Statics - TAM 210 & TAM 211

Lecture 12

February 12, 2018

Announcements

- **READ Piazza posts!** If you had difficulty with Quiz 2, start reading and creating posts. There is a direct correlation with reviewing posts and quiz grade.
- Cumulative exam, Thursday, April 5, 7-9pm, 1 Noyes Lab
  - If you need DRES accommodation, send private message to instructors on Piazza with PDF of DRES letter. You must make your own arrangements at DRES testing facilities.
  - Conflict exam request: **MUST** send private message instructors on Piazza **now or at least 2 weeks before the exam date**. Only legitimate conflicts will be allowed. See Information tab > Exam

- Upcoming deadlines:
- Tuesday (2/13)
  - PL Homework 4
- Written Assignment 2 (2/15)
  - See Schedule page for assignment instructions
- Friday (2/16)
  - Mastering Engineering Tutorial 6

Recap: General procedure for analysis

1. Read the problem carefully; write it down carefully.
2. MODEL THE PROBLEM: Draw given diagrams neatly and construct additional figures as necessary.
3. Apply principles needed.
4. Solve problem symbolically. Make sure equations are dimensionally homogeneous
5. Substitute numbers. Provide proper units *throughout*. Check significant figures. Box the final answer(s).
6. See if answer is reasonable.

Most effective way to learn engineering mechanics is to ***solve problems!***

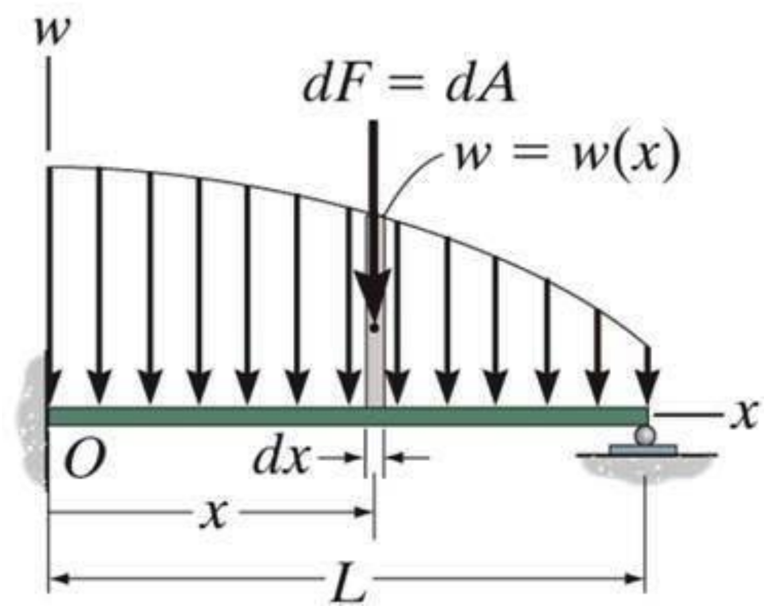
Chapter 4: Force System Resultants

Goals and Objectives

- Discuss the concept of the moment of a force and show how to calculate it in two and three dimensions
- How to find the moment about a specified axis
- Define the moment of a couple
- Finding equivalence force and moment systems
- Reduction of distributed loading

Recap

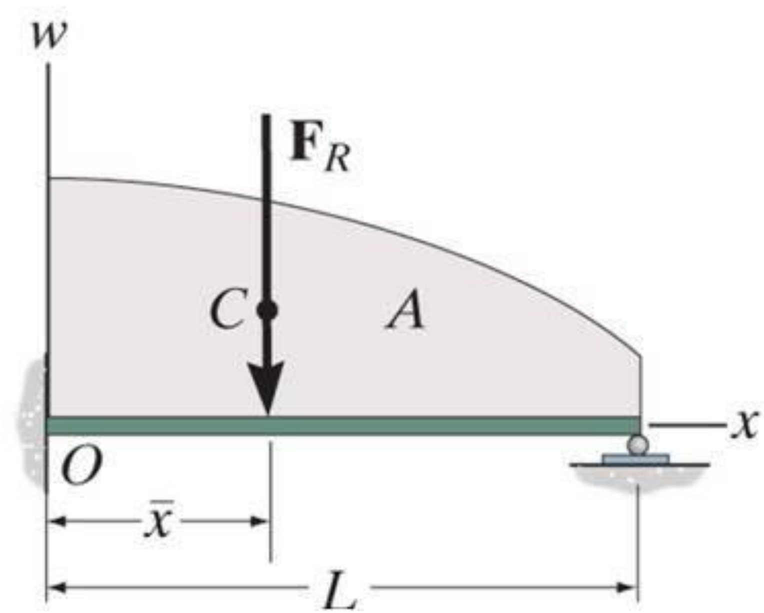
- Equivalent force system for distributed loading function $w(x)$ with units of $\frac{\text{force}}{\text{length}}$.
- Find magnitude F_R and location $\bar{x}$ of the equivalent resultant force for $\overrightarrow{\boldsymbol{F_R}}$



$$\left|\overrightarrow{\boldsymbol{F_R}}\right| = F_R = \int_0^L dF = \int_0^L w(x)\,dx = A$$

