Statics - TAM 210 & TAM 211

Lecture 17

February 23, 2018

Announcements

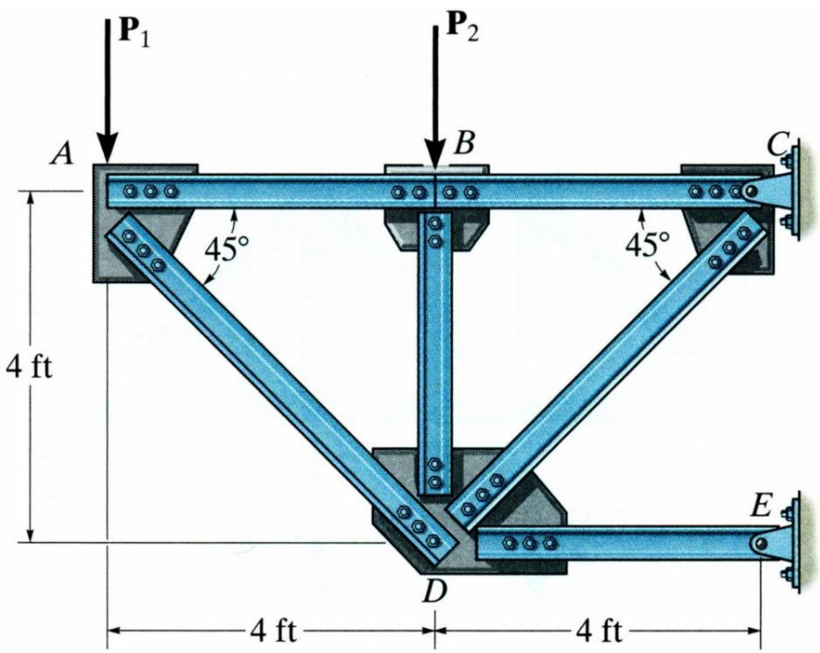
- Monday's lecture: watch for Piazza announcement over weekend for possible change
- Concept Inventory: Ungraded assessment of course knowledge
  - Extra credit: Complete #1 or #2 for 0.5 out of 100 pt of final grade each, or both for 1.5 out of 100 pt of final grade
  - #1: Sign up at CBTF (2/26-3/1 M-Th)
  - #2: (4/2-4 M-Th)
  - 50 min appointment, should take < 30 min

- Upcoming deadlines:
- Quiz 3 (2/21-23)
  - Sign up at CBTF
- Monday (2/26)
  - Mastering Engineering Tutorial 7
- Tuesday (2/27)
  - PL HW 6
- Thursday (3/1)
  - WA 3
  - See enhanced instructions

Chapter 6: Structural Analysis

Goals and Objectives

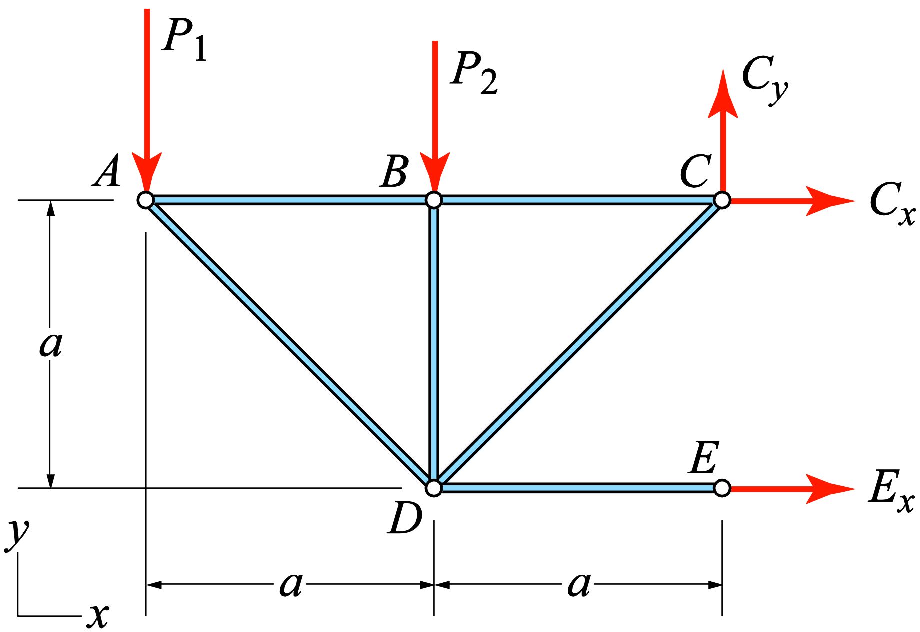
- Determine the forces in members of a truss using the method of joints
- Determine zero-force members
- Determine the forces in members of a truss using the method of sections



The truss, used to support a balcony, is subjected to the loading shown. Approximate each joint as a pin and determine the force in each member. State whether the members are in tension or compression.

