Statics - TAM 210 & TAM 211

Lecture 15

February 19, 2018

Announcements

- Structured office hours of working through practice problems will be held during Sunday office hours, starting Sunday February 24
- Students are encouraged to practice drawing FBDs, writing out equilibrium equations, and solving these by hand (especially if you have not taken a course with linear (matrix) algebra or programming in MATLAB).
  - Expending large amounts of time trying to de-bug MATLAB code is not the focus of this course. All problems can be solved by hand. Quiz questions are timed for solution by hand.

- Upcoming deadlines:
- Tuesday (2/20)
  - PL Homework 5
- Quiz 3 (2/21-23)
  - Sign up at CBTF
- Friday (2/23)
  - Mastering Engineering Tutorial 7

Chapter 5: Equilibrium of Rigid Bodies

Focus on 2D problems

Sections 5.1-5.4, 5.7

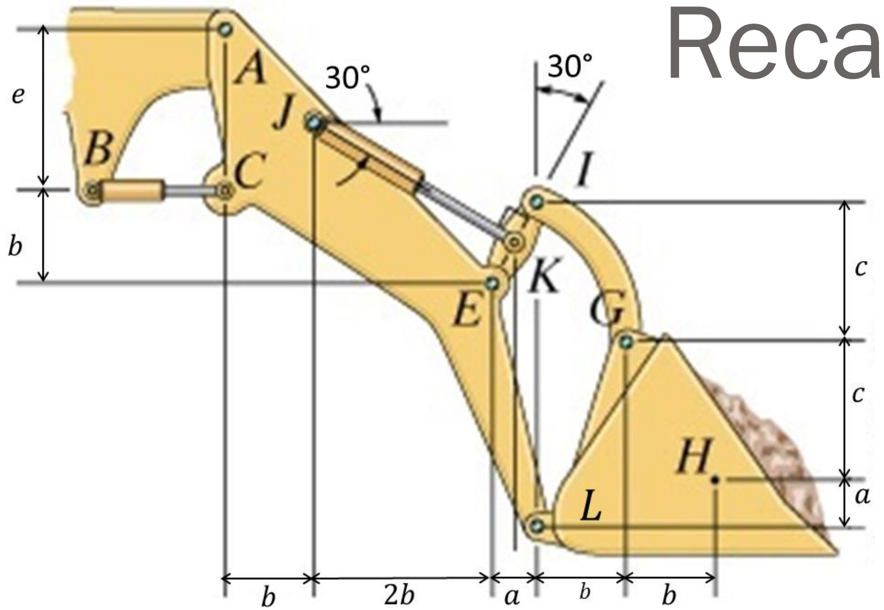
TAM 211 students will cover 3D problems (sections 5.5-5.6) in week 13

Goals and Objectives

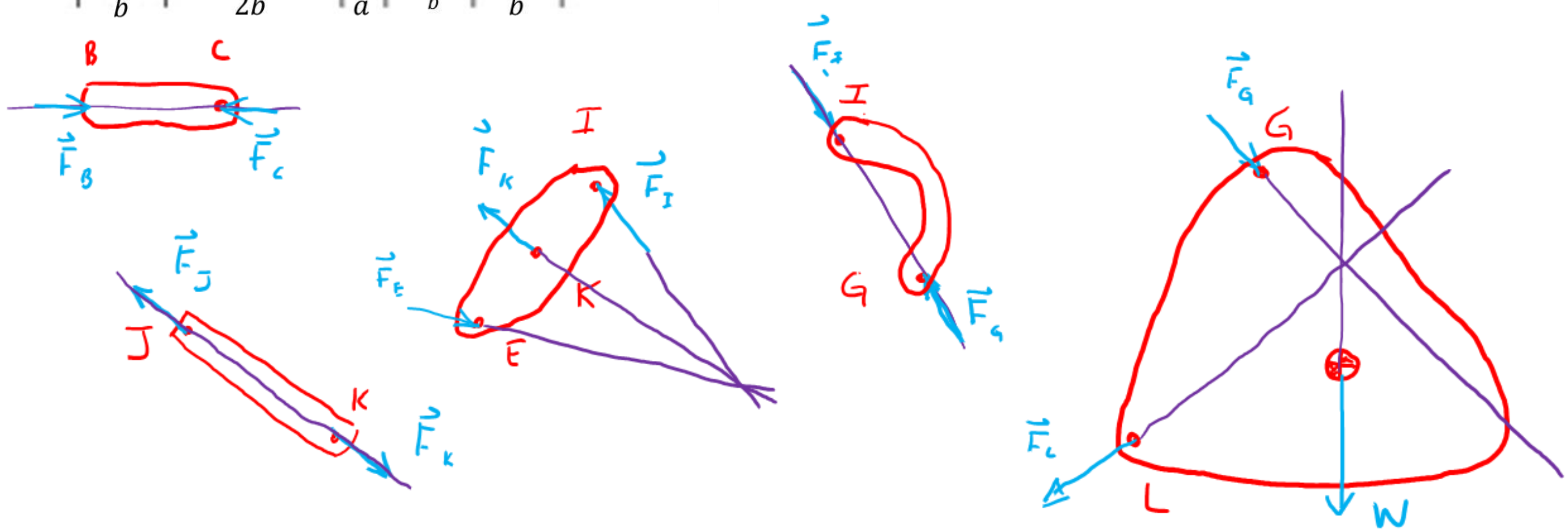
- Introduce the free-body diagram for a 2D rigid body
- Develop the equations of equilibrium for a 2D rigid body
- Solve 2D rigid body equilibrium problems using the equations of equilibrium
- Introduce concepts of
  - Reaction forces due to support
  - Two- and three-force members
  - Constraints and determinacy

Recap

Draw FBDs for each two or three force member (BC, JK, IE, IG, Bucket). Ignore weight of each link. Include dirt weight in bucket.