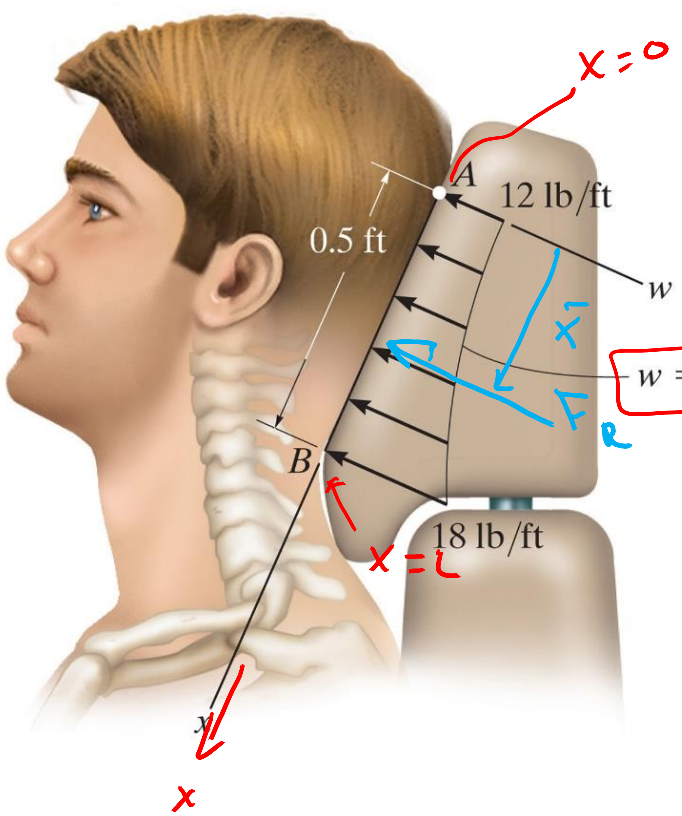
$$M_o = \int_0^L x\ w(x)\,dx = \bar{x}\,F_R$$

$$\bar{x} = \frac{M_o}{F_R} = \frac{\int_0^L x\ w(x)\,dx}{\int_0^L w(x)\,dx}$$



$\bar{x}$ = **geometric center or centroid** of area A under loading curve $w(x)$.

Find equivalent force and its location from point A for loading on headrest.



FIND: F_R & $\bar{x}$ wrt A

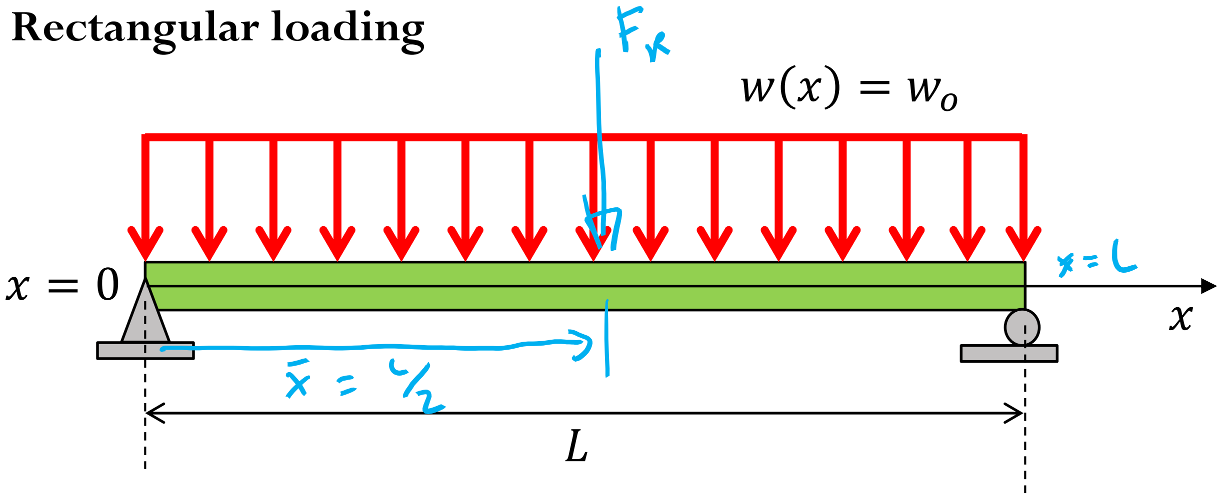
$$F_R = \int_0^L w(x)\,dx$$

$$F_R = \int_0^{0.5} 12(1+2x^2)\,dx$$

$$\bar{x} = \frac{M_A}{F_R} = \frac{\int_0^L x\,w(x)\,dx}{\int_0^L w(x)\,dx}$$

$$\bar{x} = \frac{\int_0^{0.5} x\left[12(1+2x^2)\right]dx}{\int_0^{0.5} 12(1+2x^2)\,dx}$$