Solution:

Start by setting the entire structure into **external** equilibrium. Draw the FBD.



Equilibrium requires $\sum \boldsymbol{F} = \boldsymbol{0}$ and $(\sum \boldsymbol{M})_C = \boldsymbol{0}$

$$\Sigma F_x = 0: \quad C_x + E_x = 0\,,$$

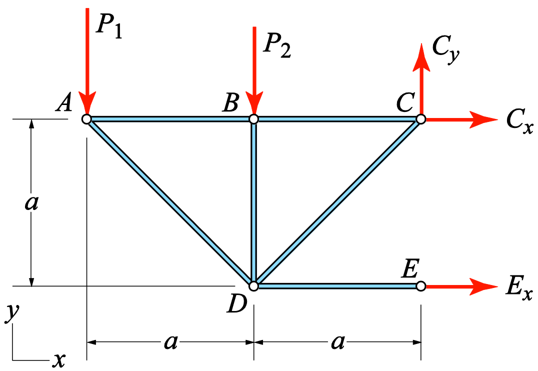
$$\Sigma F_y = 0: \quad C_y - P_1 - P_2 = 0\,,$$

$$\Sigma M_C = 0: \quad 2aP_1 + aP_2 + aE_x = 0\,.$$

Solving these equations gives the *external* reactions

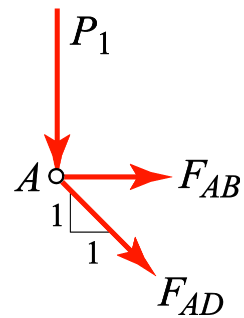
$$\boxed{C_x = 2P_1 + P_2, \quad C_y = P_1 + P_2, \quad E_x = -(2P_1 + P_2)\,.}$$

Next, start with a joint, draw the FBD, set it into *force* equilibrium *only*, and move to the next joint. Start with joints with at least 1 known force and 1-2 unknown forces.



$$C_x = 2P_1 + P_2, \quad C_y = P_1 + P_2, \quad E_x = -(2P_1 + P_2).$$

For example, start with **joint *A***:

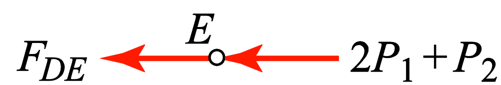


$$\Sigma F_x = 0: \quad F_{AB} + \tfrac{1}{\sqrt{2}} F_{AD} = 0,$$

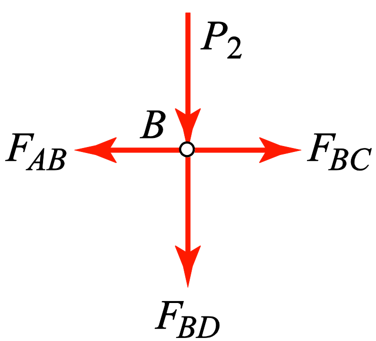
$$\Sigma F_y = 0: \quad -P_1 - \tfrac{1}{\sqrt{2}} F_{AD} = 0.$$

$$F_{AD} = -\sqrt{2}P_1, \quad F_{AB} = -\tfrac{1}{\sqrt{2}}\left(-\sqrt{2}P_1\right) = +P_1.$$

Joint E:



Joint *B*:

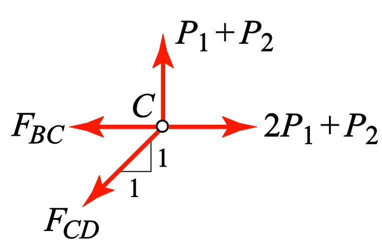


$$\Sigma F_x = 0: \quad -F_{AB} + F_{BC} = 0,$$

$$\Sigma F_y = 0: \quad -P_2 - F_{BD} = 0.$$

$$F_{BC} = F_{AB} = +P_1, \quad F_{BD} = -P_2.$$

Joint C:



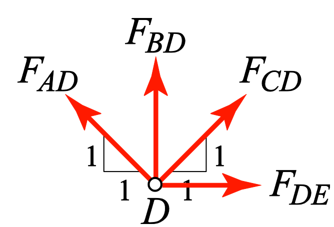
$$\Sigma F_x = 0: \quad -F_{BC} - \tfrac{1}{\sqrt{2}} F_{CD} + 2P_1 + P_2 = 0,$$

$$\Sigma F_y = 0: \quad -\tfrac{1}{\sqrt{2}} F_{CD} + P_1 + P_2 = 0.$$

$$F_{CD} = \sqrt{2}(2P_1 + P_2 - P_1) = \sqrt{2}(P_1 + P_2),$$

$$F_{CD} = \sqrt{2}(P_1 + P_2) \quad \text{(check)}.$$

Joint D: ***only needed for check***



$$\Sigma F_x = 0: \quad -\tfrac{1}{\sqrt{2}} F_{AD} + \tfrac{1}{\sqrt{2}} F_{CD} + F_{DE} = 0,$$

$$\Sigma F_y = 0: \quad \tfrac{1}{\sqrt{2}} F_{AD} + F_{BD} + \tfrac{1}{\sqrt{2}} F_{CD} = 0.$$

$$F_{DE} = \tfrac{1}{\sqrt{2}}\left(-\sqrt{2}P_1\right) - \tfrac{1}{\sqrt{2}}\sqrt{2}(P_1 + P_2) = -(2P_1 + P_2),$$

$$\tfrac{1}{\sqrt{2}}\left(-\sqrt{2}P_1\right) - P_2 + \tfrac{1}{\sqrt{2}}\sqrt{2}(P_1 + P_2) = 0 \quad \text{(check)}.$$

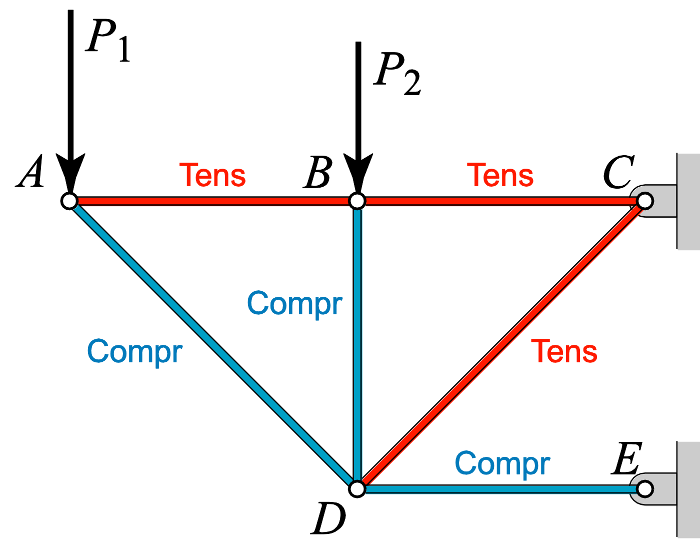
Note: The checks would not have been satisfied if the external reactions had been calculated incorrectly.

Note: The order in which the joints are set in equilibrium is usually arbitrary. Sometimes not all member loads are requested.

If provided numerical values:

$P_1 = 800$ lb

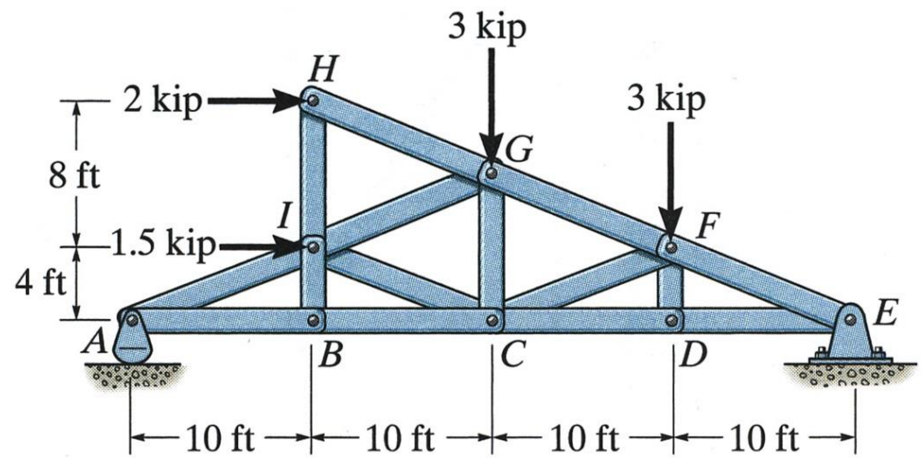
$P_2 = 0$



$$F_{AB} = P_1 = 800 \text{ lb (T)}$$
$$F_{BC} = P_1 = 800 \text{ lb (T)}$$
$$F_{AD} = -\sqrt{2}P_1 = -1130 \text{ lb (C)}$$
$$F_{BD} = -P_2 = 0$$
$$F_{CD} = \sqrt{2}(P_1 + P_2) = 1130 \text{ lb (T)}$$
$$F_{DE} = -(2P_1 + P_2) = -1600 \text{ lb (C)}$$