Directions of arrows of unknown forces/moments are arbitrary on FBD. Actual direction will be determined after solving for unknown values

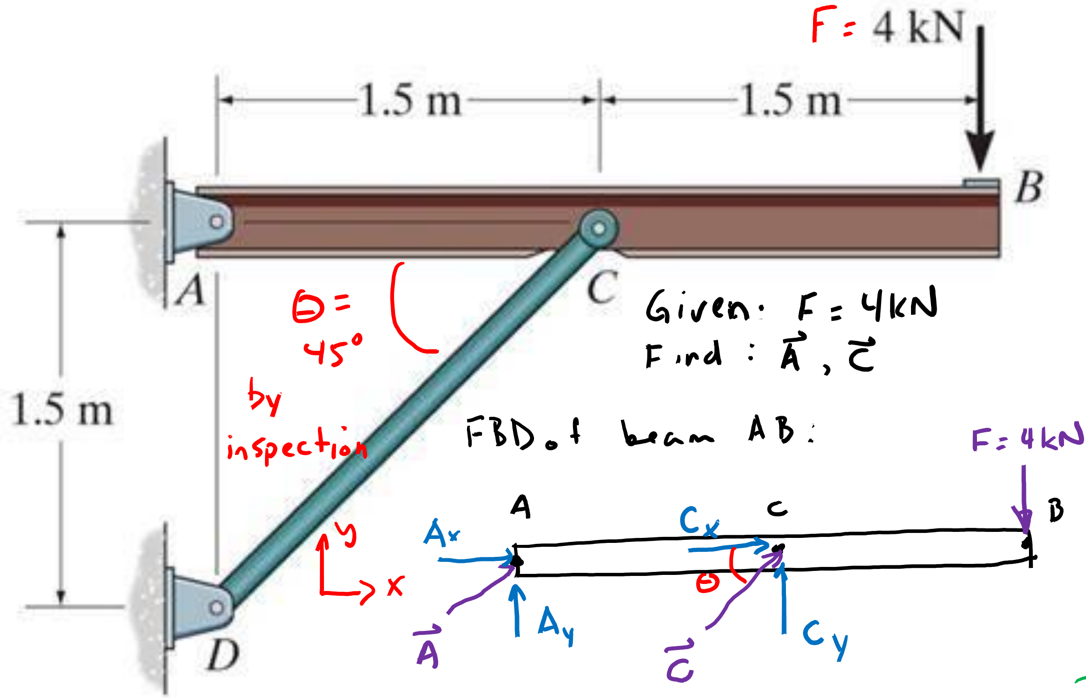


Line of action of an unknown force can be determined from 2- or 3-force members

2-force member: The 2 forces at ends are equal, opposite, collinear

3-force member: force system where the 3 forces

1. meet at the same point, or
2. are parallel



Given the 4kN load at B of the beam is supported by pins at A and C. Find the support reactions at A and C.

- Ignore weight
- use 2 & 3-force members to simplify problem.

ΣF_x: $A_x + C_x = 0 \rightarrow A_x = -C_x$

ΣF_y: $A_y + C_y - 4kN = 0 \rightarrow A_y = 4kN - C_y$

$\Sigma M_?$ Pick point to compute moment about that creates an equation with the least number of unknowns, so either A or C.

$\Sigma M = \vec{r} \times \vec{F}$ or dF if d is ⊥ distance to F

If select A,

$\circlearrowleft + \Sigma M_A$: $\vec{A}_x, \vec{A}_y, \vec{C}_x$ do not contribute to possible rotation about pt. A, since each passes thru A one's line of action