Rectangular loading

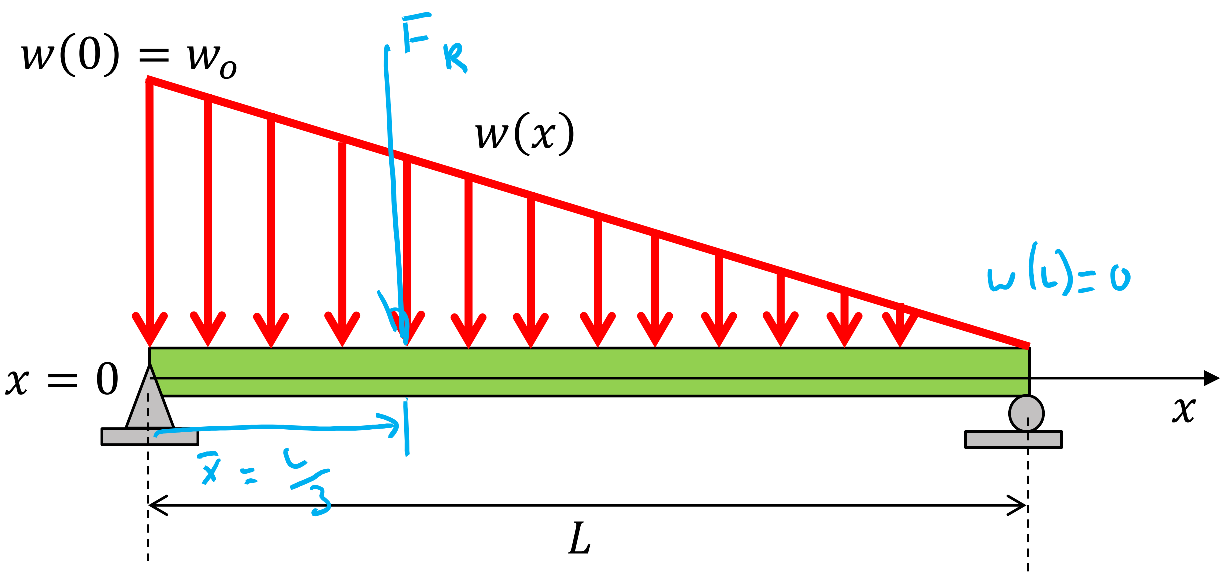


$$w(x) = w_o$$

$$\left|\vec{\boldsymbol{F}}\right| = F = w_o L$$

$$\bar{x} = \frac{L}{2}$$

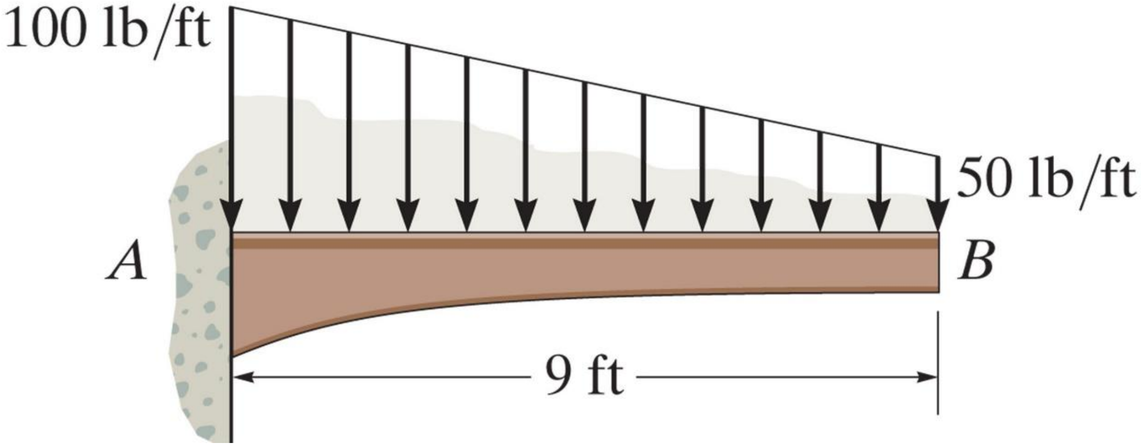
Triangular loading



$$w(x) = w_o - \frac{w_o x}{L}$$

$$F = w_o \frac{L}{2}$$

$$\bar{x} = \frac{L}{3}$$



Determine the magnitude and location of the equivalent resultant of this load.

Find: F_R & $\bar{x}$

Solution: separate distributed load into 2 simple loads (triangle + rectangle).

$$F_R = \Sigma F = F_1 + F_2$$

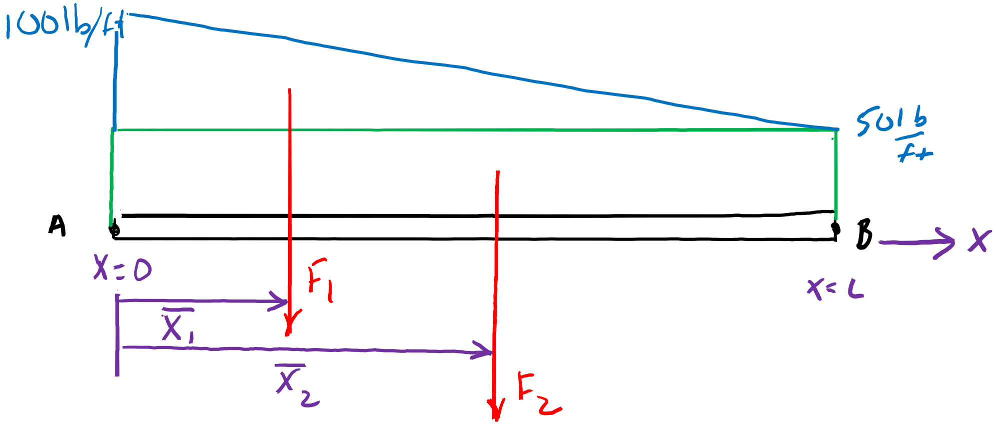
$$F_1 = w_0 \frac{L}{2} \quad , L = 9 ft$$

$$F_1 = \left(100 \frac{lb}{ft} - 50 \frac{lb}{ft}\right)\left(\frac{9 ft}{2}\right) = 225 lb$$

$$F_2 = w_0 L$$

$$F_2 = \left(50 \frac{lb}{ft}\right)(9 ft) = 450 lb$$

$$\boxed{F_R = 675 lb}$$

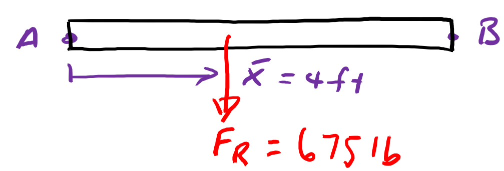


Sum moments about point A to find $\bar{x}$:

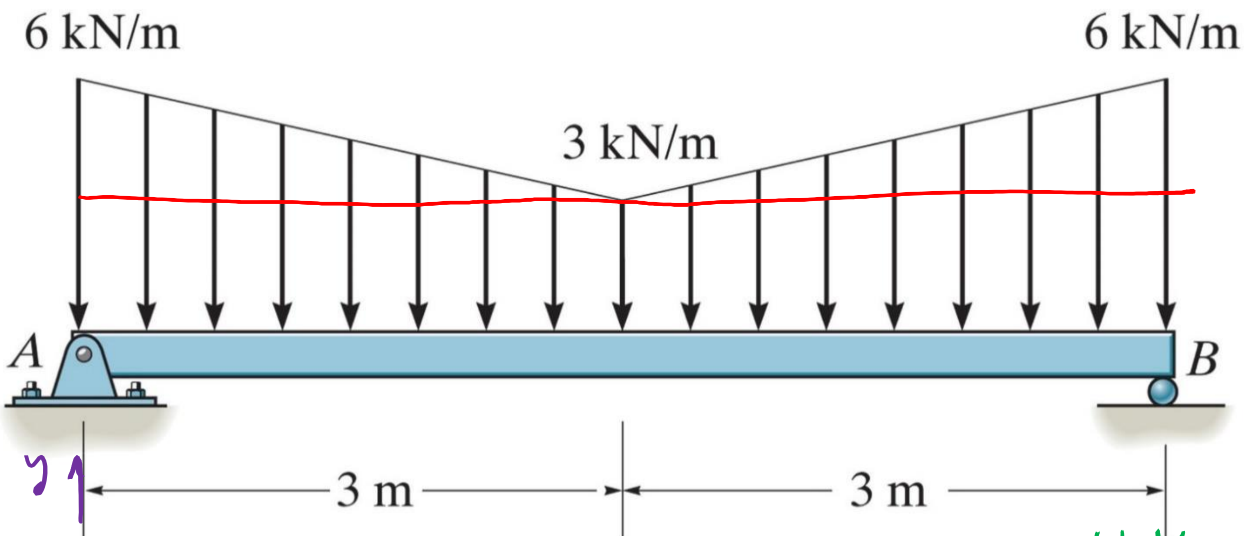
$$\bar{x}_1 F_1 + \bar{x}_2 F_2 = \bar{x} F_R$$

$$(3 ft)(225 lb) + (4.5 ft)(450 lb) = \bar{x}(675 lb)$$

$$\Rightarrow \boxed{\bar{x} = 4 ft}$$



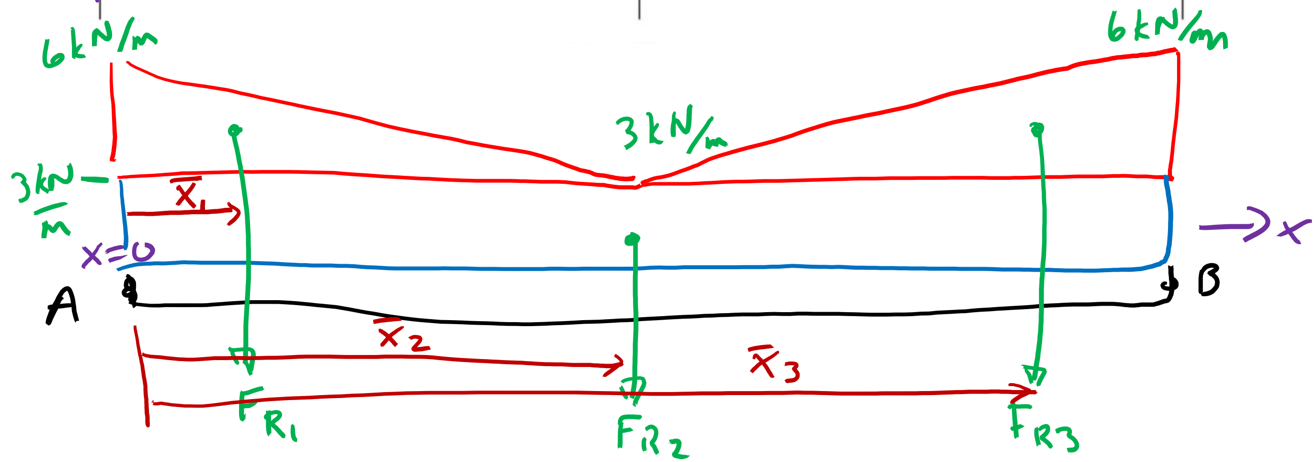
See Example 4.23 in text for full derivation



Replace the distributed loading by an equivalent resultant force and couple moment acting at point A.

Find: $\vec{F}_R$ & $\vec{M}_A$

Draw FBD and split into simple loads



$\vec{F}_R = F_{R_1} + F_{R_2} + F_{R_3}$

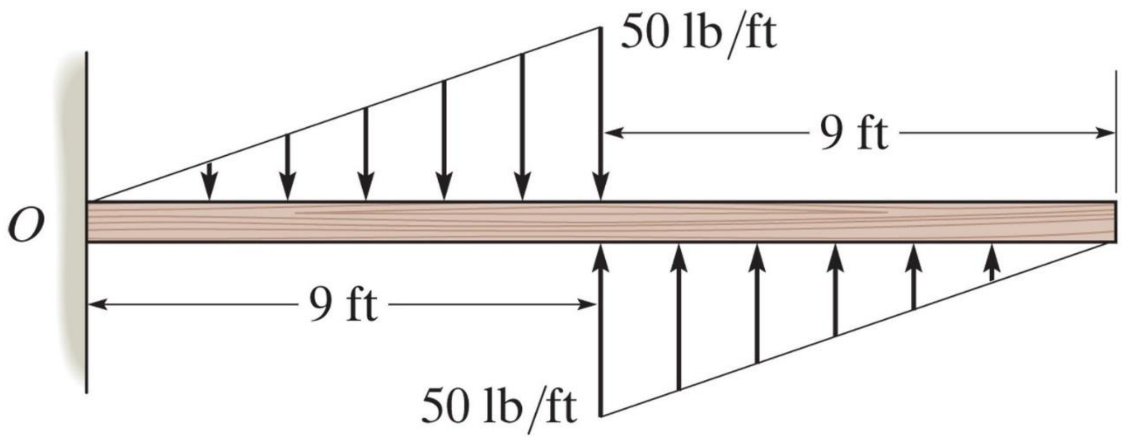
$F_R = \frac{3m}{2}\left(3\frac{kN}{m}\right) + 6m\left(3\frac{kN}{m}\right) + \frac{3m}{2}\left(\frac{3kN}{m}\right)$

