Statics - TAM 210 & TAM 211

Lecture 17

February 23, 2018

Announcements

- Monday's lecture: watch for Piazza announcement over weekend for possible change
- Concept Inventory: Ungraded assessment of course knowledge
  - Extra credit: Complete #1 or #2 for 0.5 out of 100 pt of final grade each, or both for 1.5 out of 100 pt of final grade
  - #1: Sign up at CBTF (2/26-3/1 M-Th)
  - #2: (4/2-4 M-Th)
  - 50 min appointment, should take < 30 min

- Upcoming deadlines:
- Quiz 3 (2/21-23)
  - Sign up at CBTF
- Monday (2/26)
  - Mastering Engineering Tutorial 7
- Tuesday (2/27)
  - PL HW 6
- Thursday (3/1)
  - WA 3
  - See enhanced instructions

Chapter 6: Structural Analysis

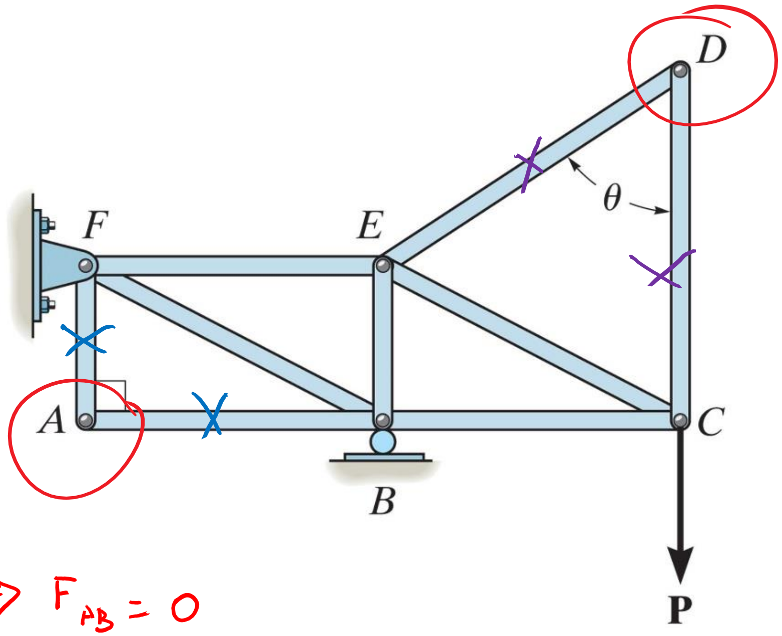
Goals and Objectives

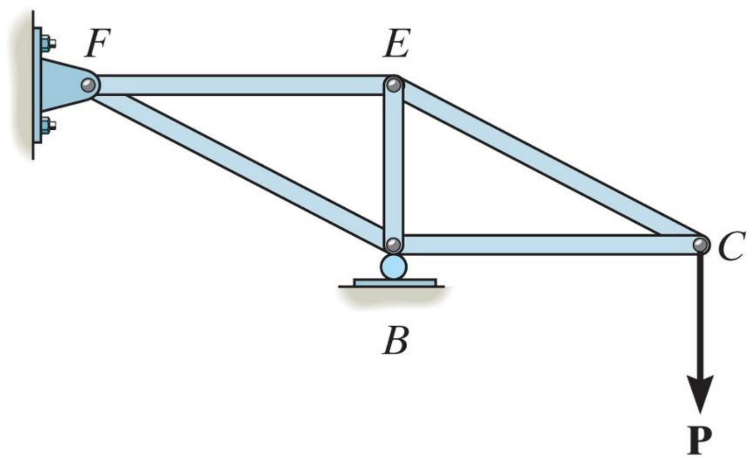
- Determine the forces in members of a truss using the method of joints
- Determine zero-force members
- Determine the forces in members of a truss using the method of sections

Recap: Zero-force members

Two situations:

- Joint with **ONLY** two non-collinear members, no external or support reaction applied to the joint → **Both members are zero-force members**.





⇒ $F_{AB} = 0$

$F_{AF} = 0$

$F_{DC} = 0$

$F_{DE} = 0$

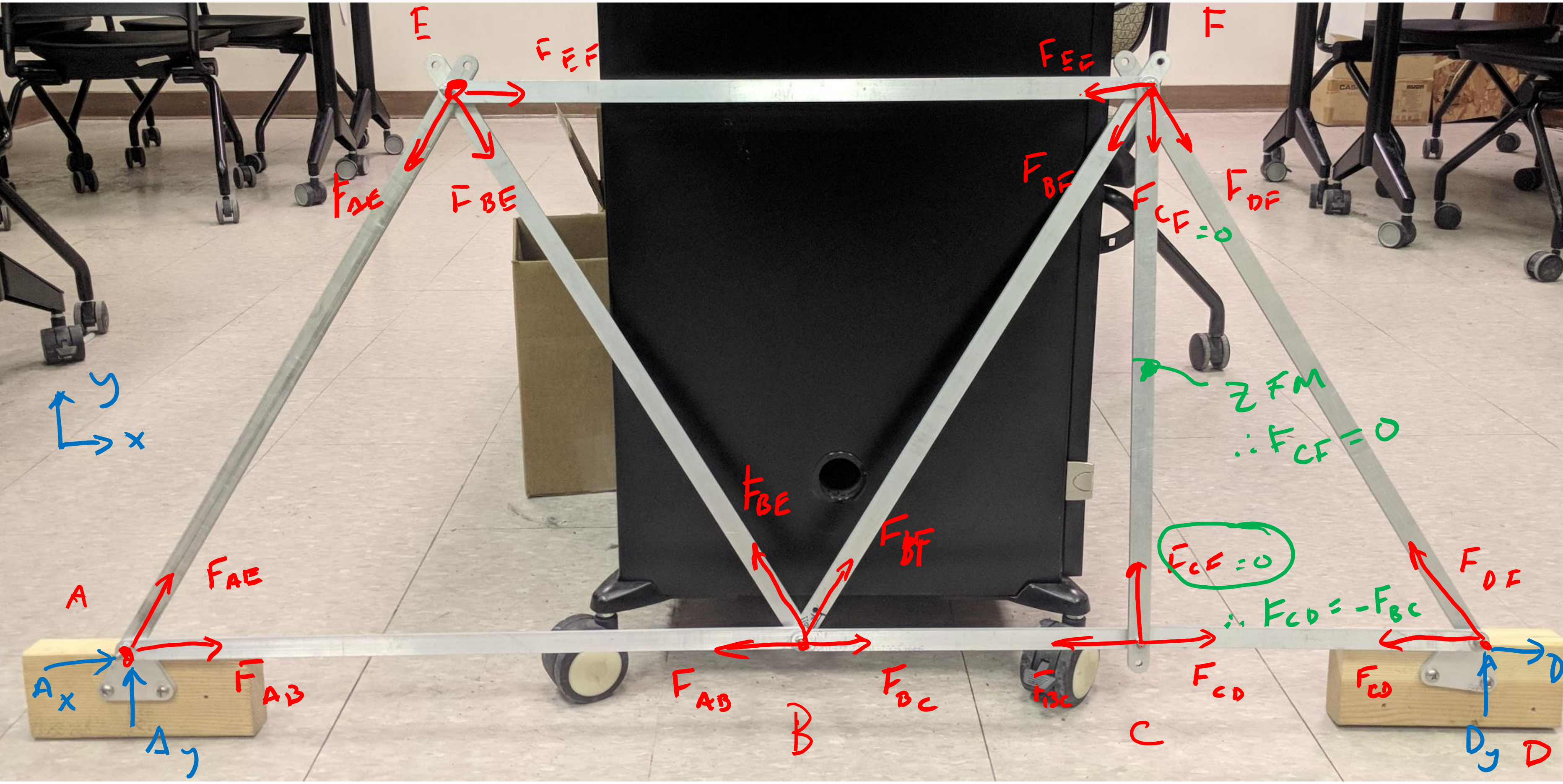
Why not also point E?