$(1.5m)C_y + (3m)(-4kN) = 0 \rightarrow$ $C_y = 8kN$

$\therefore$ $A_y = -4kN$

From FBD: $C_y = C\sin\theta$, $\theta = 45°$

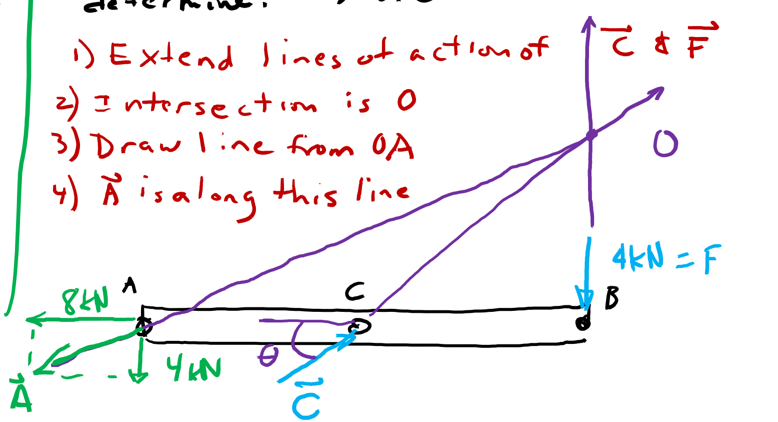
$\Rightarrow C = 11.3\ kN$

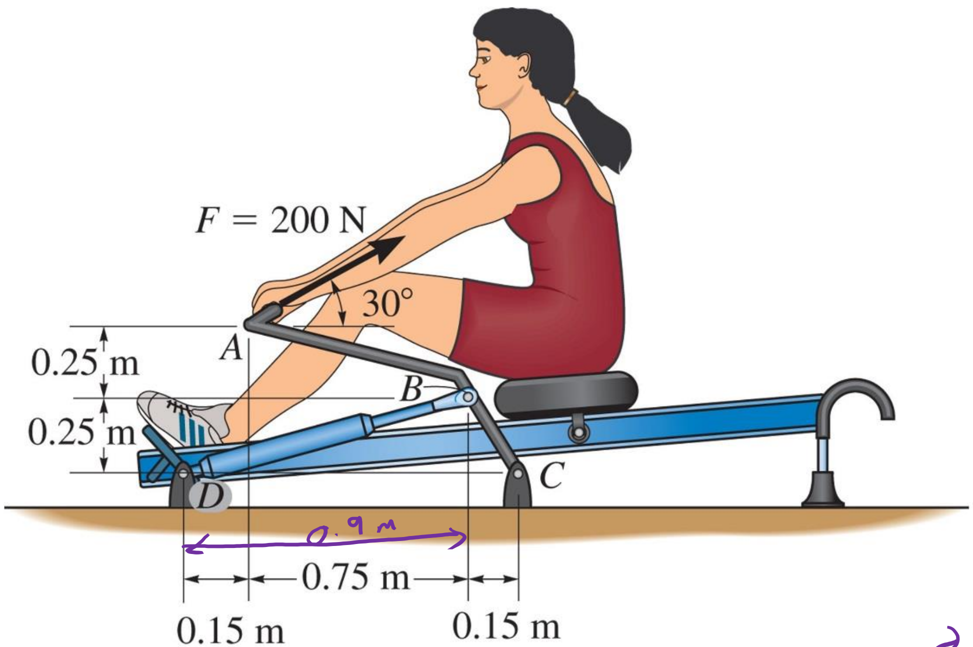
$\therefore C_x = C\cos\theta \rightarrow$ $C_x = 8kN$

$\Rightarrow$ $A_x = -8kN$

What is orientation of $\vec{A}$? use 3-force member principle to determine: "3 forces meet at same pt"

1) Extend lines of action of $\vec{C}$ & $\vec{F}$
2) Intersection is O
3) Draw line from OA
4) $\vec{A}$ is along this line



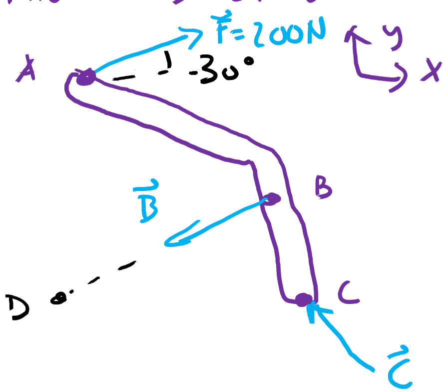


The woman exercises on the rowing machine. If she exerts a holding force of F = 200 N on the handle ABC, determine the reaction force at pin C and the force developed along the hydraulic cylinder BD on the handle.

Sample problem: Not covered in class

Find: $\vec{C}$, $\vec{B}$

① Draw FBD of 3 force members



② Determine x, y components of each force

a) $\vec{F} = 200N(\cos 30°)\hat{i} + 200N(\sin 30°)\hat{j}$

b) $\vec{B} = |B|\vec{u}_{BD}$, $\vec{u}_{BD} = \frac{\vec{r}_{BD}}{|\vec{r}_{BD}|}$

$\therefore \vec{B} = B\left[\frac{-0.9\hat{i} - 0.25\hat{j}}{\sqrt{0.9^2 + 0.25^2}}\right]$

Sum moments about C to solve for B

$\Sigma M_C: \vec{r}_{CB} \times \vec{B} + \vec{r}_{CA} \times \vec{F} = 0$

$\Rightarrow$ **B = 628 N**

Solve for reaction forces at C:

$\Sigma F_x: 200N(\cos 30°) + 628N\left(\frac{-0.9}{\sqrt{\ }}\right) = 0$

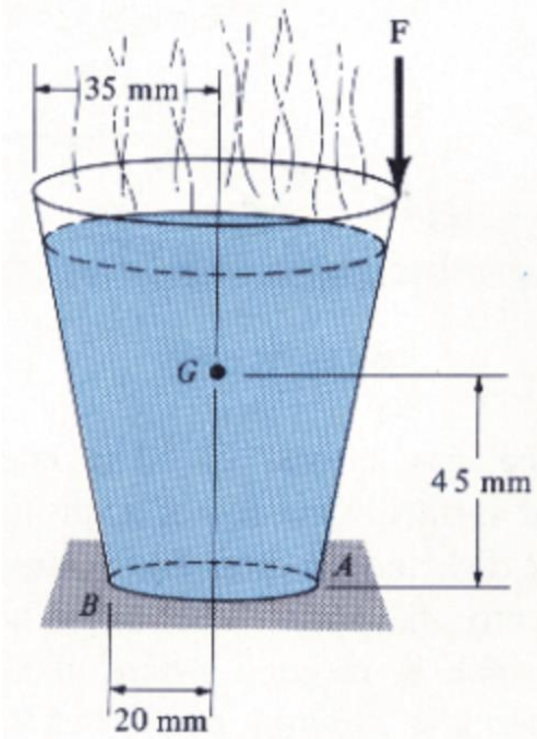
$\rightarrow$ **$C_x = -432N$**

$\Sigma F_y: 200N(\sin 30°) + 628N\left(\frac{-0.25}{\sqrt{\ }}\right) = 0$

$\rightarrow$ **$C_y = 68.1N$**

$\vec{B} = 628\left(\frac{-0.9}{\sqrt{\ }}\hat{i} - \frac{0.25}{\sqrt{\ }}\hat{j}\right)N$

