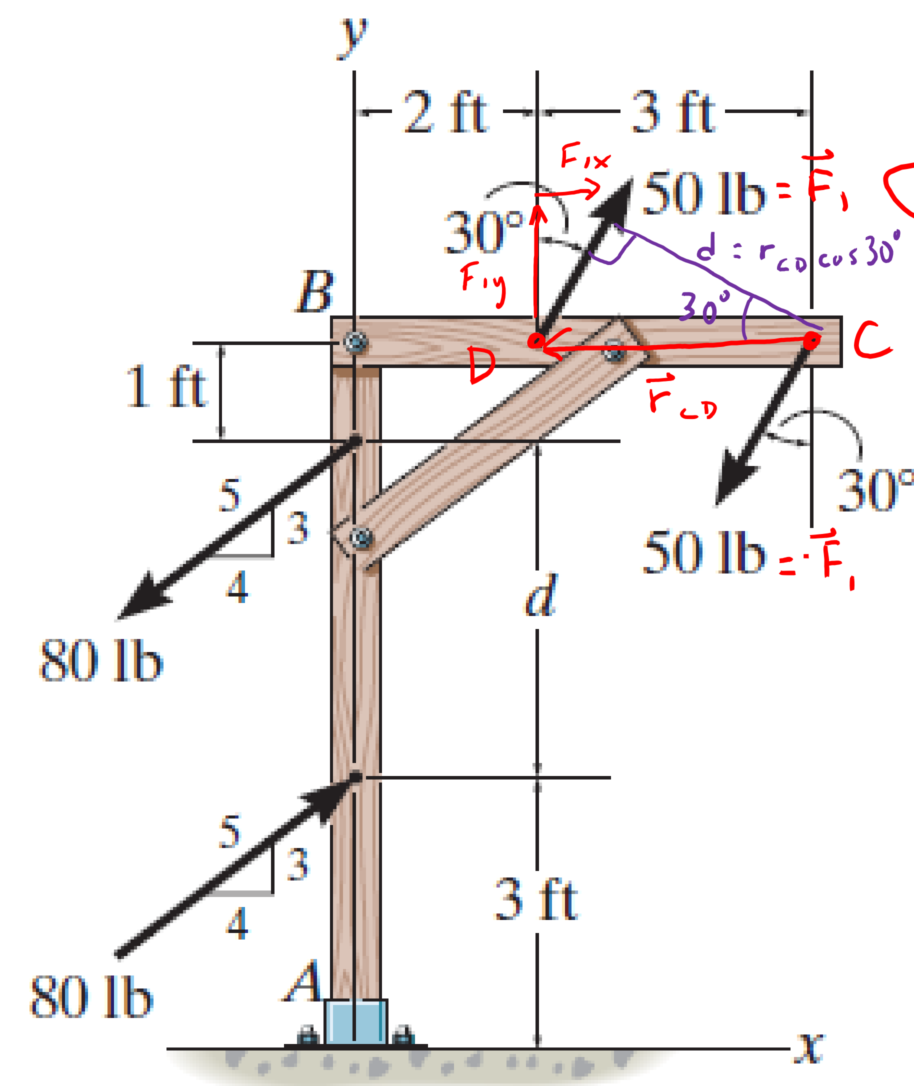


- Upcoming deadlines:
- Quiz 2 (2/7-9)
  - Reserve testing time at CBTF
  - Lectures 5-9
- Friday (2/9)
  - Mastering Engineering Tutorial 5
- Tuesday (2/13)
  - PL Homework 4

Chapter 4: Force System Resultants

Goals and Objectives

- Discuss the concept of the moment of a force and show how to calculate it in two and three dimensions
- How to find the moment about a specified axis
- Define the moment of a couple
- Finding equivalence force and moment systems
- Reduction of distributed loading



Two couples act on the beam with the geometry shown and d = 4 ft. Find the resultant couple

$\vec{M}_R = \sum \vec{M}_i$

In response to student question about couple moment when $\vec{r}$ is not $\perp$ to $\vec{F}$:

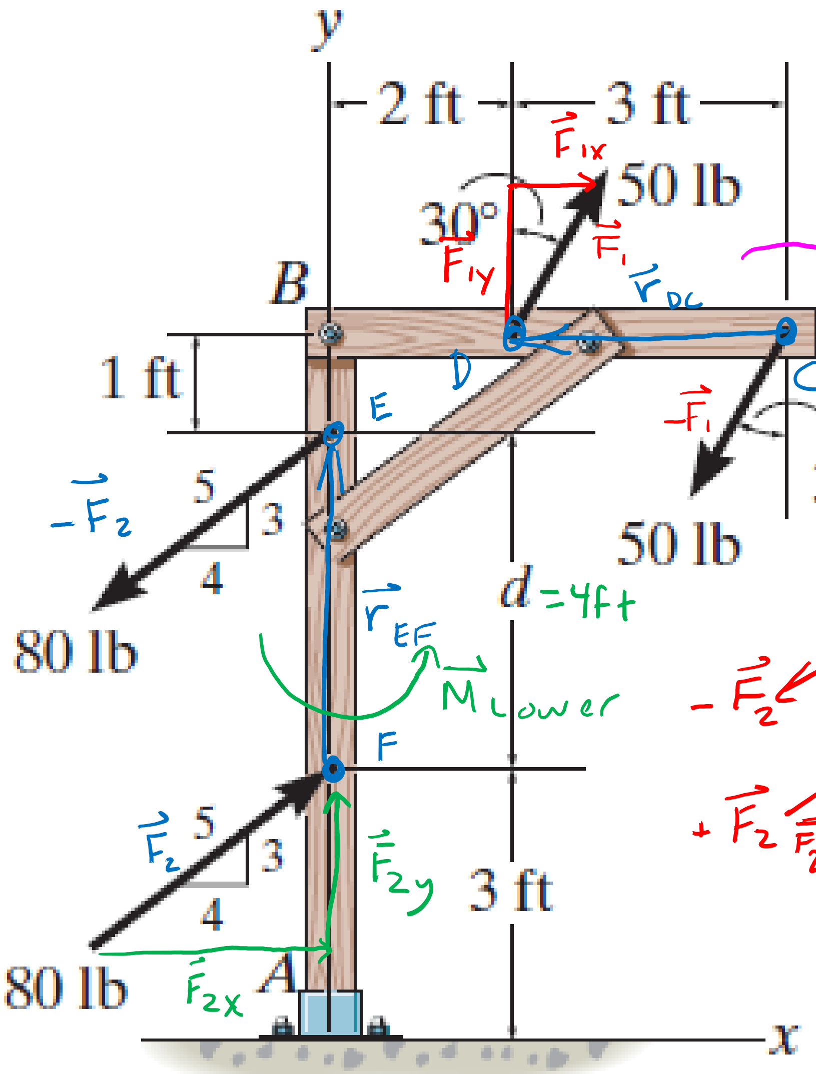
For upper beam, what is the Moment due to the 50lb force couple? Find: $\vec{M}_{upper}$