Recap:

Picture of truss structure from this week's Discussion Section. Recall: a truss is composed of only slender, straight 2-force members and these members are joined by pins (pin joints – only forces & no moment); so all resultant forces in the member are directed along the axis of the member and concentrated only at the end joints (Lecture 15). Neglect the weight of the truss members (or split weight and include as part of the resultant force). Note joints A and D are connected to ground by hinge or pin supports (Lecture 12).

Method of Joints (Lecture 15): Use to solve for the resultant forces on one or more pin joints in the truss. Procedure: (i) Created FBD of each joint – assume unknown forces point in the direction of tension (or away from the pin). {When drawing forces: draw only one single resultant force per truss member, but both orthogonal components of support reaction force.} (ii) Start with joint with at least 1 known force and 1-2 unknown forces (since only have 2 eqns of equilibrium for a pin joint $\sum F_x = 0$ and $\sum F_y = 0$).

Assume no added loads on truss in this current embodiment.



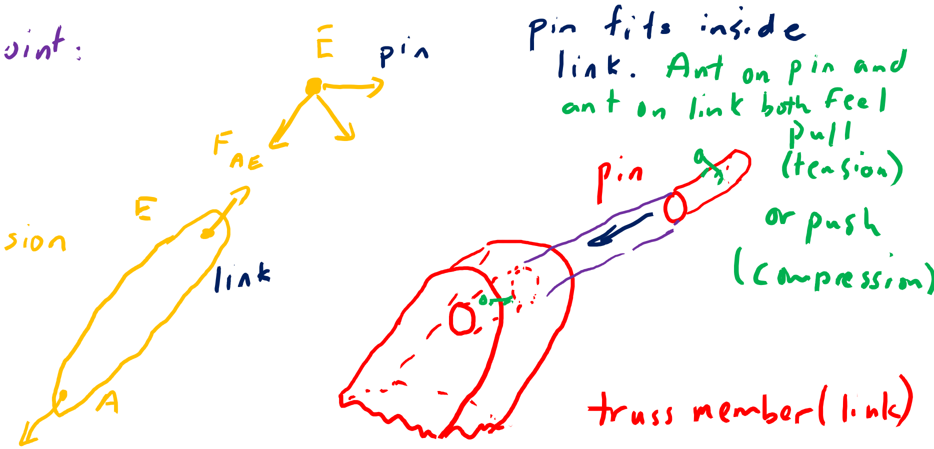
Draw FBD of each joint:

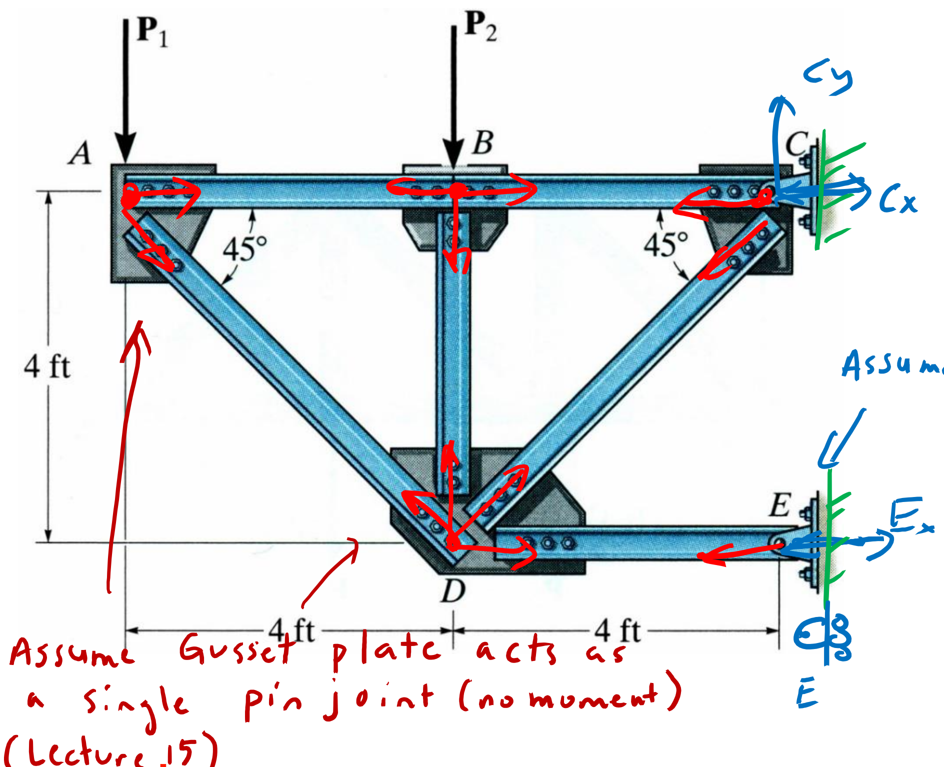
Truss Structure:
- 2-force members
- A & D are pinned supports – draw in both orthogonal reaction forces