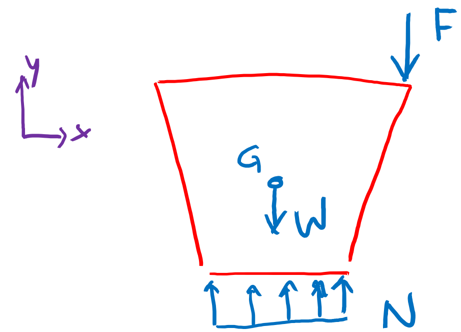
$\vec{C} = -432\hat{i} + 68.1\hat{j}\ N$



The cup is filled with 125 g of liquid. The mass center is located at G. If a vertical force F is applied to the rim of the cup, determine its magnitude so the cup is on the verge of tipping over.

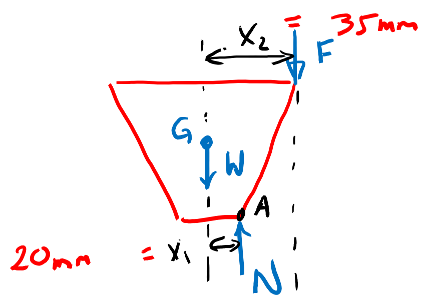
Sample problem: Not covered in class

Draw FBD



The system is not in equilibrium unless F = 0.

On verge of tipping means that N acts at a specific point.



$\Sigma F_x: \quad 0$

$\Sigma F_y: \quad N - W - F = 0$

$\Sigma M_A: \quad x_1 W - (x_2 - x_1) F = 0$

$$F = \left(\frac{x_1}{x_2 - x_1}\right) W$$

$$F = \frac{20 \text{ mm}}{(35 - 20) \text{ mm}} \left(0.125 g \cdot 9.8 \frac{m}{s^2}\right)$$

$$\boxed{F = 1.635 \text{ N}}$$

Chapter 6: Structural Analysis

Goals and Objectives

- Determine the forces in members of a truss using the method of joints
- Determine zero-force members
- Determine the forces in members of a truss using the method of sections

Simple trusses

Trusses are commonly used to support roofs.

A more challenging question is, that for a given load, how can we design the trusses' geometry to minimize cost?

Scaffolding

An understanding of statics is critical for predicting and analyzing possible modes of failure.

Buckling of slender members in compression is always a consideration in structural analysis.

Simple trusses

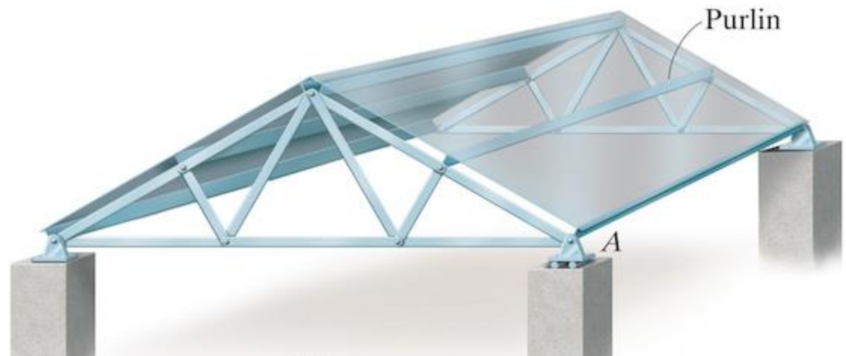
Truss:

- Structure composed of slender members joined together at end points
- Transmit loads to supports

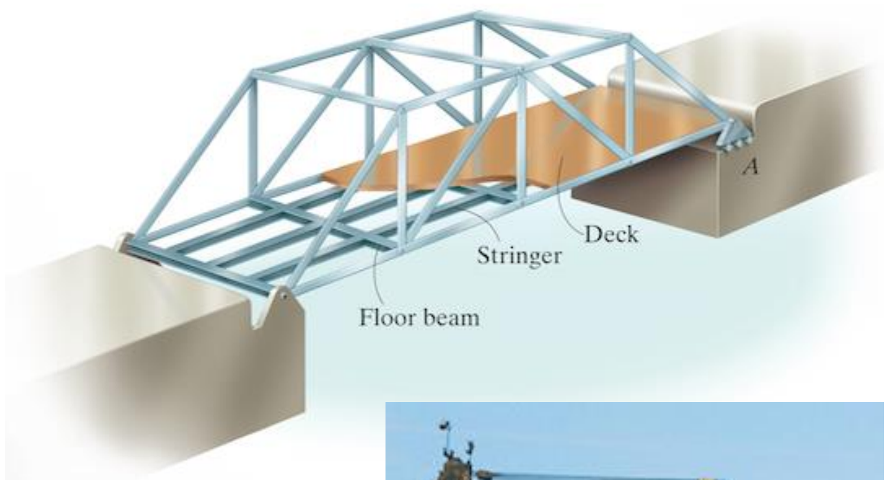
Assumption of trusses

- Loading applied at joints, with negligible weight (If weight included, vertical and split at joints)
- Members joined by smooth pins