$= 27\ kN\ (-\hat{j}) = F_R$

Sum moments about pt A

$+\circlearrowleft\ \vec{M}_A = -\bar{x}_1 F_{R_1} - \bar{x}_2 F_{R_2} - \bar{x}_3 F_{R_3}$

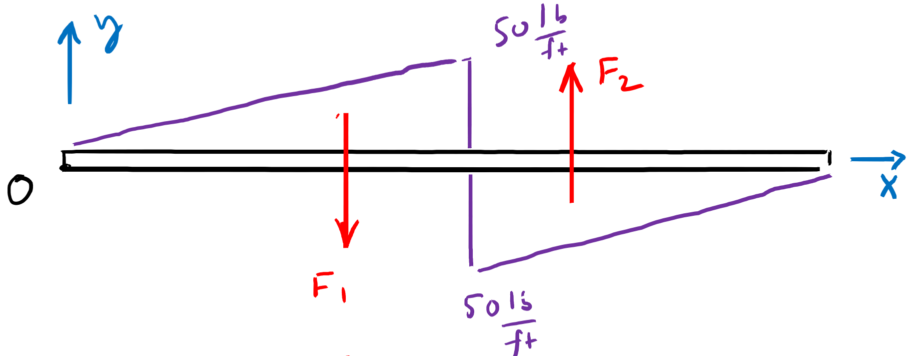
$\vec{M}_A = -\frac{L_1}{3}F_{R_1} - \frac{L_1+L_2}{2}F_{R_2} - \left(L_1 + \frac{2}{3}L_2\right)F_{R_3} = -81\ kN \cdot m\ \hat{k}$



Replace the loading by an equivalent resultant force and couple moment acting at point O.

Find: F_R, $\vec{M}_o$

Soln: Draw FBD



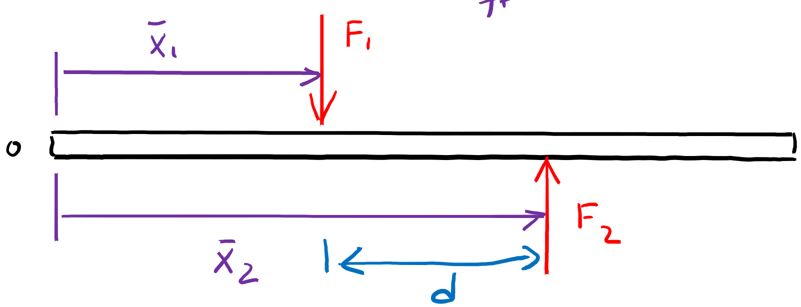
Sum Forces:

$$\vec{F}_R = \Sigma F_y = F_1 + F_2$$

$$= -\frac{9ft}{2}\left(50\frac{lb}{ft}\right)\hat{j} + \frac{9ft}{2}\left(50\frac{lb}{ft}\right)\hat{j}$$

$$\Rightarrow \vec{F}_R = 0$$

$$\Rightarrow \vec{F}_1 = -\vec{F}_2$$



$$+\circlearrowleft \vec{M}_o = \vec{r} \times \vec{F} = -\bar{x}_1 F + \bar{x}_2 F_2$$

$$= -\left(\frac{2}{3}9ft\right)F_1 + \left(9ft + \frac{1}{3}9ft\right)F_2$$

$$= -(6ft)F_1 + (12ft)F_2$$

But $\vec{F}_1 = -\vec{F}_2$, $|\vec{F}_1| = |\vec{F}_2| = F$

$$= (6ft)F = (6ft)\left(\frac{9ft}{2}\right)\left(50\frac{lb}{ft}\right) = 1350\,lb{\cdot}ft\ (+\hat{k})$$

Or if note that $\vec{F}_1$ & $\vec{F}_2$ create a couple moment, then can say

$\Sigma\vec{M}_o = \vec{r} \times \vec{F} = dF$, where $F = F_1$ or F_2

$\therefore \vec{M}_o = (6ft)\left(\frac{9ft}{2}\right)\left(50\frac{lb}{ft}\right) = 1350\,lb{\cdot}ft\ (+\hat{k})$ ✓ same

Chapter 5: Equilibrium of Rigid Bodies

Goals and Objectives

- Introduce the free-body diagram for a rigid body
- Develop the equations of equilibrium for a rigid body
- Solve rigid body equilibrium problems using the equations of equilibrium

Equilibrium of a Rigid Body

Static equilibrium:

$\sum \vec{\boldsymbol{F}} = \boldsymbol{0}$ (zero forces = no translation)

$\sum(\vec{\boldsymbol{M}}) = \boldsymbol{0}$ (zero moment = no rotation)

Maintained by reaction forces and moments

Forces from supports / constraints are exactly enough to produce zero forces and moments