$$\vec{M}_{upper} = \vec{r} \times \vec{F}$$
$$= \vec{r}_{CD} \times \vec{F}_1$$
$$= (-3\text{ft}\ \hat{i}) \times 50(\sin 30\,\hat{i} + \cos 30\,\hat{j})\text{lb}$$
$$= -130\ \text{ft}\cdot\text{lb}\ \hat{k}$$

Alternatively, where d is $\perp$ distance

$$|M_{upper}| = dF$$
$$= (|r_{CD}| \cos 30°\ \text{ft})(50\ \text{lb})$$
$$= (3 \cos 30°)\,50\ \text{ft}\cdot\text{lb}$$
$$= 130\ \text{ft}\cdot\text{lb ccw}\ (-\hat{k})$$

$\vec{M}_{upper} = -130\ \text{ft}\cdot\text{lb}\ \hat{k}$ ✓ same



Two couples act on the beam with the geometry shown and d = 4 ft. Find the resultant couple

$\vec{M}_{upper} = -130 \text{ ft}\cdot\text{lb}\ \hat{k}$ cw

Lower beam: Find $\vec{M}_{Lower}$

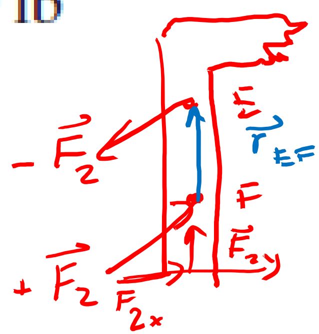
$\vec{M}_L = \vec{r}_{EF} \times \vec{F}_2$

$= d\hat{j} \times (F_{2x}\hat{i} + F_{2y}\hat{j})$

$= 4\text{ft}\,\hat{j} \times (80\tfrac{4}{5}\hat{i} + 80\tfrac{3}{5}\hat{j})\text{ lb}$

(Note: $\hat{j}\times\hat{i} = \hat{k}$, $\hat{j}\times\hat{j} = 0$)

$= 256 \text{ ft}\cdot\text{lb}\ \hat{k}$ ccw



∴ Resultant couple

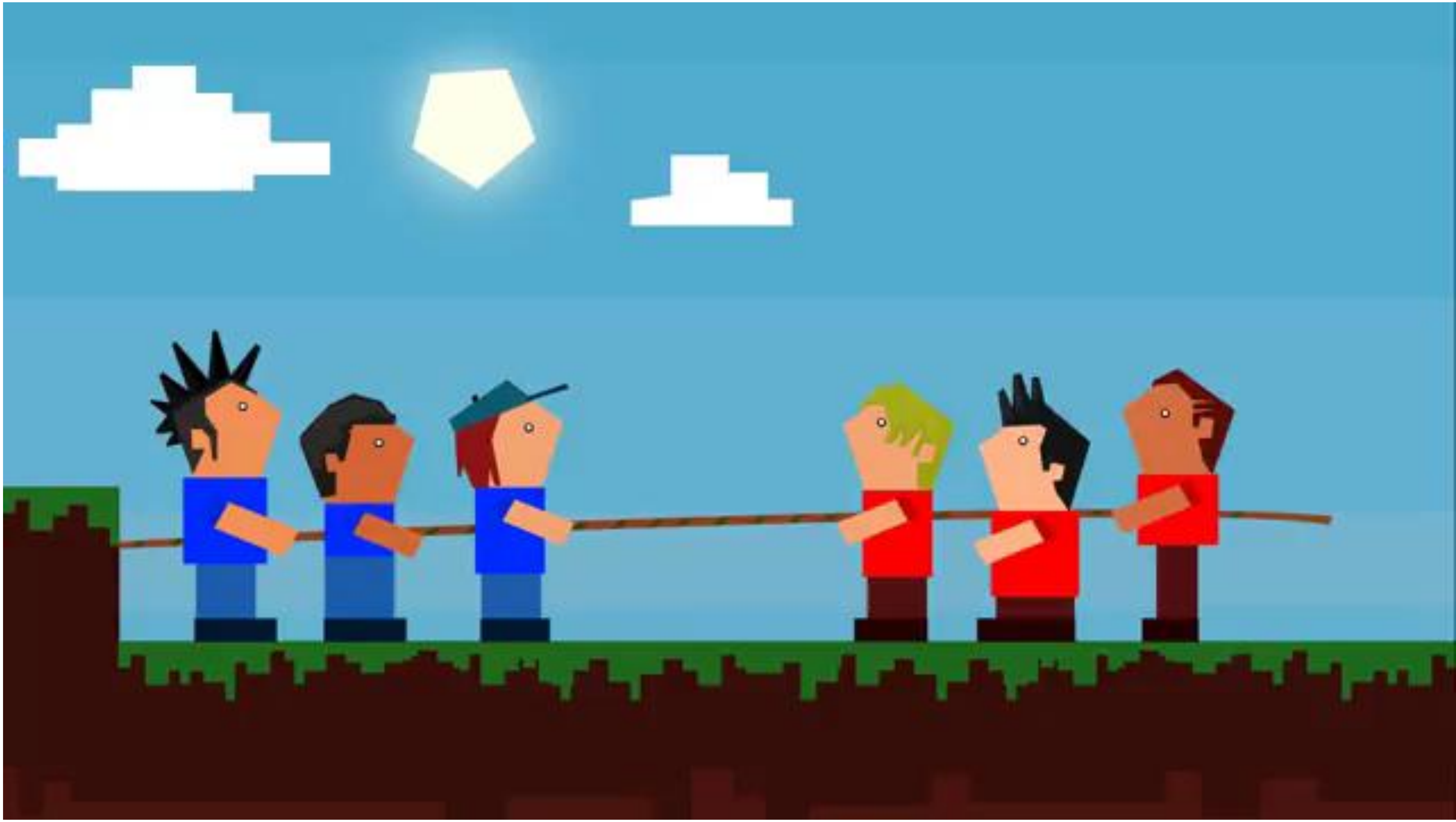
$\vec{M}_R = \vec{M}_{upper} + \vec{M}_{Lower}$