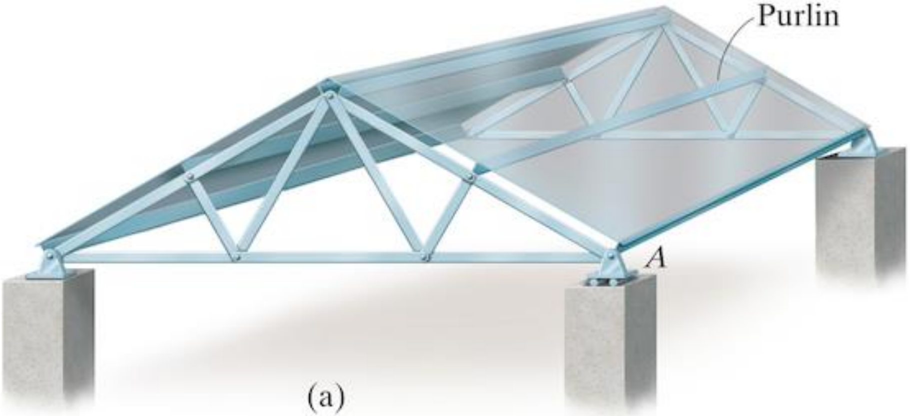
Result: **all truss members are two-force members,** and therefore the force acting at the end of each member will be directed along the axis of the member

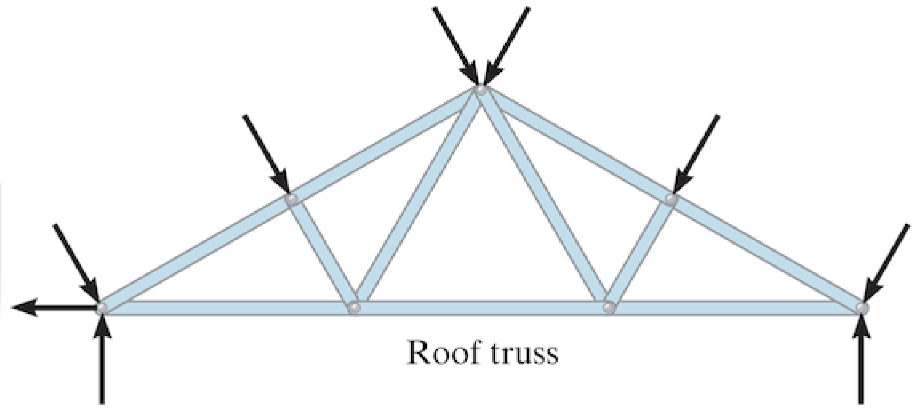


(a)



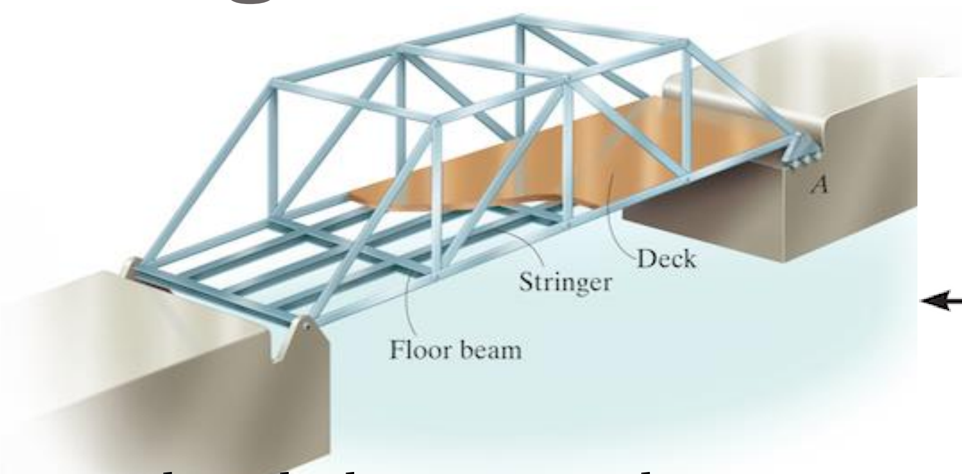
Roof trusses

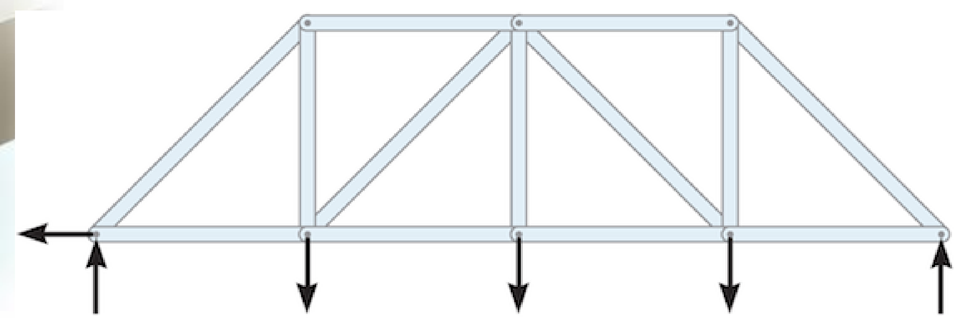




Load on roof transmitted to purlins, and from purlins to roof trusses at joints.