No added external loads

Assuming tension

Link CF is a zero-force member. $\therefore$ $F_{CF} = 0$, $F_{CD} = -F_{BC}$



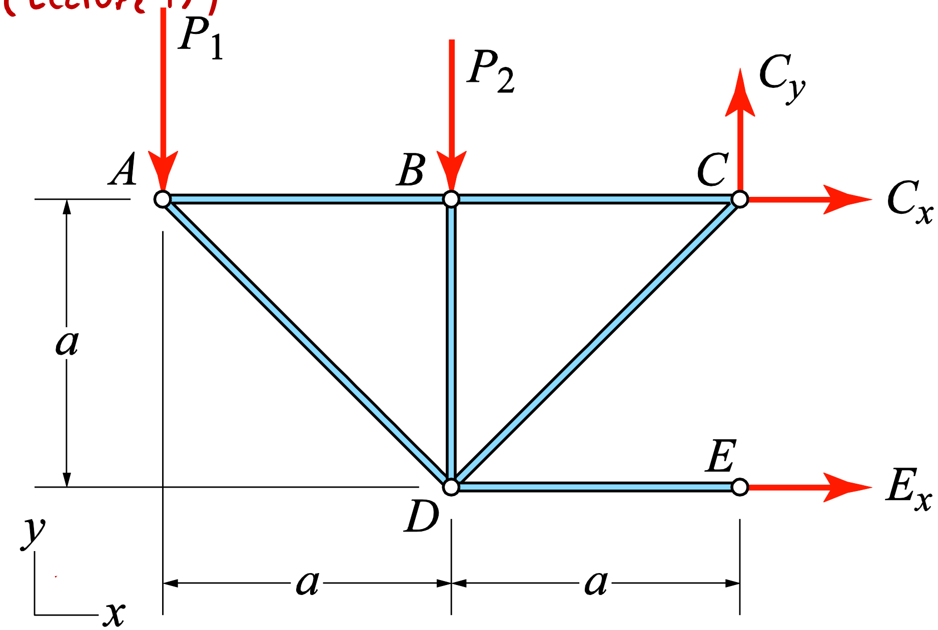


The truss, used to support a balcony, is subjected to the loading shown. Approximate each joint as a pin and determine the force in each member. State whether the members are in tension or compression.

Assume E is supported by Roller

Solution:

Start by setting the entire structure into **external** equilibrium. Draw the FBD.



Equilibrium requires $\sum \boldsymbol{F} = \boldsymbol{0}$ and $(\sum \boldsymbol{M})_C = \boldsymbol{0}$

$$\Sigma F_x = 0: \quad C_x + E_x = 0\,,$$

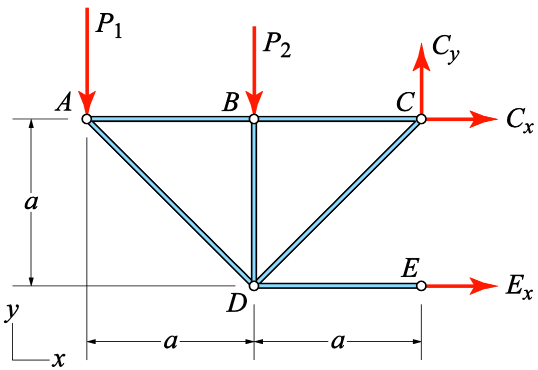
$$\Sigma F_y = 0: \quad C_y - P_1 - P_2 = 0\,,$$

$$\Sigma M_C = 0: \quad 2aP_1 + aP_2 + aE_x = 0\,.$$

Solving these equations gives the *external* reactions

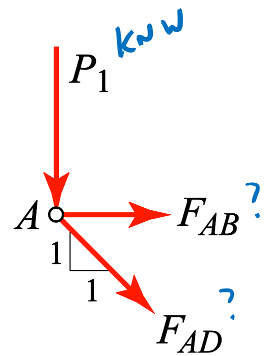
$$\boxed{C_x = 2P_1 + P_2, \quad C_y = P_1 + P_2, \quad E_x = -(2P_1 + P_2)\,.}$$

Next, start with a joint, draw the FBD, set it into *force* equilibrium *only*, and move to the next joint. Start with joints with at least 1 known force and 1-2 unknown forces.



$$C_x = 2P_1 + P_2,\quad C_y = P_1 + P_2,\quad E_x = -(2P_1 + P_2).$$

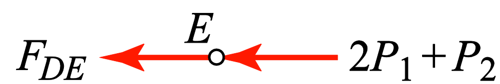
For example, start with **joint *A***:



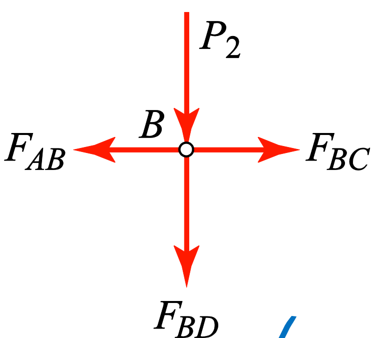
$$\Sigma F_x = 0:\quad F_{AB} + \tfrac{1}{\sqrt{2}} F_{AD} = 0,$$
$$\Sigma F_y = 0:\quad -P_1 - \tfrac{1}{\sqrt{2}} F_{AD} = 0.$$

$$F_{AD} = -\sqrt{2}P_1,\quad F_{AB} = -\tfrac{1}{\sqrt{2}}\left(-\sqrt{2}P_1\right) = +P_1.$$