$= -130\hat{k} + 256\hat{k}$ ft·lb

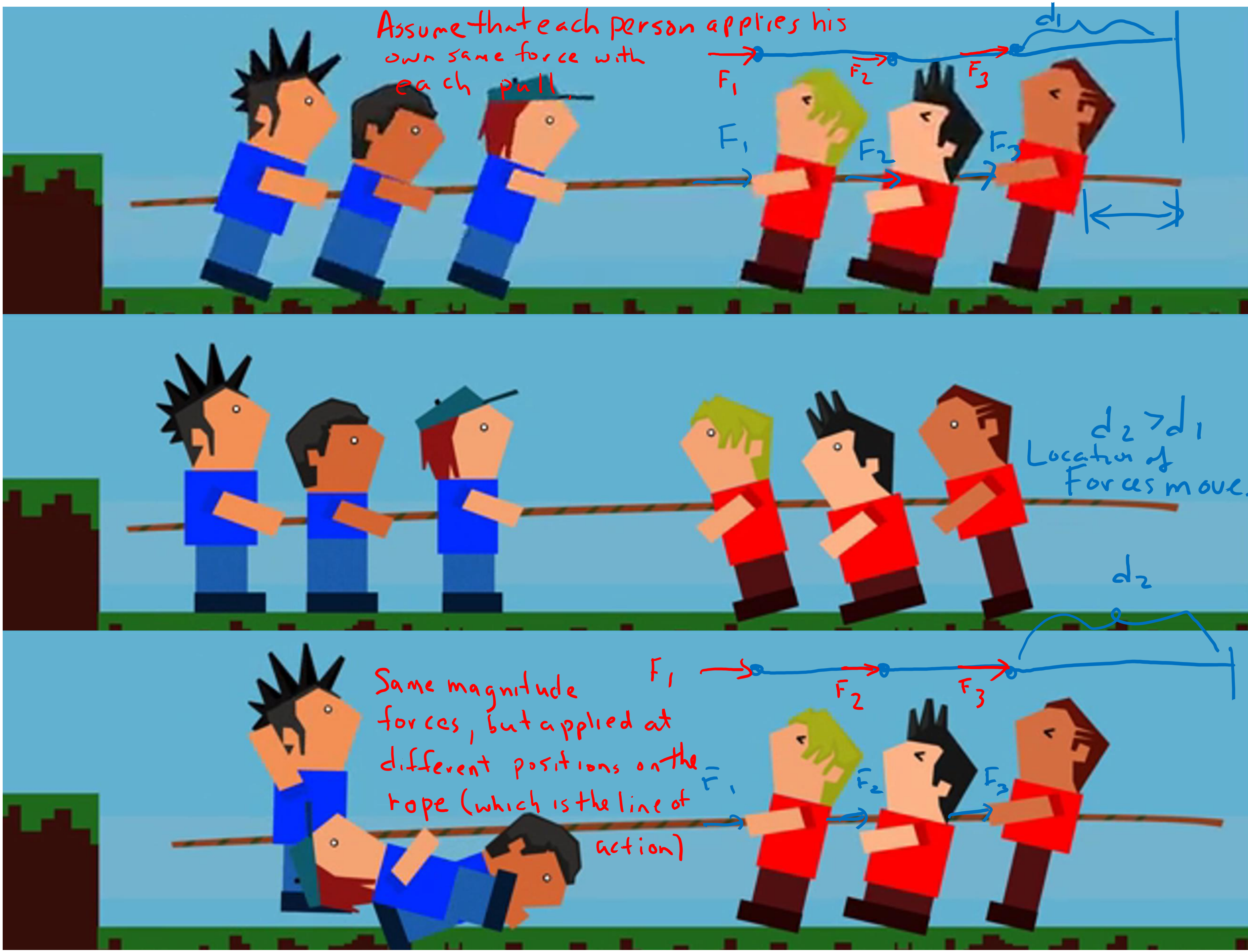
$\boxed{\vec{M}_R = 126\hat{k} \text{ ft}\cdot\text{lb}}$ ccw

Q: Which components of forces contribute to rotation of the body?

Moving a force on its line of action

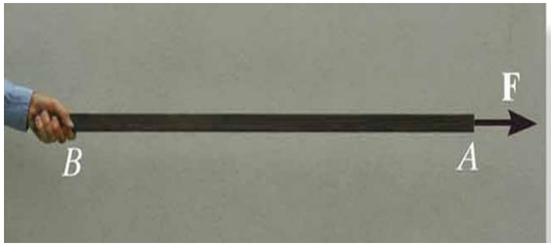


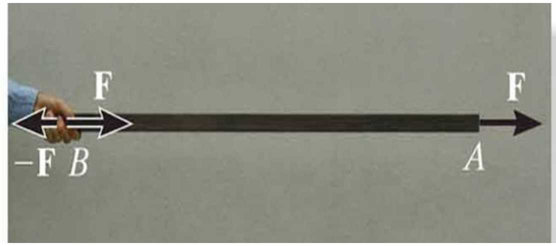
https://www.wikihow.com/Win-at-Tug-of-War

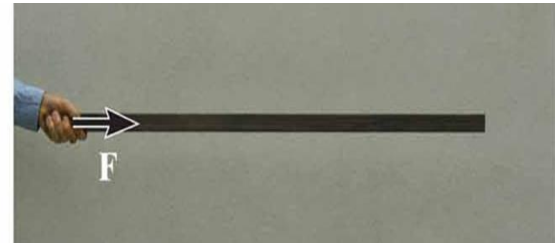


Assume that each person applies his own same force with each pull
d1
F1
F2
F3
d2 > d1
Location of Forces moves
d2
Same magnitude forces, but applied at different positions on the rope (which is the line of action)