Assumption of rigid body

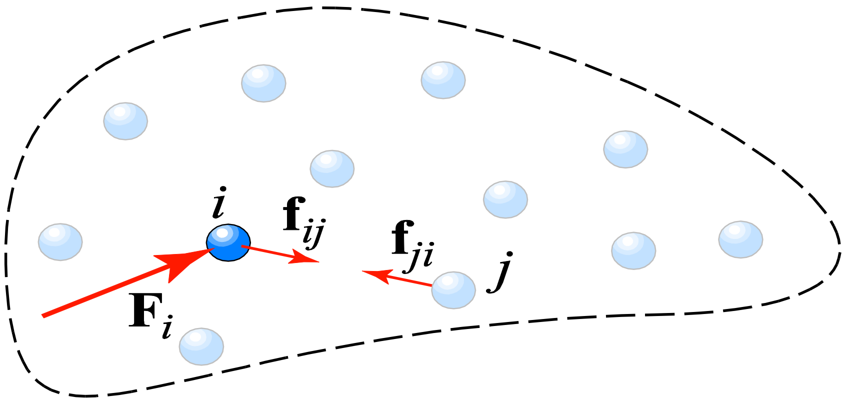
Shape and dimensions of body remain **unchanged** by application of forces.

More precisely:
All **deformations of bodies** are small enough to be ignored in analysis.

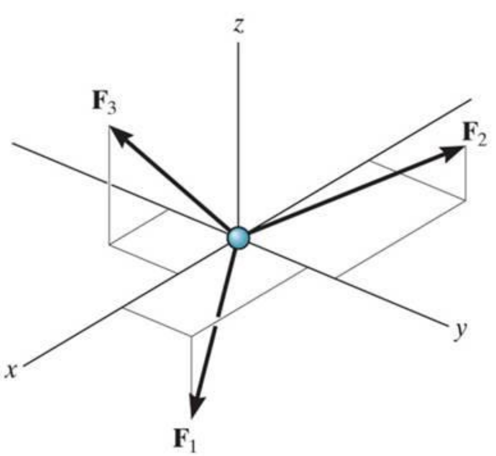
Equilibrium of a Rigid Body

Equilibrium of a rigid body is of central importance in statics. We regard a rigid body as a collection of particles.

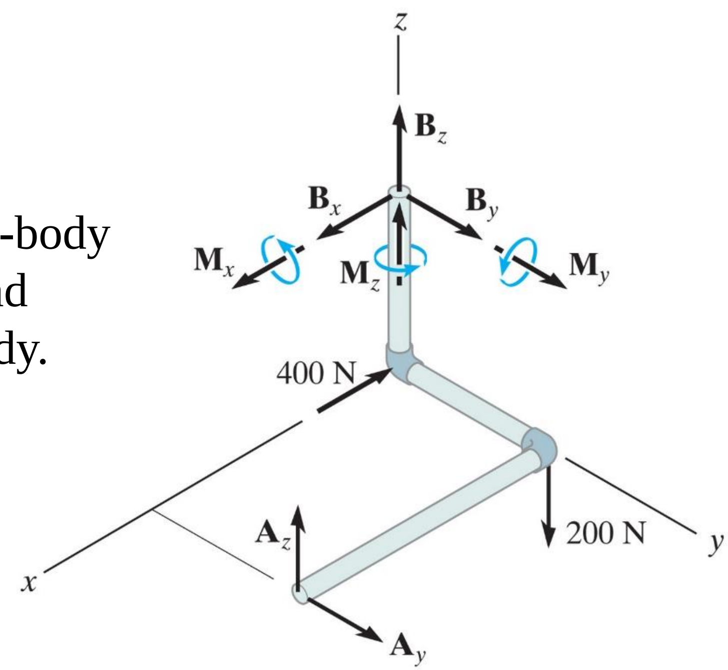
$\overrightarrow{\boldsymbol{F}_i}$ = resultant external force on particle $\boldsymbol{i}$
$\overrightarrow{\boldsymbol{f}_{ij}}$ = internal force on particle $\boldsymbol{i}$ by particle $\boldsymbol{j}$
$\overrightarrow{\boldsymbol{f}_{ij}}$ = internal force on particle $\boldsymbol{j}$ by particle $\boldsymbol{i}$



Note that $\overrightarrow{\boldsymbol{f}_{ij}} = \overrightarrow{\boldsymbol{f}_{ij}}$ by Newton's third law. Therefore the internal forces will not appear in the equilibrium equations.

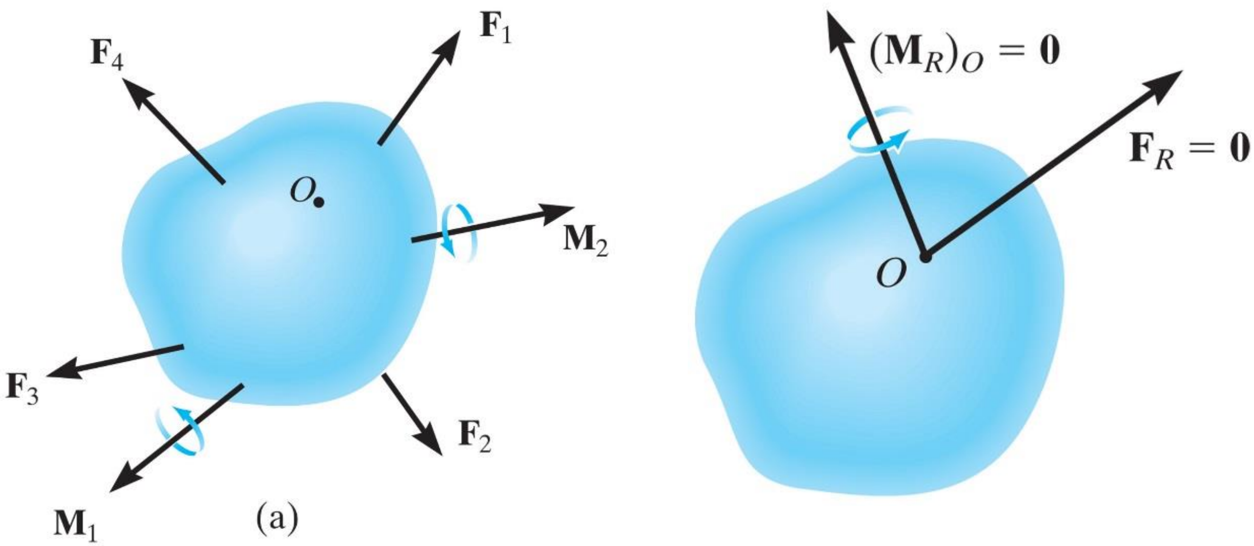


In contrast to the forces on a particle, the forces on a rigid-body are not usually concurrent and may cause rotation of the body.