Joint E:



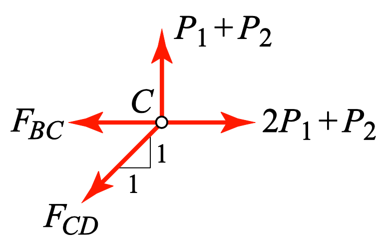
Joint *B*:



$$\Sigma F_x = 0:\quad -F_{AB} + F_{BC} = 0,$$
$$\Sigma F_y = 0:\quad -P_2 - F_{BD} = 0.$$

$$F_{BC} = F_{AB} = +P_1,\quad F_{BD} = -P_2.$$

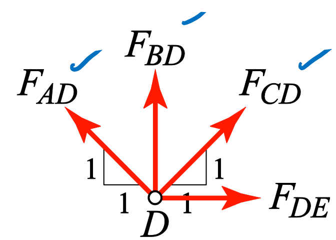
Joint C:



$$\Sigma F_x = 0:\quad -F_{BC} - \tfrac{1}{\sqrt{2}} F_{CD} + 2P_1 + P_2 = 0,$$
$$\Sigma F_y = 0:\quad -\tfrac{1}{\sqrt{2}} F_{CD} + P_1 + P_2 = 0.$$

$$F_{CD} = \sqrt{2}(2P_1 + P_2 - P_1) = \sqrt{2}(P_1 + P_2),$$
$$F_{CD} = \sqrt{2}(P_1 + P_2)\quad \text{(check)}.$$

Joint D: *only needed for check*



$$\Sigma F_x = 0:\quad -\tfrac{1}{\sqrt{2}} F_{AD} + \tfrac{1}{\sqrt{2}} F_{CD} + F_{DE} = 0,$$
$$\Sigma F_y = 0:\quad \tfrac{1}{\sqrt{2}} F_{AD} + F_{BD} + \tfrac{1}{\sqrt{2}} F_{CD} = 0.$$

$$F_{DE} = \tfrac{1}{\sqrt{2}}\left(-\sqrt{2}P_1\right) - \tfrac{1}{\sqrt{2}}\sqrt{2}(P_1 + P_2) = -(2P_1 + P_2),$$
$$\tfrac{1}{\sqrt{2}}\left(-\sqrt{2}P_1\right) - P_2 + \tfrac{1}{\sqrt{2}}\sqrt{2}(P_1 + P_2) = 0\quad \text{(check)}.$$

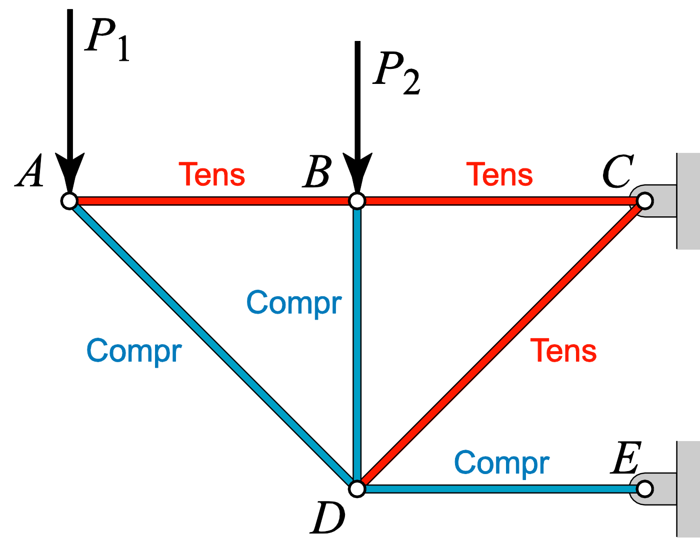
Note: The checks would not have been satisfied if the external reactions had been calculated incorrectly.

Note: The order in which the joints are set in equilibrium is usually arbitrary. Sometimes not all member loads are requested.

If provided numerical values:

$P_1 = 800$ lb

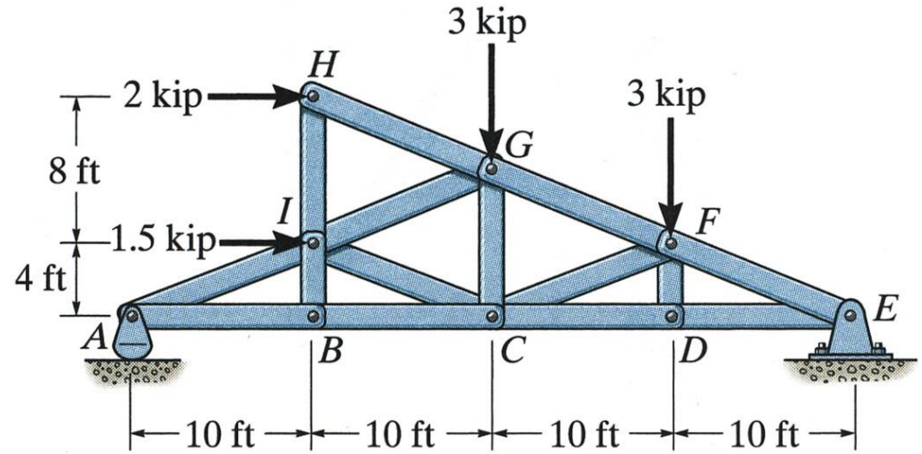
$P_2 = 0$



$$F_{AB} = P_1 = 800 \text{ lb (T)}$$
$$F_{BC} = P_1 = 800 \text{ lb (T)}$$
$$F_{AD} = -\sqrt{2}P_1 = -1130 \text{ lb (C)}$$
$$F_{BD} = -P_2 = 0$$
$$F_{CD} = \sqrt{2}(P_1 + P_2) = 1130 \text{ lb (T)}$$
$$F_{DE} = -(2P_1 + P_2) = -1600 \text{ lb (C)}$$

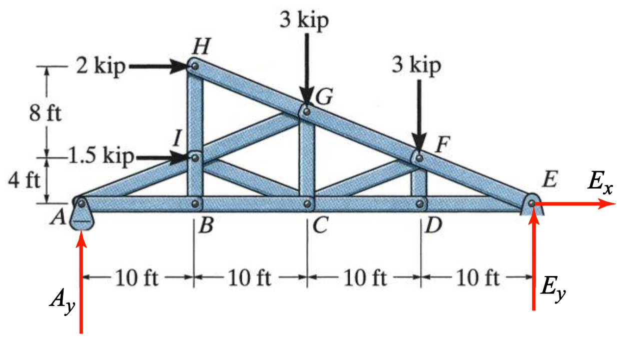
Note that, in the absence of P_2, member BD is a zero-force member

Determine the force in member *FG* of the truss and state if the member is in tension or compression.