Moving a force on its line of action





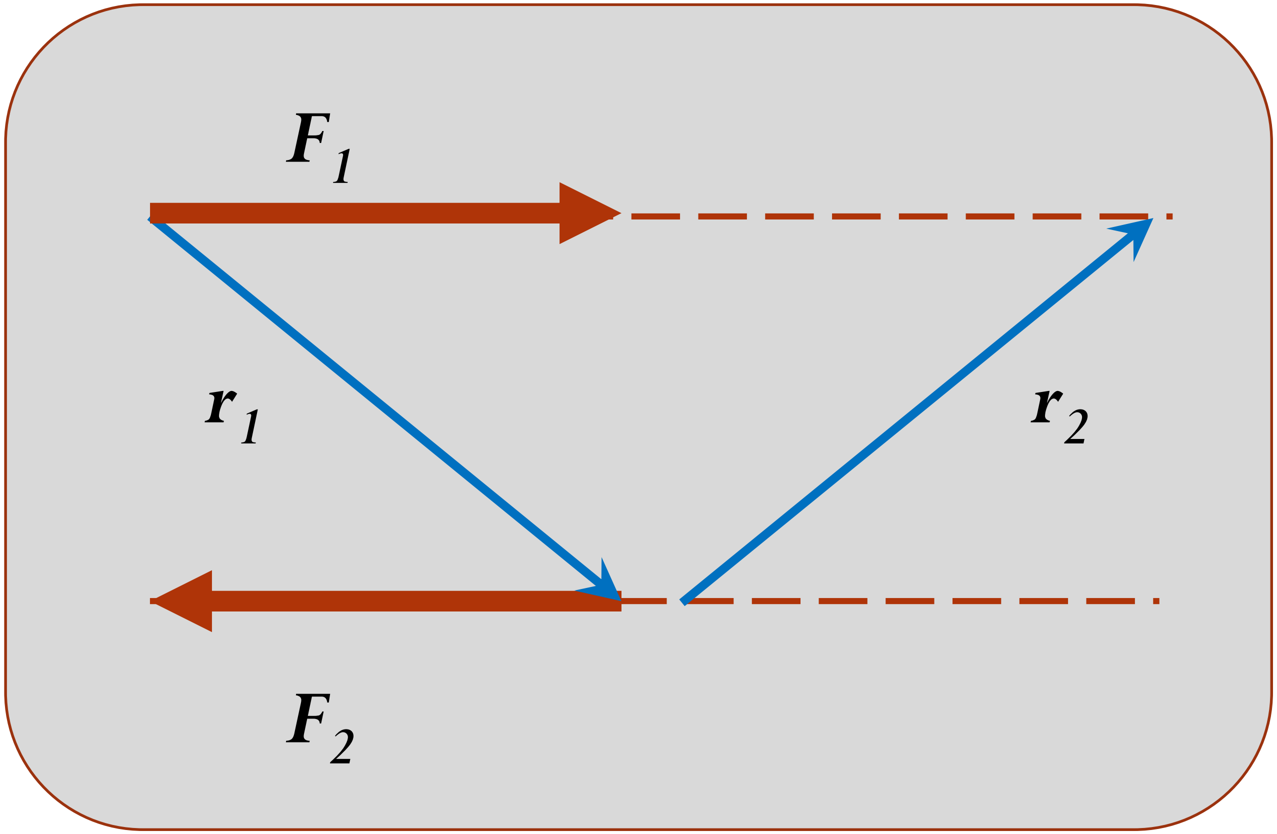


Moving a force from A to B, when both points are on the vector's line of action, does not change the **external effect**.

Hence, a force vector is called a **sliding vector**.

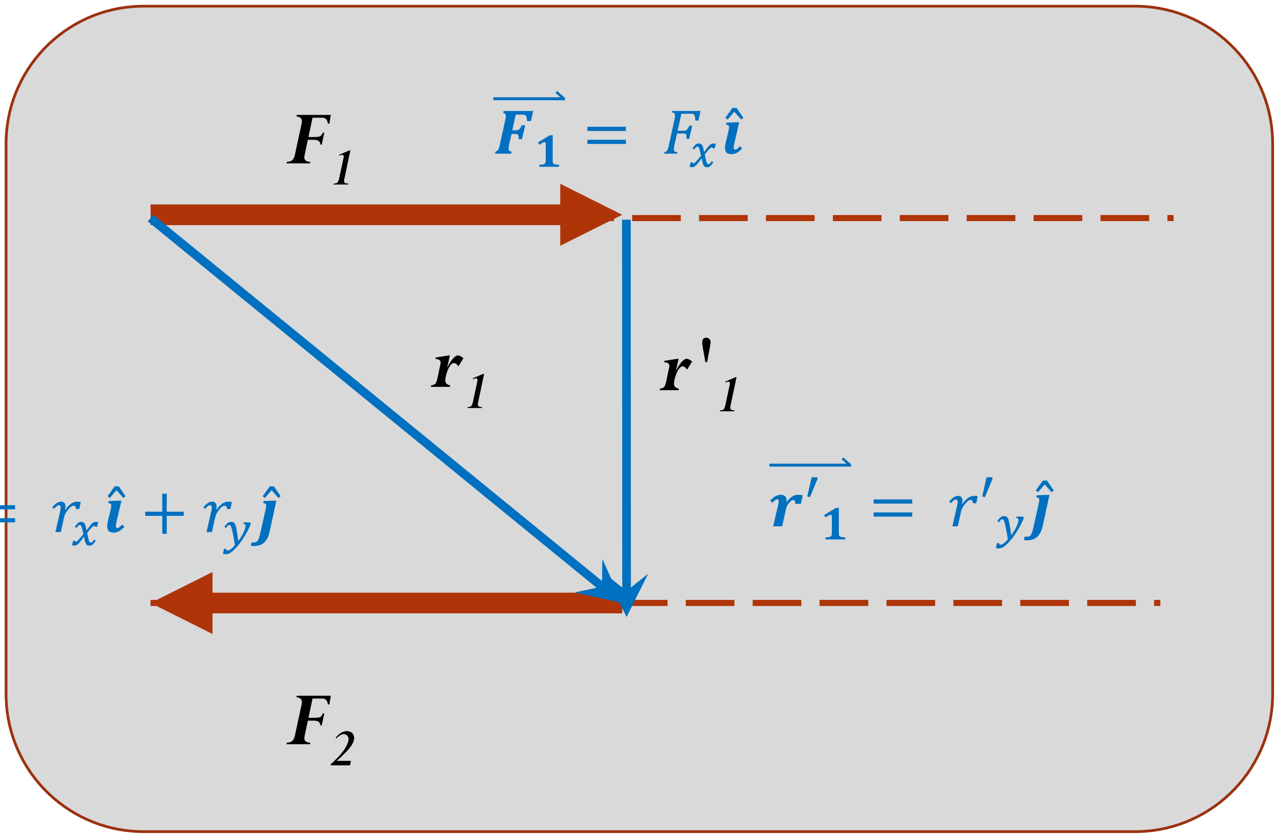
However, the **internal effect** of the force on the body does depend on where the force is applied.