Equilibrium of a Rigid Body

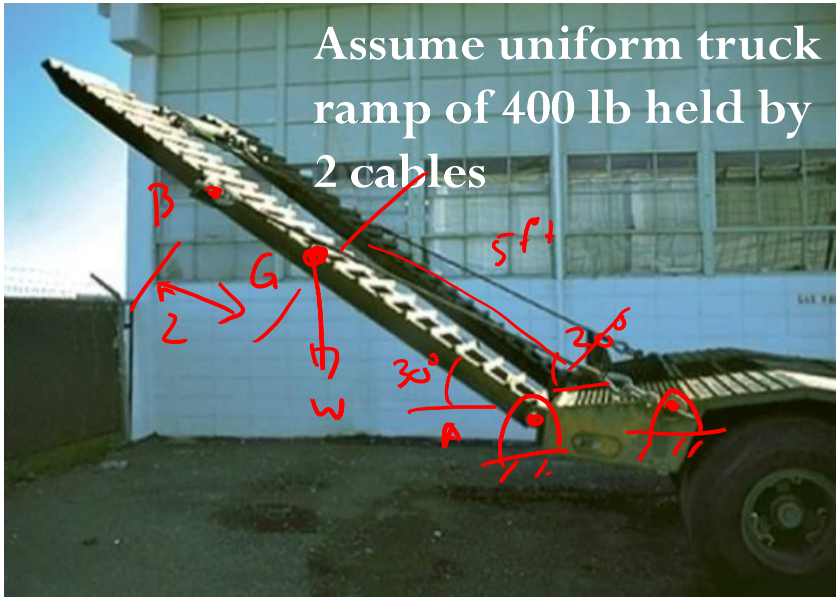
We can reduce the force and couple moment system acting on a body to an equivalent resultant force and a resultant couple moment at an arbitrary point O.



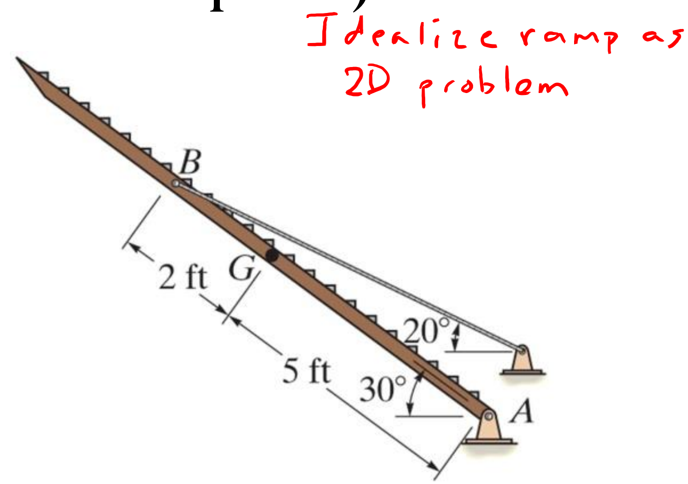
$$\overrightarrow{F_R} = \sum \vec{F} = 0$$

$$(\overrightarrow{M_R})_O = \sum \overrightarrow{M_O} = 0$$

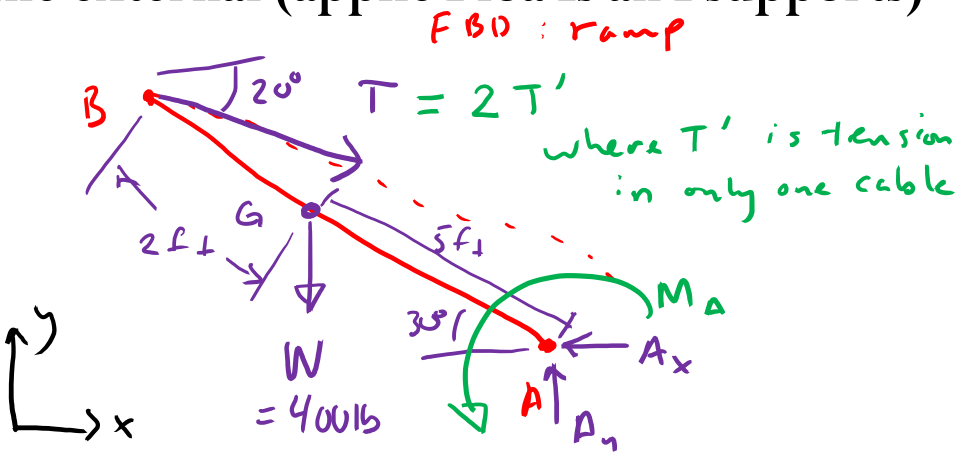
Process of solving rigid body equilibrium problems