Bridge trusses

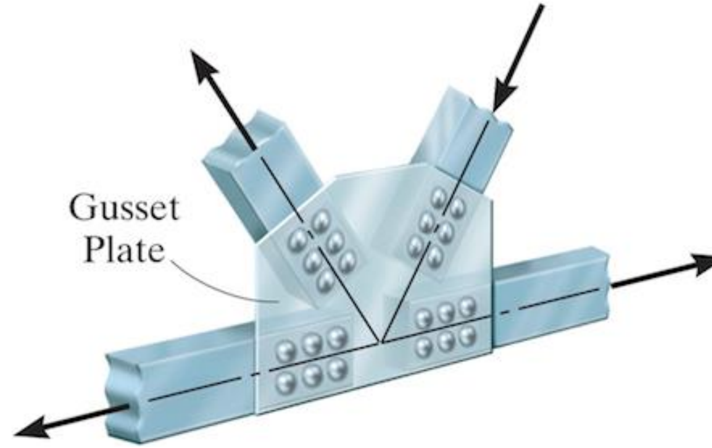


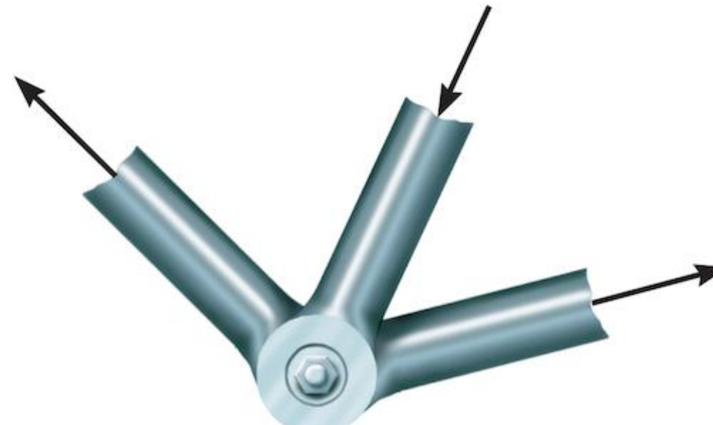


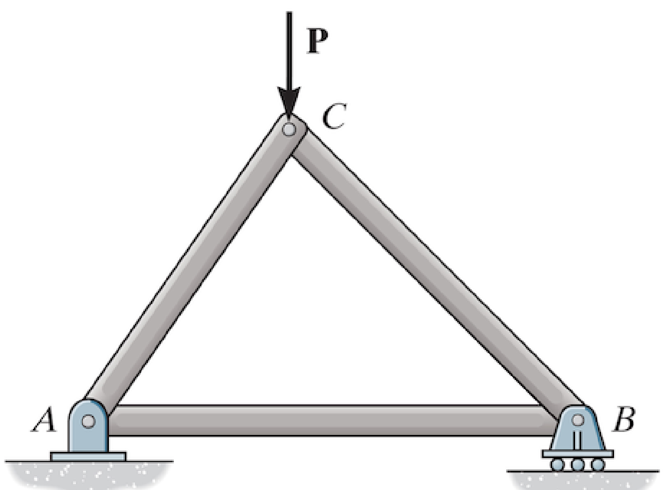
Load on deck transmitted to stringers, and from stringers to floor beams, and from floor beams to bridge trusses at joints.

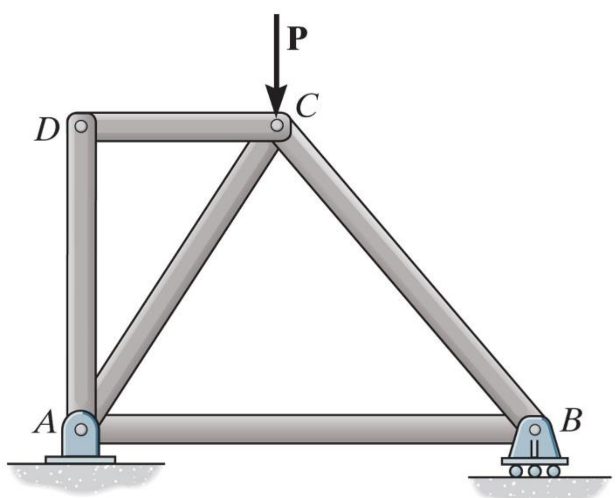
Truss joints

- Bolting or welding of the ends of the members to a gusset plates or passing a large bolt through each of the members
- Properly aligned gusset plates equivalent to pins (i.e., no moments) from coplanar, concurrent forces
- Simple trusses built from triangular members









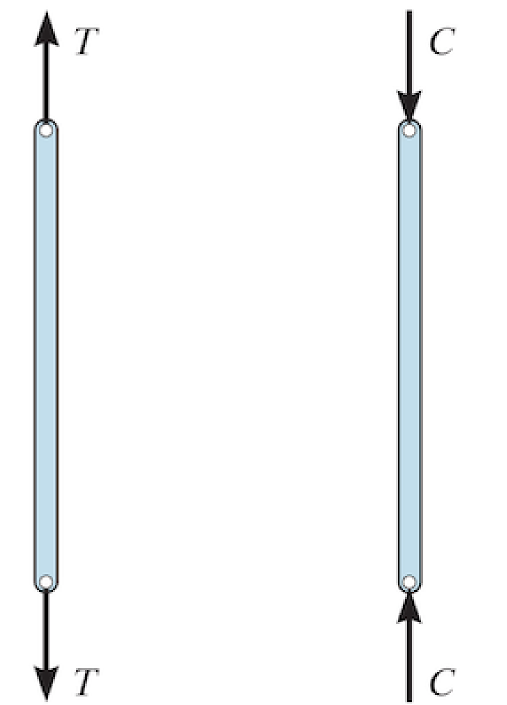
Method of joints

- Entire truss is in equilibrium if and only if all individual pieces (truss members and connecting pins) are in equilibrium.
- Truss members are two-force members: equilibrium satisfied by equal, opposite, collinear forces.
  - Tension: member has forces elongating.
  - Compression: member has forces shortening.
- Pins in equilibrium: $\sum F_x = 0$ and $\sum F_y = 0$

2 eqns!