$\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ form a couple.



$\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ form a couple.

Moving a force on its line of action



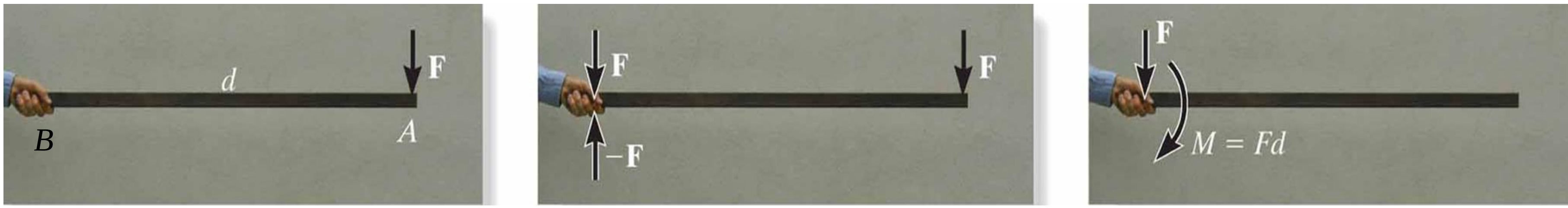
$$\overrightarrow{\boldsymbol{r_1}} = r_x\hat{\boldsymbol{\imath}} + r_y\hat{\boldsymbol{\jmath}}$$

$$r_y = r'_y$$

$$\overrightarrow{\boldsymbol{M}} = \overrightarrow{\boldsymbol{r_1}} \times \overrightarrow{\boldsymbol{F_1}} = -r_yF_x\hat{\boldsymbol{k}}$$

$$\overrightarrow{\boldsymbol{r'_1}} \times \overrightarrow{\boldsymbol{F_1}} = -r'_yF_x\hat{\boldsymbol{k}}$$
$$= -r_yF_x\hat{\boldsymbol{k}} = \overrightarrow{\boldsymbol{M}}$$

Moving a force off of its line of action

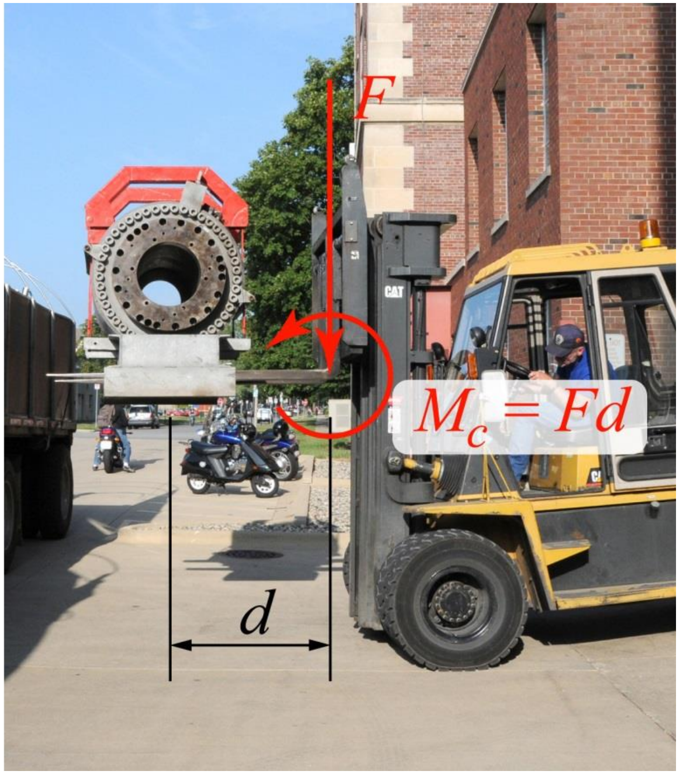


The two force systems are equipollent since the resultant force is the same in both systems, and the resultant moment with respect to any point P is the same in both systems.

So moving a force off its line of action means you have to “add” a new couple. Since this new couple moment is a **free vector**, it can be applied at any point on the body.

Are these systems the same?

Force system I



Force system II

A – YES

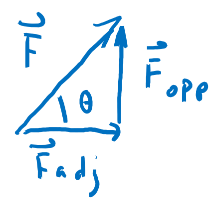
B – NO

Equipollent (or equivalent) force systems

A force **system** is a collection of **forces** and **couples** applied to a body.

Two force systems are said to be **equipollent** (or equivalent) if they have the **same resultant force** AND the **same resultant moment** with respect to any point O.