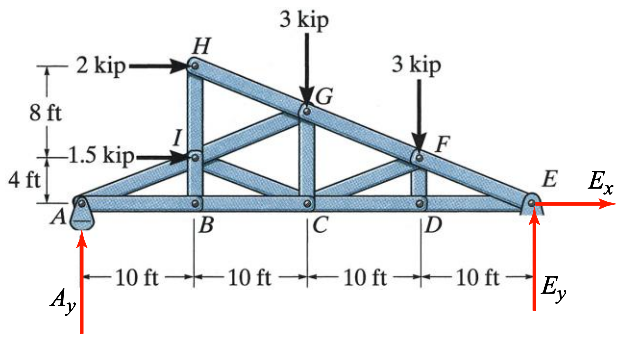
Note that, in the absence of P_2, member BD is a zero-force member

Determine the force in member *FG* of the truss and state if the member is in tension or compression.



Solution:

Draw a free-body diagram of the entire truss



Equilibrium of the entire structure requires:

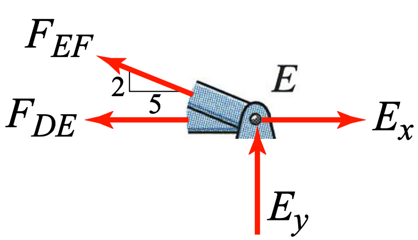
$$2+1.5+E_x=0\,,$$

$$-3-3+A_y+E_y=0\,,$$

$$-40A_y-4(1.5)-12(2)+20(3)+10(3)=0\,.$$

$A_y\ =\ 1.5$ kips,	$E_x\ =\ -3.5$ kips,	$E_y\ =\ 4.5$ kips.

Joint E:



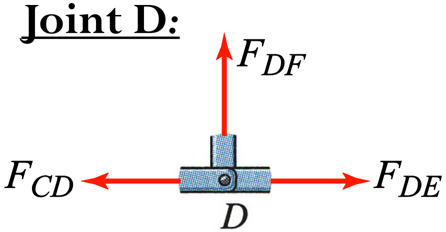
$$\Sigma F_x=0:\quad -\tfrac{5}{\sqrt{29}}F_{EF}-F_{DE}+E_x=0\,,$$

$$\Sigma F_y=0:\quad \tfrac{2}{\sqrt{29}}F_{EF}+E_y=0\,.$$

$$F_{EF}\ =\ -\tfrac{\sqrt{29}}{2}E_y\ =\ -\tfrac{\sqrt{29}}{2}(4.5)\ =\ -12.12\text{ kips (C)},$$

$$F_{DE}\ =\ -\tfrac{5}{\sqrt{29}}F_{EF}+E_x\ =\ -\tfrac{5}{\sqrt{29}}(-12.12)+(-3.5)\ =\ 7.75\text{ kips (T)}.$$

Joint D:

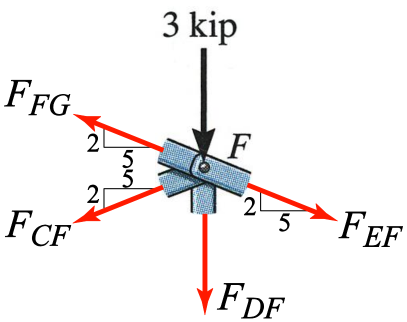


$$\Sigma F_x=0:\quad -F_{CD}+F_{DE}=0\,,\qquad F_{CD}\ =\ F_{DE}\ =\ 7.75\text{ kips (T)},$$

$$\Sigma F_y=0:\quad F_{DF}=0\,.\qquad F_{DF}\ =\ 0.$$

Note: Member *DF* turned out to be a zero-force member. Under what conditions would this result not be true?

Joint F:



$$\Sigma F_x = 0: \quad -\tfrac{5}{\sqrt{29}} F_{FG} - \tfrac{5}{\sqrt{29}} F_{CF} + \tfrac{5}{\sqrt{29}} F_{EF} = 0\,,$$

$$\Sigma F_y = 0: \quad \tfrac{2}{\sqrt{29}} F_{FG} - \tfrac{2}{\sqrt{29}} F_{CF} - F_{DF} - \tfrac{2}{\sqrt{29}} F_{EF} - 3 = 0\,.$$