Procedure for analysis:

- Free-body diagram for each joint
- Start with joints with at least 1 known force and 1-2 unknown forces.
- Generates two equations, 1-2 unknowns for each joint.
- Assume the unknown force members to be in *tension; i.e. the forces "pull" on the pin*. Numerical solutions will yield positive scalars for members in tension and negative scalar for members in compression.



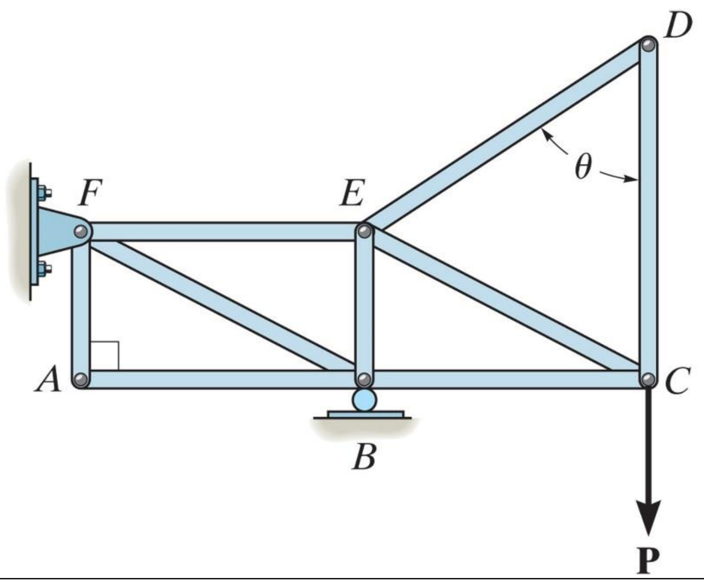
Tension Compression

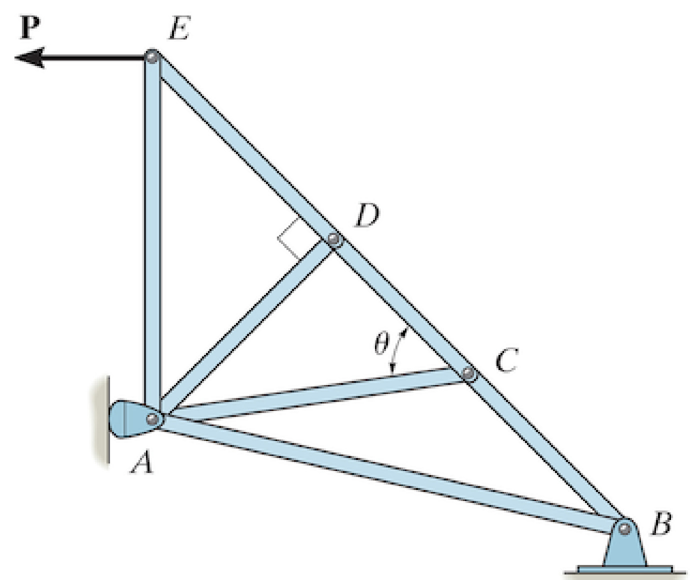
Zero-force members

- Particular members in a structure may experience no force for certain loads.
- Zero-force members are used to increase stability
- Identifying members with zero-force can expedite analysis. ⇒ can quickly say $F_{member} = 0$

Two situations:

- Joint with two non-collinear members, no external or support reaction applied to the joint → **Both members are zero-force members**.
- Joint with two collinear member, plus third non-collinear, no external or support reaction applied to non-collinear member → **Non-collinear member is a zero-force member**.