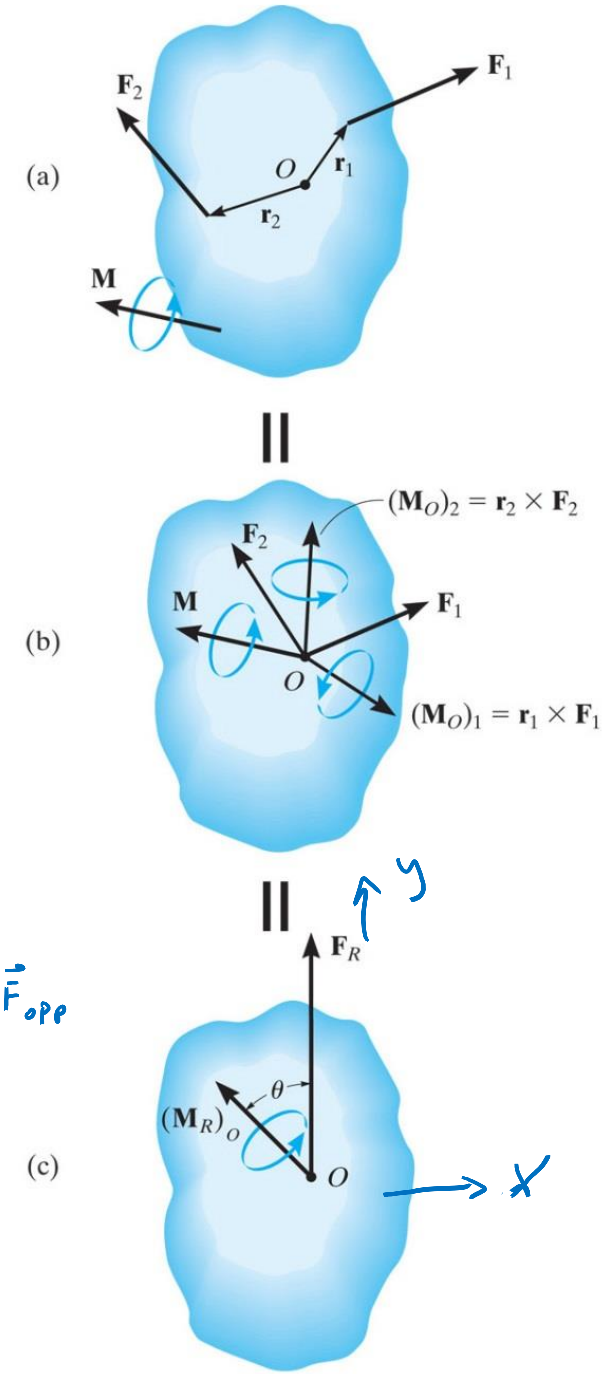
Reducing a force system to a single resultant force $\boldsymbol{F}_R$ and a single resultant couple moment $(\boldsymbol{M}_R)_o$:

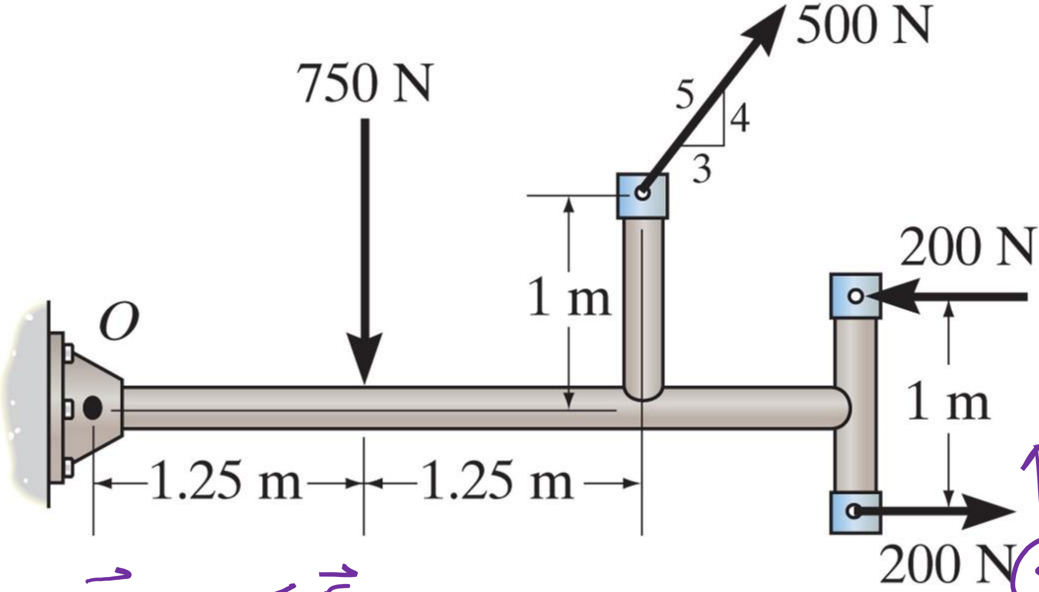


$$\overrightarrow{\boldsymbol{F}_R} = \Sigma F_x \hat{\boldsymbol{\imath}} + \Sigma F_y \hat{\boldsymbol{\jmath}} + \Sigma F_z \widehat{\boldsymbol{k}}$$

$$|\overrightarrow{\boldsymbol{F}_R}| = \sqrt{F_x^2 + F_y^2 + F_z^2} \qquad \theta = \tan^{-1}\frac{F_{opp}}{F_{adj}}$$

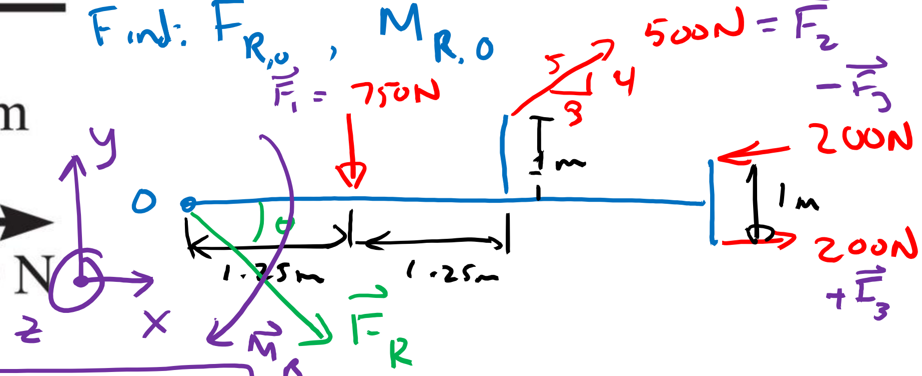
$$(\boldsymbol{M}_R)_o = \sum \boldsymbol{M}_o + \sum \boldsymbol{M}$$





Replace the forces and couple system acting on the member by an equivalent force and couple moment acting at point O.