1. Create idealized model (modeling and assumptions)



2. Draw free body diagram showing ALL the external (applied loads and supports)



3. Apply equations of equilibrium

$$\overrightarrow{F_R} = \sum \vec{F} = 0$$

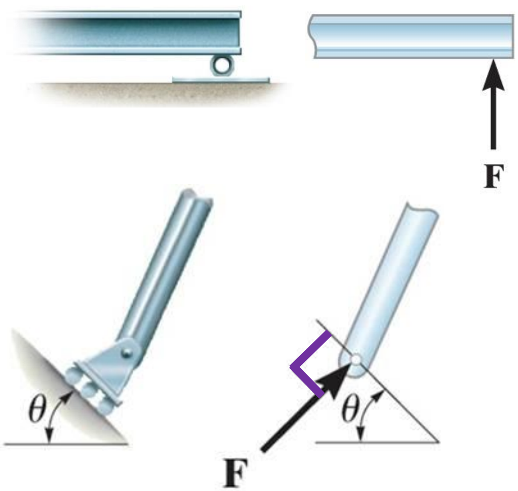
$$(\overrightarrow{M_R})_A = \sum \overrightarrow{M_A} = 0 = f(T, W)$$

In this case, let's sum moments about pt A

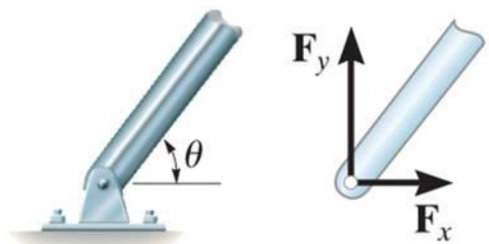
See Example 5.11 in text for full derivation

Equilibrium in two-dimensional bodies
(Support reactions)

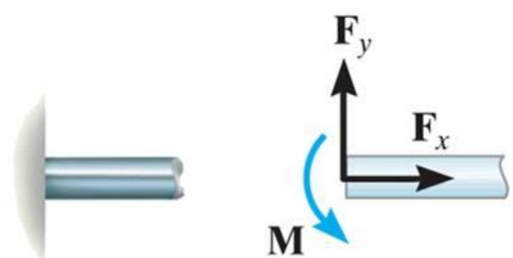
Roller



Smooth pin or hinge



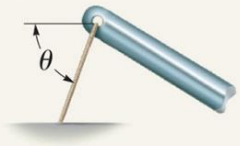
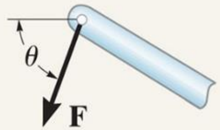
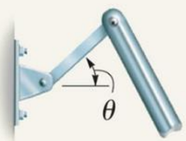
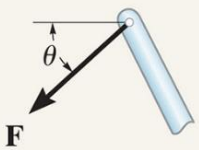
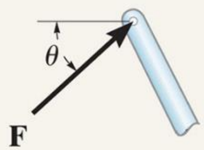
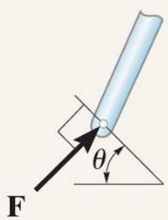
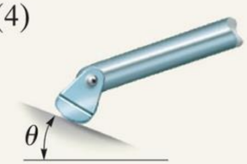
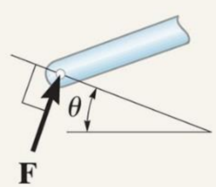
Fixed support



- If a support prevents the translation of a body in a given direction, then a force is developed on the body on that direction
- If a rotation is prevented, a couple moment is exerted on the body

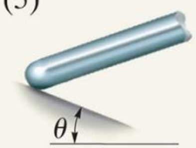
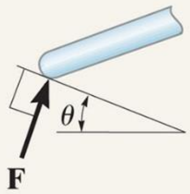
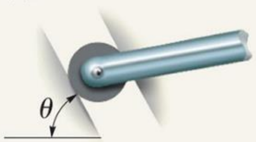
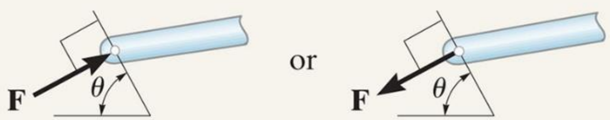
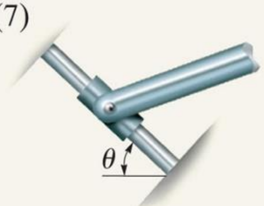
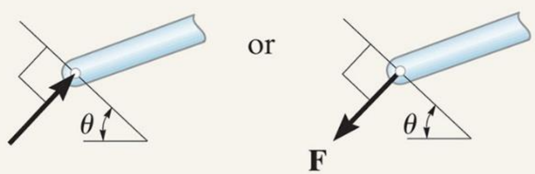
Types of connectors

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link	 or 	One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

Types of connectors

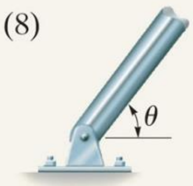
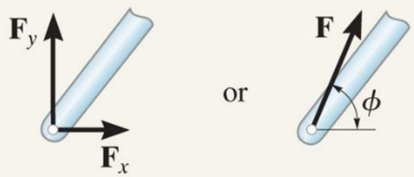
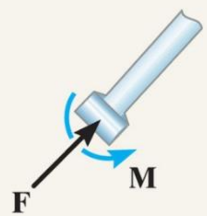
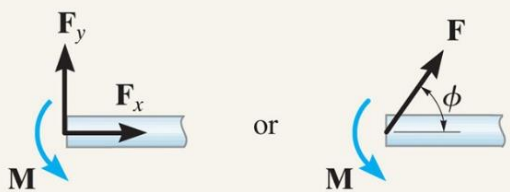
TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(5)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6)  roller or pin in confined smooth slot		One unknown. The reaction is a force which acts perpendicular to the slot.
(7)  member pin connected to collar on smooth rod		One unknown. The reaction is a force which acts perpendicular to the rod.

continued

Types of connectors

TABLE 5–1 Continued

Types of Connection	Reaction	Number of Unknowns
(8)  smooth pin or hinge		Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected to collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support		Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.