Solution:

Draw a free-body diagram of the entire truss



Equilibrium of the entire structure requires:

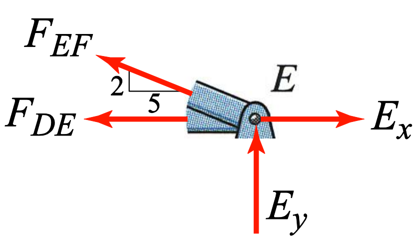
$$2 + 1.5 + E_x = 0\,,$$

$$-3 - 3 + A_y + E_y = 0\,,$$

$$-40A_y - 4(1.5) - 12(2) + 20(3) + 10(3) = 0\,.$$

$$\boxed{A_y = 1.5 \text{ kips}, \quad E_x = -3.5 \text{ kips}, \quad E_y = 4.5 \text{ kips.}}$$

Joint E:



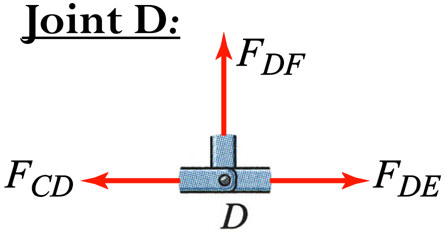
$$\Sigma F_x = 0: \quad -\tfrac{5}{\sqrt{29}} F_{EF} - F_{DE} + E_x = 0\,,$$

$$\Sigma F_y = 0: \quad \tfrac{2}{\sqrt{29}} F_{EF} + E_y = 0\,.$$

$$F_{EF} = -\tfrac{\sqrt{29}}{2} E_y = -\tfrac{\sqrt{29}}{2}(4.5) = -12.12 \text{ kips (C)},$$

$$F_{DE} = -\tfrac{5}{\sqrt{29}} F_{EF} + E_x = -\tfrac{5}{\sqrt{29}}(-12.12) + (-3.5) = 7.75 \text{ kips (T)}.$$

Joint D:

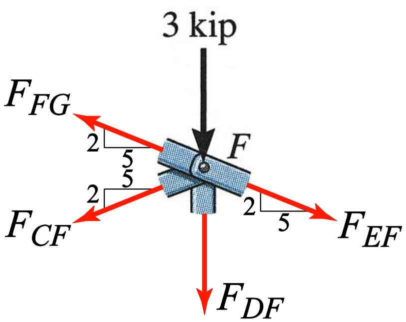


$$\Sigma F_x = 0: \quad -F_{CD} + F_{DE} = 0\,, \qquad F_{CD} = F_{DE} = 7.75 \text{ kips (T)},$$

$$\Sigma F_y = 0: \quad F_{DF} = 0\,. \qquad F_{DF} = 0.$$

Note: Member *DF* turned out to be a zero-force member. Under what conditions would this result not be true?

Joint F:



$$\Sigma F_x = 0: \quad -\tfrac{5}{\sqrt{29}} F_{FG} - \tfrac{5}{\sqrt{29}} F_{CF} + \tfrac{5}{\sqrt{29}} F_{EF} = 0\,,$$

$$\Sigma F_y = 0: \quad \tfrac{2}{\sqrt{29}} F_{FG} - \tfrac{2}{\sqrt{29}} F_{CF} - F_{DF} - \tfrac{2}{\sqrt{29}} F_{EF} - 3 = 0\,.$$

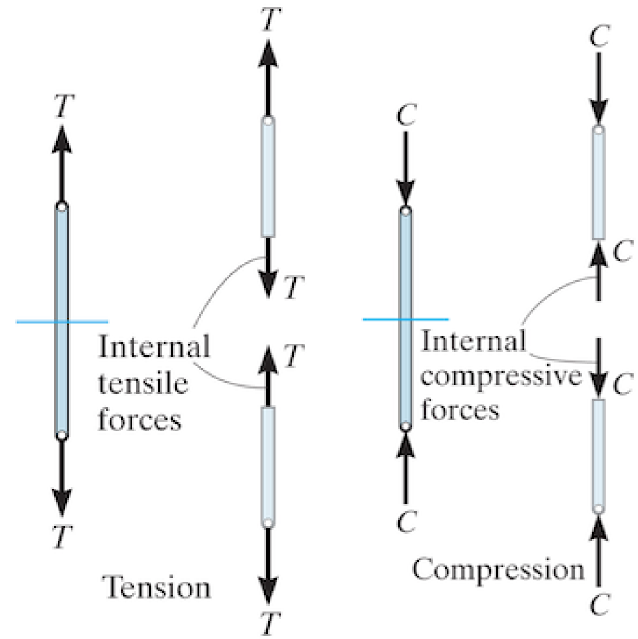
$$F_{FG} + F_{CF} = F_{EF}\,,$$

$$F_{FG} - F_{CF} = F_{EF} + \tfrac{\sqrt{29}}{2}\left(3 + F_{DF}\right).$$

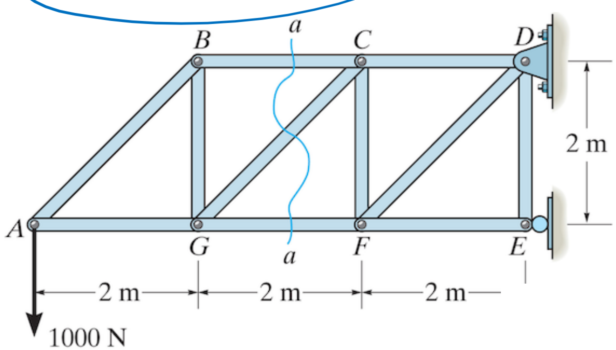
$$\begin{aligned} F_{FG} &= F_{EF} + \tfrac{1}{2} \cdot \tfrac{\sqrt{29}}{2}\left(3 + F_{DF}\right) \\ &= -12.12 + \tfrac{\sqrt{29}}{4}(3+0) \\ &= -8.08 \text{ kips (C).} \end{aligned}$$