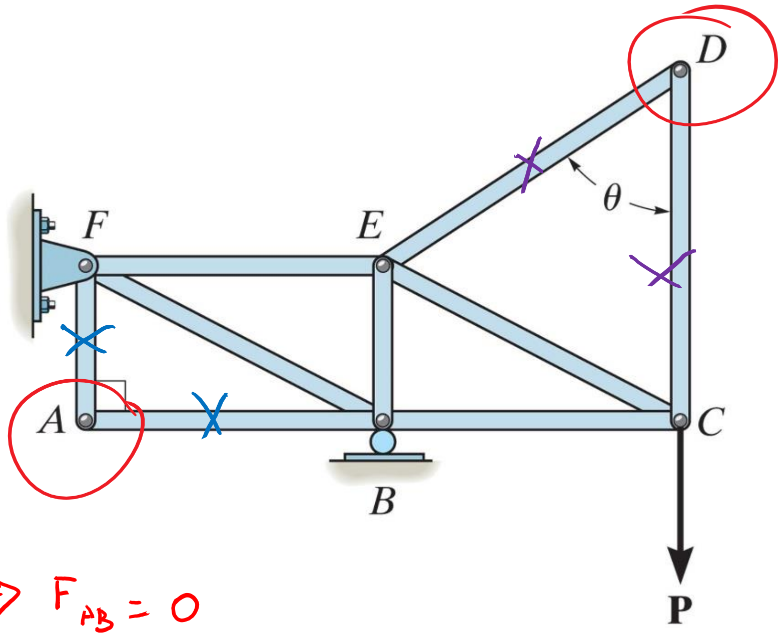


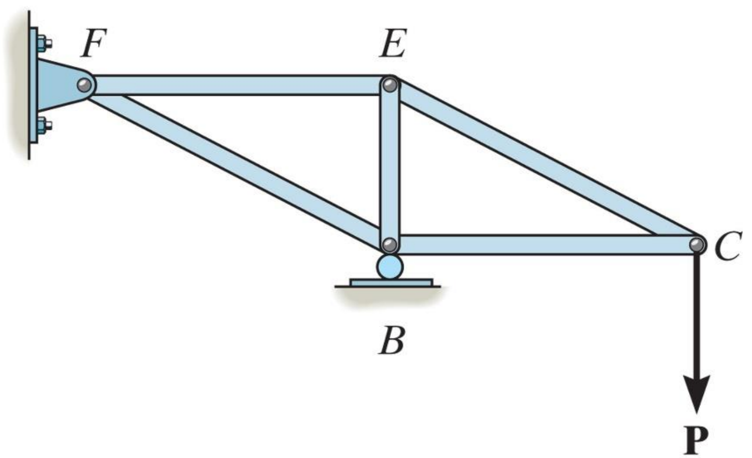


Zero-force members

Two situations:

- Joint with two non-collinear members, no external or support reaction applied to the joint → **Both members are zero-force members**.





⇒ $F_{AB} = 0$

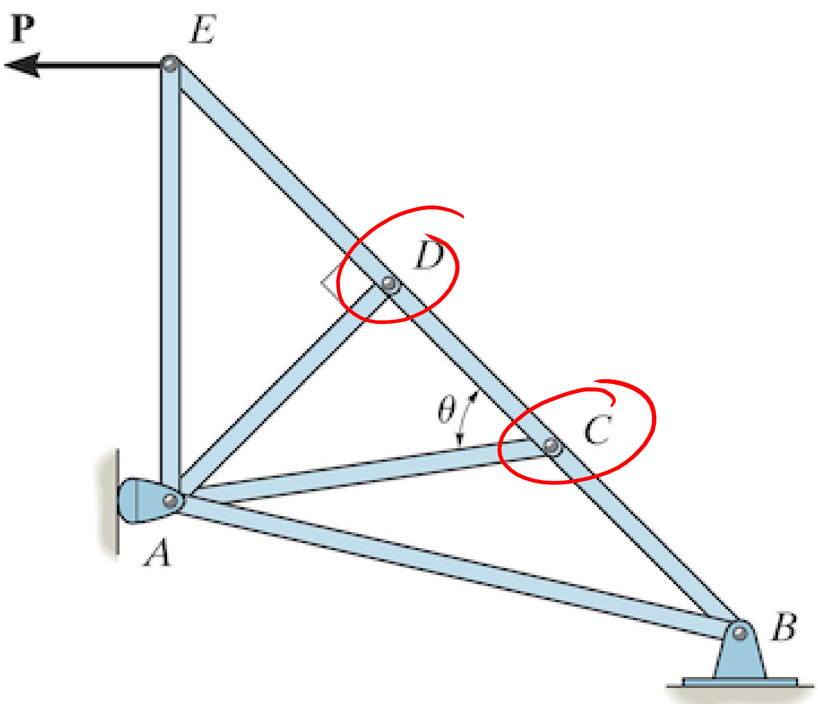
$F_{AF} = 0$

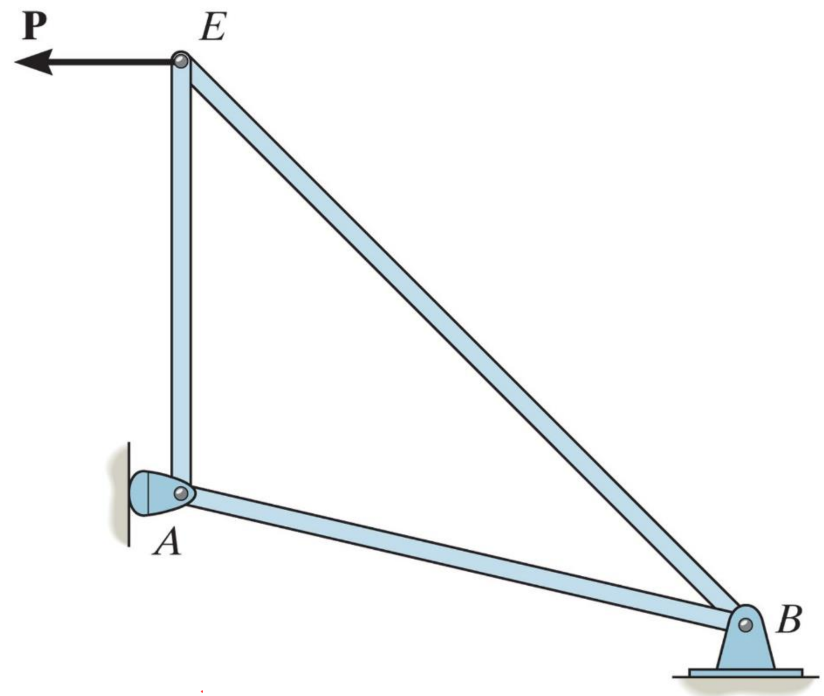
$F_{DC} = 0$

$F_{DE} = 0$

Zero-force members

- Joint with two collinear member, plus third non-collinear, no external or support reaction applied to non-collinear member → **Non-collinear member is a zero-force member**.





$\Rightarrow F_{DA} = 0$

$F_{DC} = F_{DE}$

$F_{CA} = 0$

$F_{CB} = F_{CD}$