Find: $\vec{F}_{R,O}$, $\vec{M}_{R,O}$



$\vec{F}_{R,P} = \sum \vec{F}$

$F_x: \frac{3}{5} 500N - 200N + 200N = \boxed{300N\,\hat{i} = F_{Rx}}$

$F_y: \frac{4}{5} 500N - 750N = \boxed{-350N\,\hat{j} = F_{Ry}}$

$|\vec{F}_R| = \sqrt{300^2 + (-350^2)} = 461\,N$

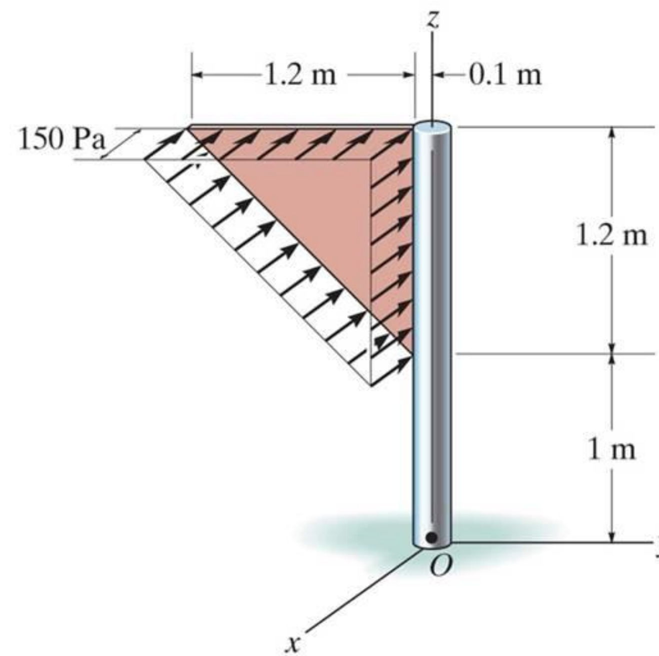
$\theta = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right) = 49.4°$

$\vec{M}_R = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{M}_{200N,\,couple}$

$\vec{M}_R = (-1.25m)(750N)(-\hat{k}) + (2.5m\,\hat{j}) \times \left(\frac{3}{5}500N\,\hat{i} + \frac{4}{5}500N\,\hat{j}\right) + (1m)(200N)\,\hat{k}$

$= \boxed{-37.5\,Nm\,\hat{k} = \vec{M}_R}$

Reduction to a simple distributed load

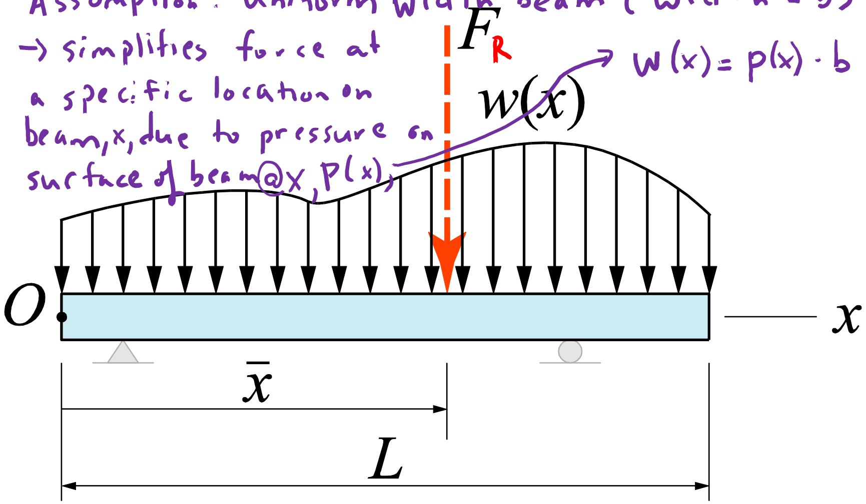


The lumber places a distributed load (due to the weight of the wood) on the beams. To analyze the load's effect on the steel beams, it is often helpful to reduce this distributed load to a single force. How would you do this?

To be able to design the joint between the sign and the sign post, we need to determine a single equivalent resultant force and its location.

Reduction to a simple distributed load

Assumption: Uniform width beam (width = b)
→ simplifies force at a specific location on beam, x, due to pressure on surface of beam @ x, p(x), → w(x) = p(x) · b



In structural analysis, often presented with **distributed load** $w(x)$ (force/unit length) and need to find equivalent loading $\boldsymbol{F}$. ↑ $\vec{F}_{Result}$

Ex: winds, fluids, weight on body's surface.

(x) is a func– position x along the beam
: force / Length $\left[\frac{N}{m}\right]$ or $\left[\frac{lb}{ft}\right]$

By equipollence, we require that $\sum \boldsymbol{F}$ be the same in both systems, i.e.,

$$|\vec{\boldsymbol{F}}| = \int_0^L w(x)\, dx = A$$

$\vec{F}_R$ = area under the loading function curve

and $\sum \boldsymbol{M}_P$ with respect to any point P be the same in both systems, i.e.,

$$\int_0^L w(x)\, x\, dx = \bar{x}\, \boldsymbol{F}$$

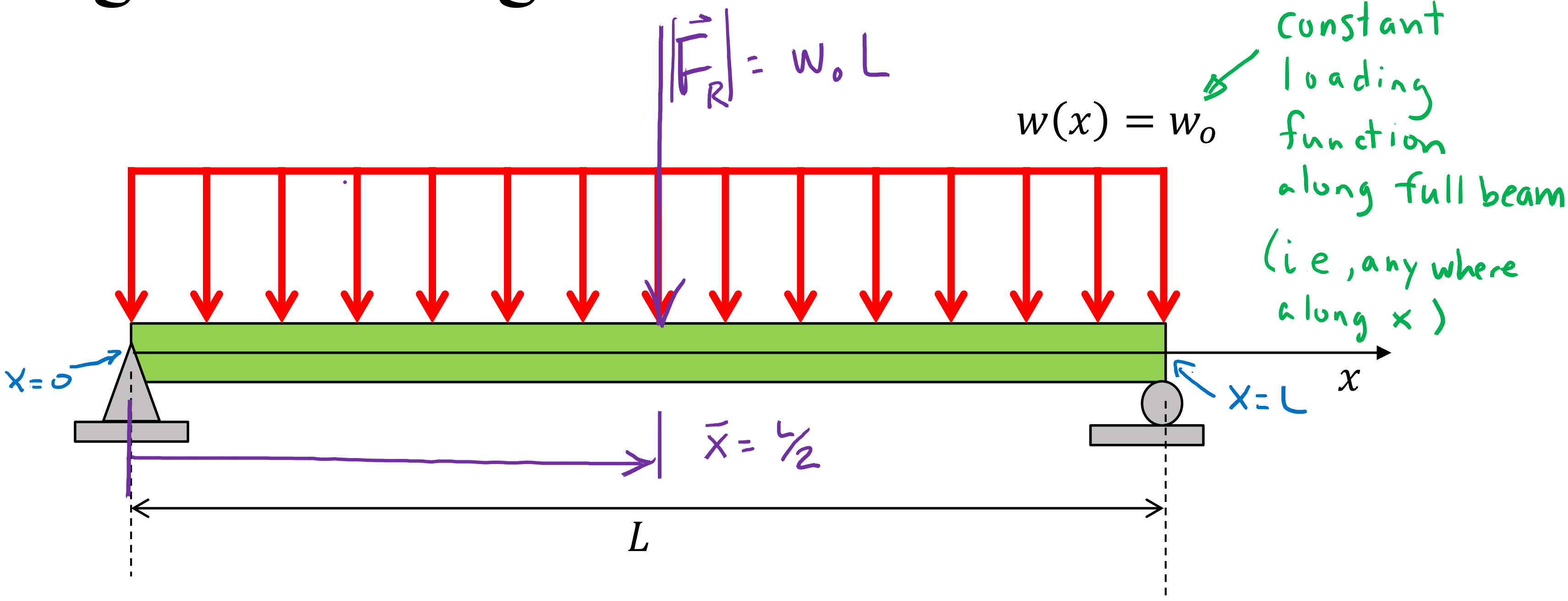
$\bar{x}$ is location at which $\vec{F}_R$ is applied