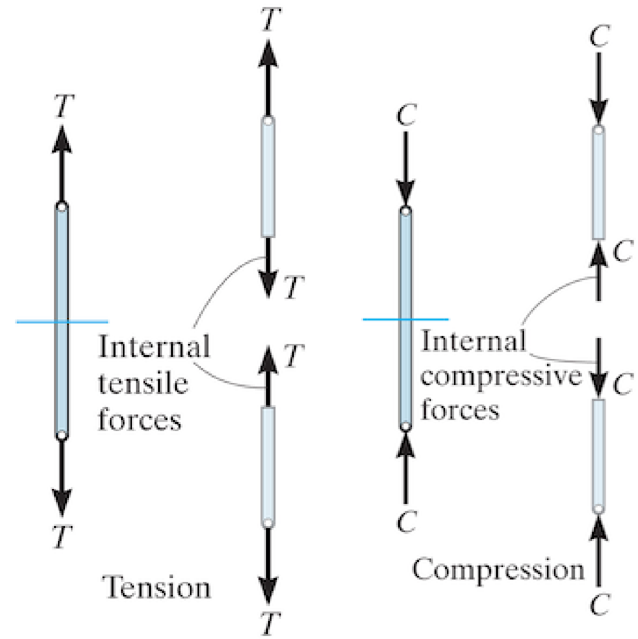
$$F_{FG} + F_{CF} = F_{EF}\,,$$

$$F_{FG} - F_{CF} = F_{EF} + \tfrac{\sqrt{29}}{2}(3 + F_{DF})\,.$$

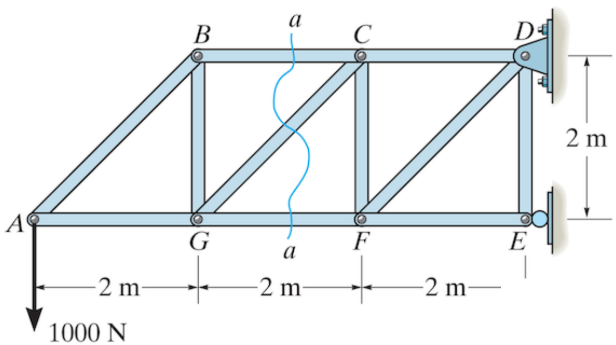
$$\begin{aligned} F_{FG} &= F_{EF} + \tfrac{1}{2} \cdot \tfrac{\sqrt{29}}{2}(3 + F_{DF}) \\ &= -12.12 + \tfrac{\sqrt{29}}{4}(3 + 0) \\ &= -8.08 \text{ kips (C)}. \end{aligned}$$

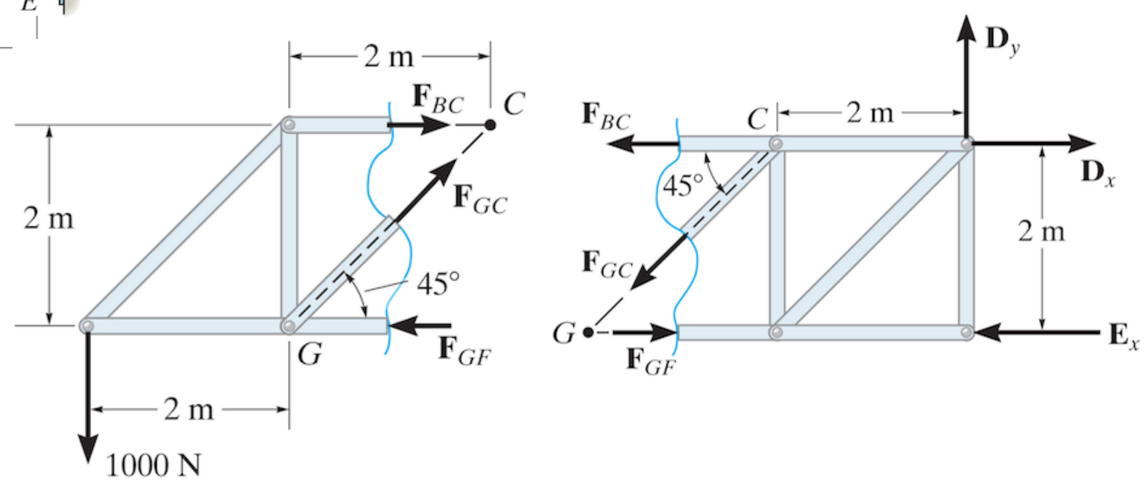
Note: Nine scalar equations of equilibrium were needed to obtain this answer. Might there be a shorter way?