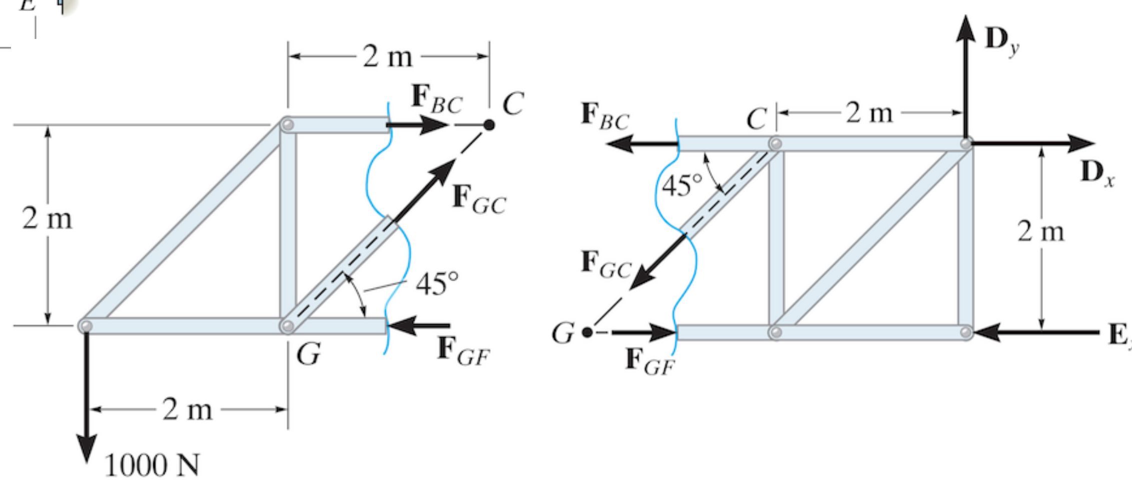
Note: Nine scalar equations of equilibrium were needed to obtain this answer. Might there be a shorter way? Method of sections

Method of sections



- Determine external support reactions
- "Cut" the structure at a section of interest into two separate pieces and set either part into force and moment equilibrium (your cut should be such that you have **no more than** three unknowns)

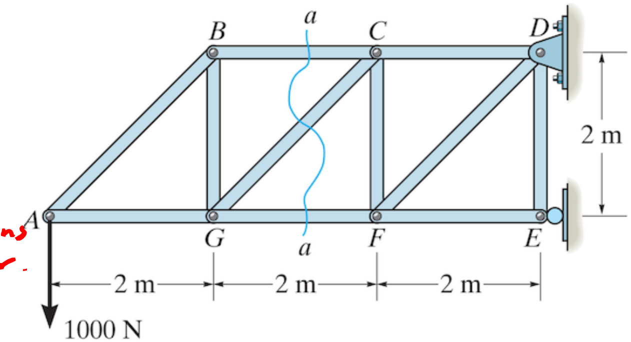


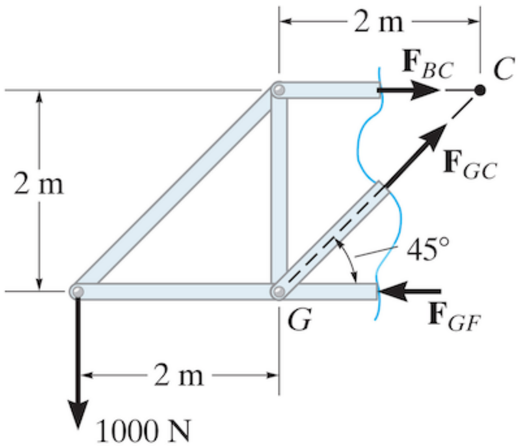


- Determine equilibrium equations (e.g., moment around point of intersection of two lines)
- Assume all internal loads are tensile.

Method of sections

- Determine equilibrium equations (e.g., moment around point of intersection of two lines) ⇐ Reduces # unknowns in eqn. Solve faster.
- Assume all internal loads are tensile.

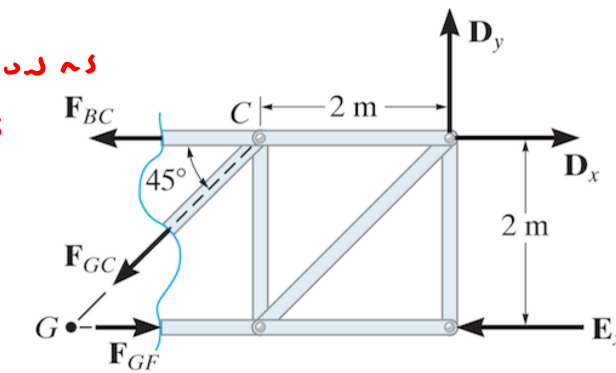




Left section : 3 unknowns

Right section : 6 unkns

⇒ solve left section first!