Combining both equations gives:

$$\bar{x} = \frac{\int_0^L w(x) x\, dx}{\int_0^L w(x) dx}$$

$\bar{x}$ = **geometric center or centroid** of area A under loading curve $w(x)$.

Rectangular loading

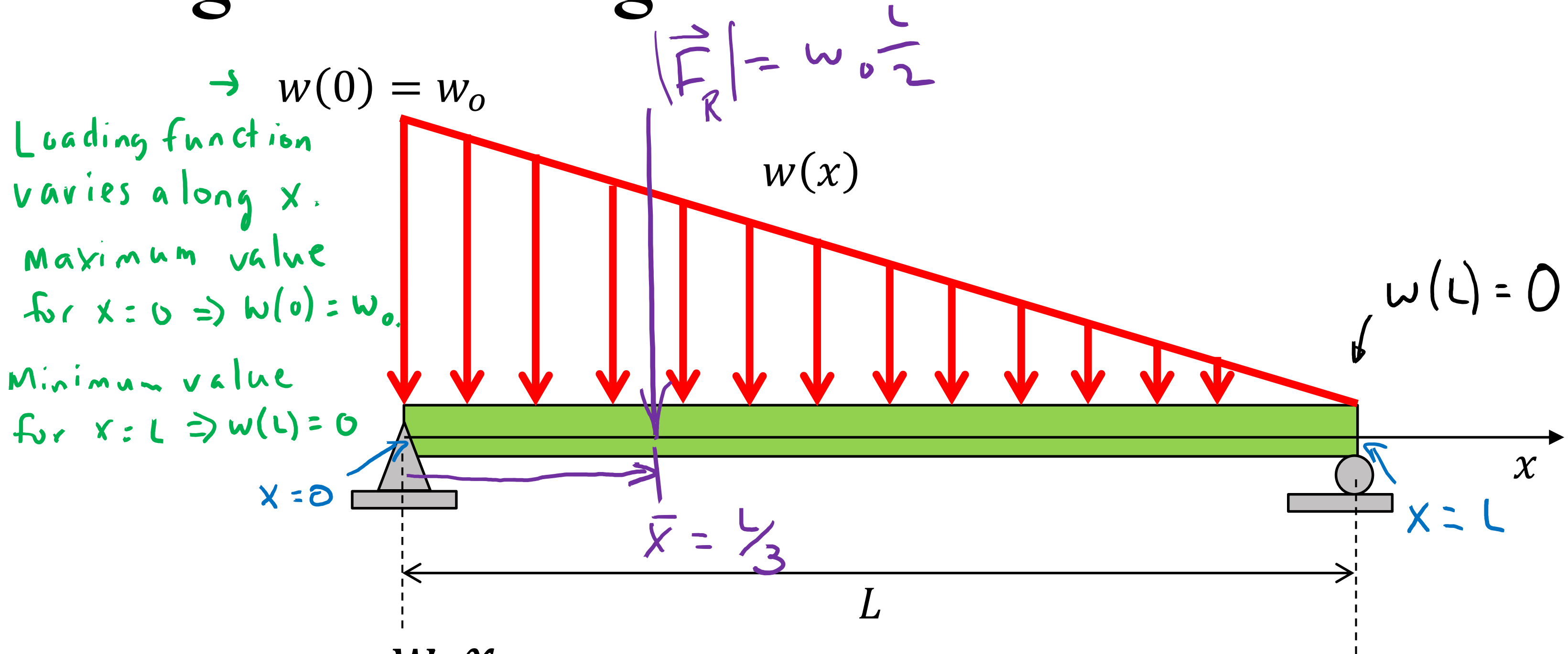


$$w(x) = w_o \qquad \boldsymbol{F} = \int_0^L w(x)\,dx \qquad \bar{x} = \frac{\int_0^L w(x)\,x\,dx}{\int_0^L w(x)\,dx}$$

$$|\vec{F}_R| = \int_0^L w_o\,dx = w_o L$$

$$\bar{x} = \frac{\int_0^L w_o x\,dx}{\int_0^L w_o\,dx} = \frac{w_o \frac{L^2}{2}}{w_o L} = \frac{L}{2}$$

Triangular loading



$$w(x) = w_o - \frac{w_o x}{L}$$

$$|\vec{F}_R| = \int_0^L \left(w_o - \frac{w_o x}{L}\right) dx = w_o L - w_o \frac{L}{2} = w_o \frac{L}{2}$$

$$\bar{x} = \frac{\int_0^L \left(w_o - \frac{w_o x}{L}\right) x\, dx}{\int_0^L \left(w_o - \frac{w_o x}{L}\right) dx} = \frac{w_o \frac{L^2}{2} - \frac{w_o}{L} \frac{L^3}{3}}{w_o \frac{L}{2}} = \frac{\frac{w_o L^2}{6}}{\frac{w_o L}{2}} = \frac{L}{3}$$