Method of sections



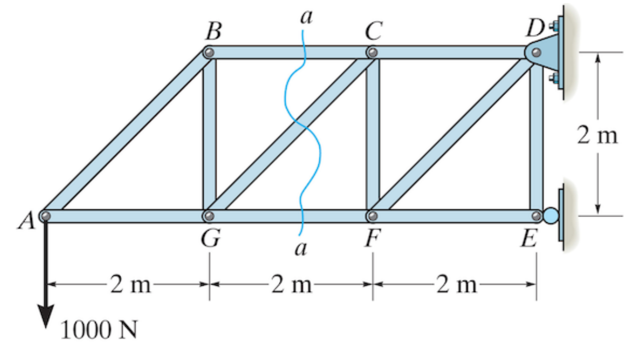
- Determine external support reactions
- "Cut" the structure at a section of interest into two separate pieces and set either part into force and moment equilibrium (your cut should be such that you have **<u>no more than</u>** three unknowns)





- Determine equilibrium equations (e.g., moment around point of intersection of two lines)
- Assume all internal loads are tensile.

Method of sections



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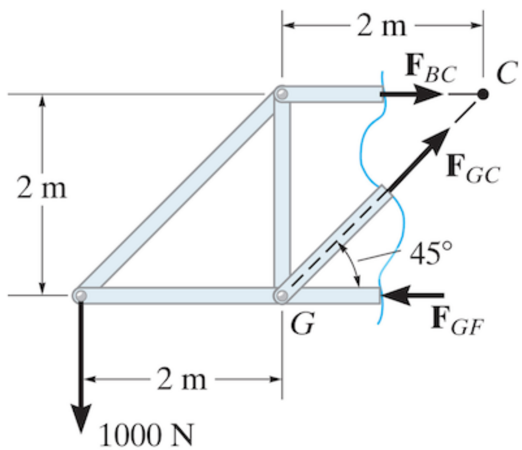
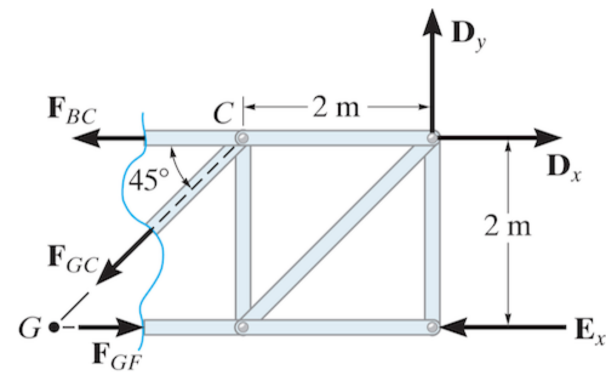
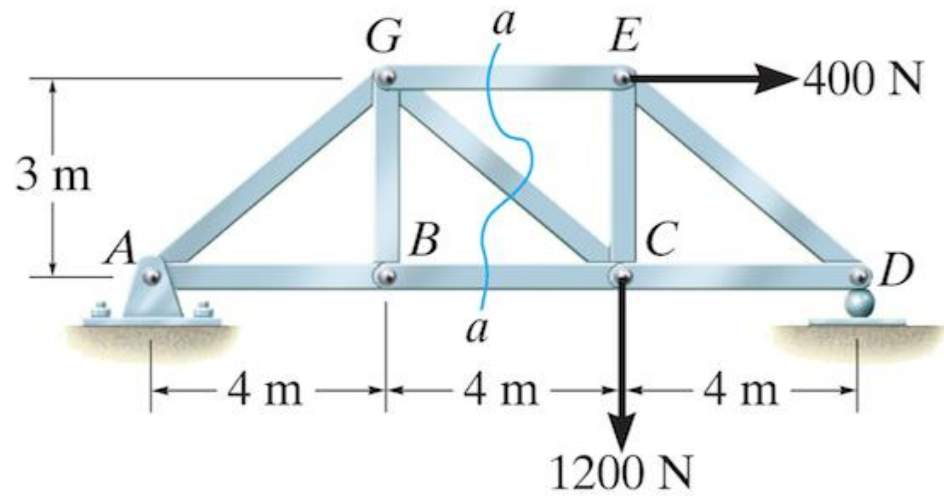
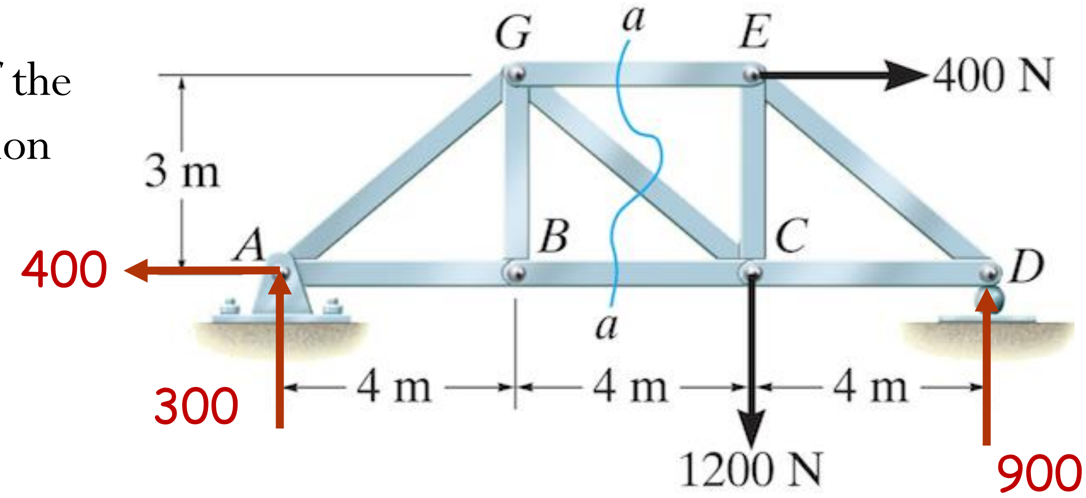