$+\circlearrowleft \Sigma M_C : -2m(F_{GF}) + 4m(1000N) = 0$

ΣF_x

ΣF_y ⇒ F_{BC}, F_{GC}

$+\circlearrowleft \Sigma M_G :$

$2m(F_{BC}) - 2m(D_x) + 2m(D_y) = 0$

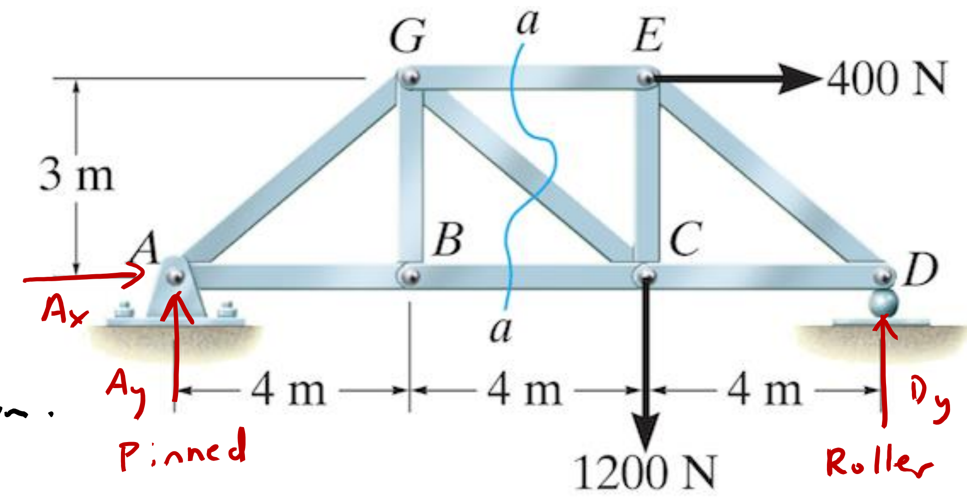
E_x passes thru G, so no effect on moment.

ΣF_x

ΣF_y ⇒ D_x, D_y, E_x

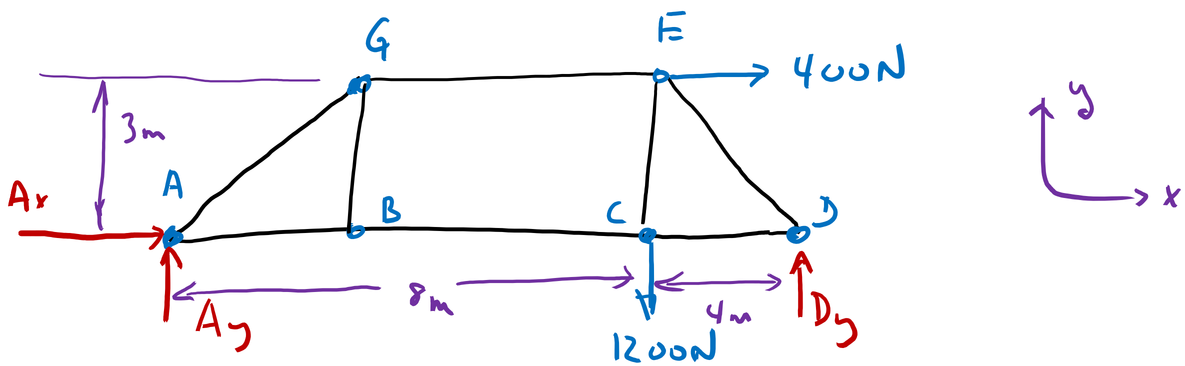
Determine the force in member BC of the truss and state if the member is in tension or compression.

[Problem solved outside of class.]



① Solve for reaction forces @ A & D.

Draw FBD. Solve Eqns of Equilibrium.



$\Sigma F_x: \; A_x + 400\,N = 0 \Rightarrow \boxed{A_x = -400\,N\,\hat{i} \;\; (\leftarrow)}$

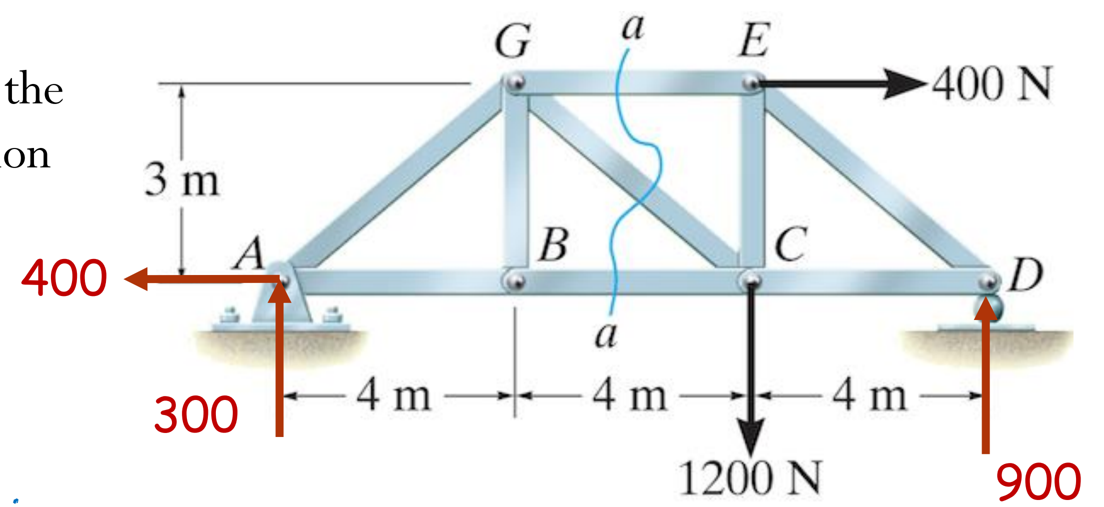
$\Sigma F_y: \; A_y - 1200\,N + D_y = 0 \Rightarrow A_y = 1200\,N - D_y$

$\circlearrowleft \Sigma M_A$: pick A since want eqn w/ least # unknowns (solve faster)

$-(3m)\,400\,N - (8m)\,1200\,N + (12\,m)\,D_y = 0$

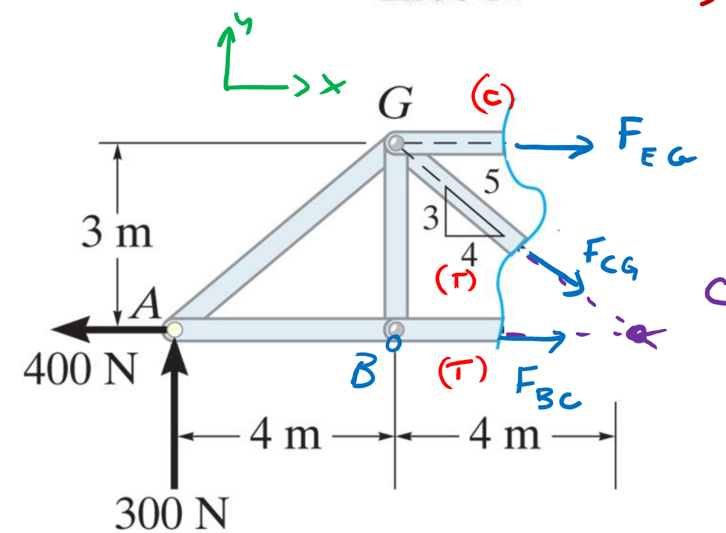
$\therefore \boxed{D_y = 900\,N\,\hat{j} \; (\uparrow), \quad A_y = 300\,N\,\hat{j} \; (\uparrow)}$

Determine the force in member BC of the truss and state if the member is in tension or compression.