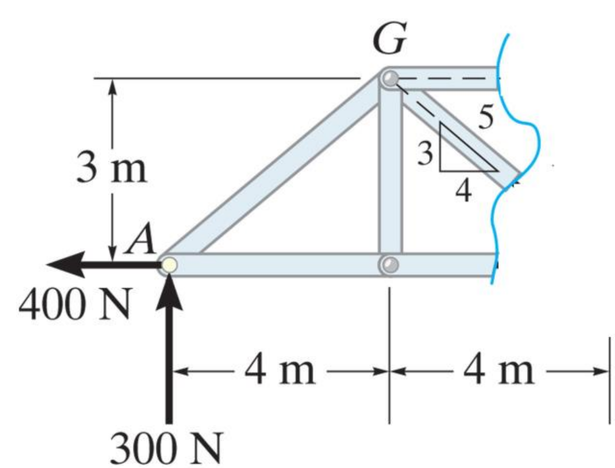


Determine the force in member GC of the truss and state if the member is in tension or compression.

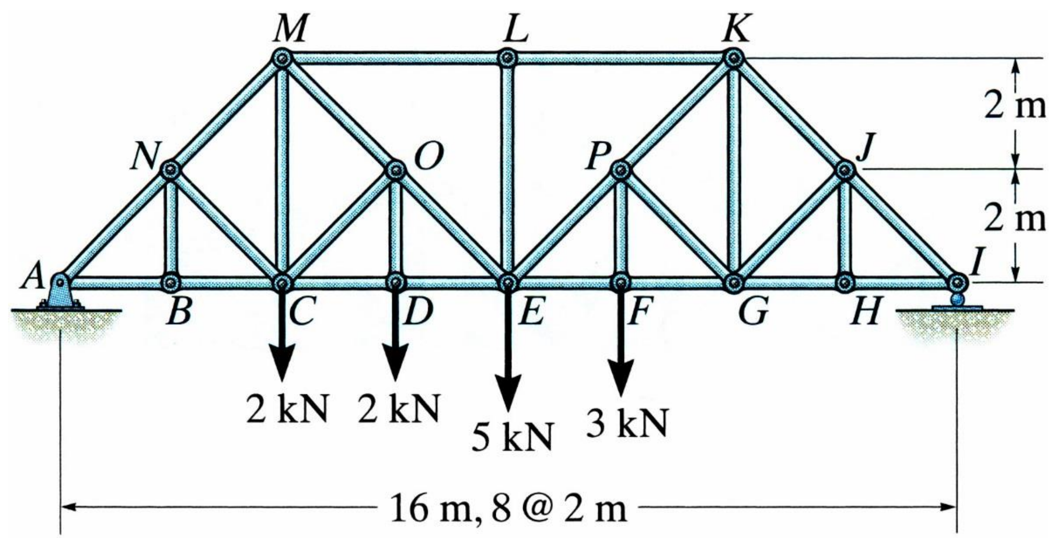


Determine the force in member GC of the truss and state if the member is in tension or compression.



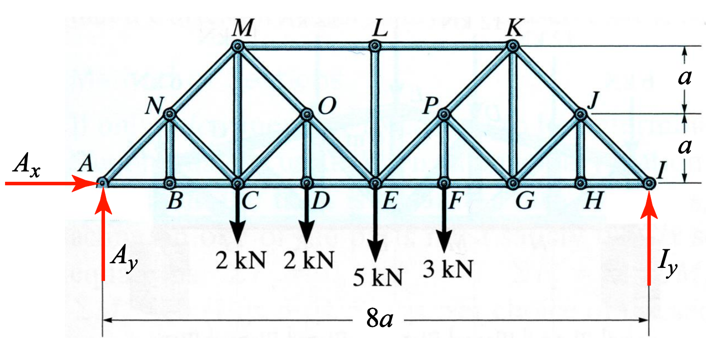


Determine the force in members OE, LE, LK of the Baltimore truss and state if the member is in tension or compression.



Solution:

Use method of sections, since cutting LK, LE, OE, and DE will separate the truss into two pieces. Note that LE is a zero-force member. Draw free-body diagram of entire structure, and set into external equilibrium:

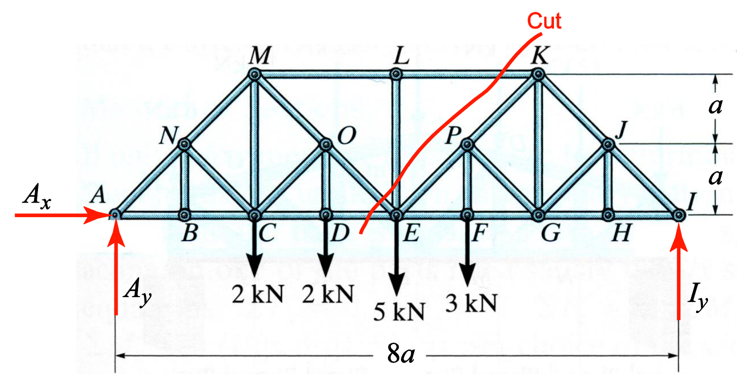


$$\Sigma F_x = 0: \qquad A_x = 0,$$

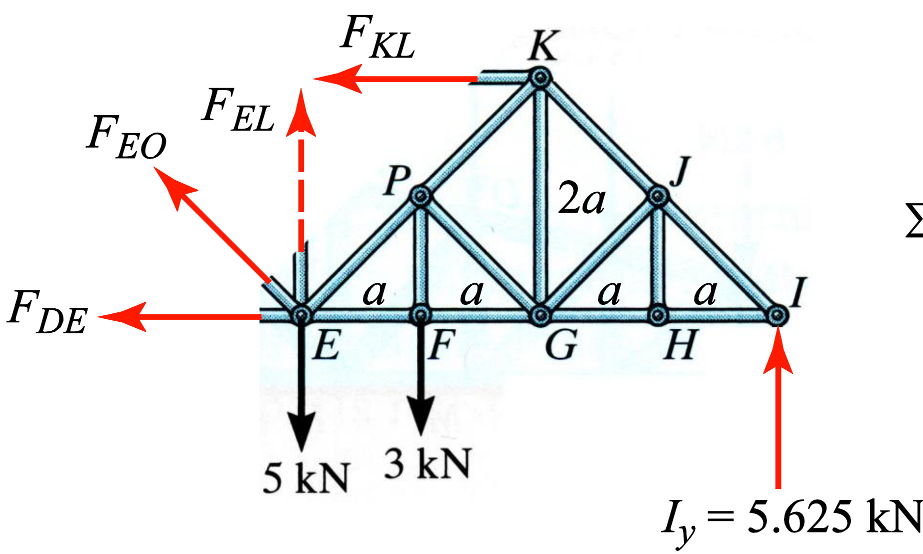
$$\Sigma F_y = 0: \qquad A_y + I_y - 2 - 2 - 5 - 3 = 0,$$

$$\Sigma M_A = 0: \quad -2a(2) - 3a(2) - 4a(5) - 5a(3) + 8aI_y = 0.$$

$$\boxed{A_x = 0, \quad A_y = 6.375\text{ kN}, \quad I_y = 5.625\text{ kN.}}$$



Normally, introducing four unknowns would make the problem intractable. However, *LE* is a *zero-force* member. Set *either* remaining section into equilibrium. Here, there is no real preference, but the right half will be fine



$$\Sigma F_x = 0: \quad -F_{DE} - \tfrac{1}{\sqrt{2}} F_{EO} - F_{KL} = 0\,,$$

$$\Sigma F_y = 0: \quad \tfrac{1}{\sqrt{2}} F_{EO} + 0 - 5 - 3 + I_y = 0\,,$$

$$\Sigma M_E = 0: \quad 0(5) + a(-3) + 4aI_y + 2aF_{KL} = 0\,.$$

$F_{DE} = +7.38$ kN (T),	$F_{EL} = 0$ (zero-force),
$F_{EO} = +3.36$ kN (T),	$F_{KL} = -9.75$ kN (C).