(2) Use method of Sections to solve for force in BC (F_{BC})

- Extend two lines at cut to find point of intersection.
- Draw unknown truss forces in cut members.
- Solve Eq of Eq around created point (pt C in this case):



$$+\circlearrowleft \Sigma M_C: \; -(3m)F_{EG} - (8m)\,300\,N = 0$$

$$\Rightarrow F_{EG} = -800N$$

$$\Sigma F_x: \; -400N + F_{EG} + F_{BC} + F_{CG}\left(\tfrac{4}{5}\right) = 0$$

$$\Sigma F_y: \; 300N - F_{CG}\left(\tfrac{3}{5}\right) = 0 \;\Rightarrow F_{CG} = 500N$$

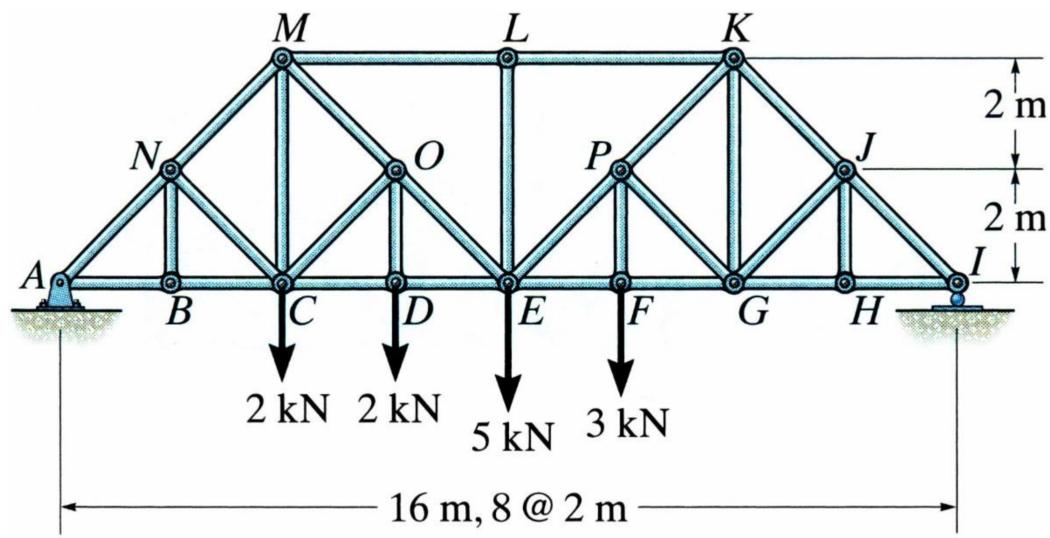
$$\therefore F_{BC} = 400N - 500\left(\tfrac{4}{5}\right) - (-800N)$$

$$\Rightarrow \boxed{F_{BC} = 800N\,\hat{i} \;\; (\rightarrow)}$$

Since F_{BC}, F_{CG} are positive, FBD is correct, so BC & CG are in tension. F_{EG} is negative therefore F_{EG} is drawn incorrectly in the original FBD, so it should point in the opposite direction.

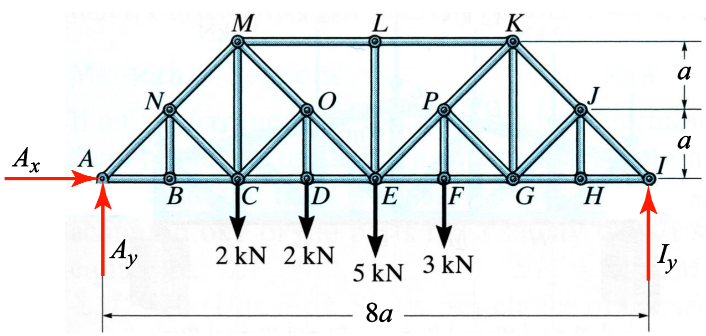
∴ member EG is in compression

Determine the force in members OE, LE, LK of the Baltimore truss and state if the member is in tension or compression.



Solution:

Use method of sections, since cutting LK, LE, OE, and DE will separate the truss into two pieces. Note that LE is a zero-force member. Draw free-body diagram of entire structure, and set into external equilibrium:

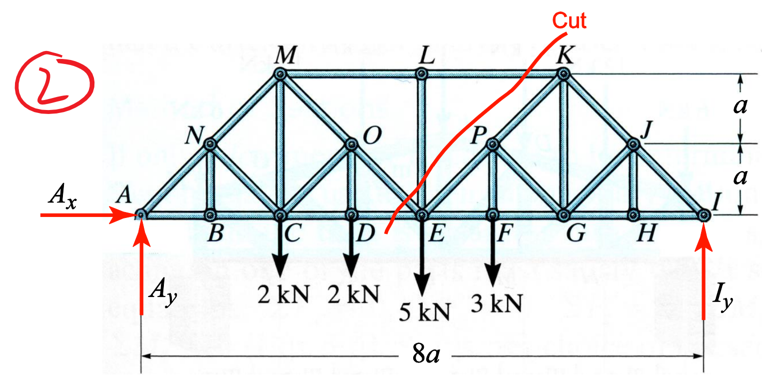


$$\Sigma F_x = 0: \qquad A_x = 0,$$

$$\Sigma F_y = 0: \qquad A_y + I_y - 2 - 2 - 5 - 3 = 0,$$

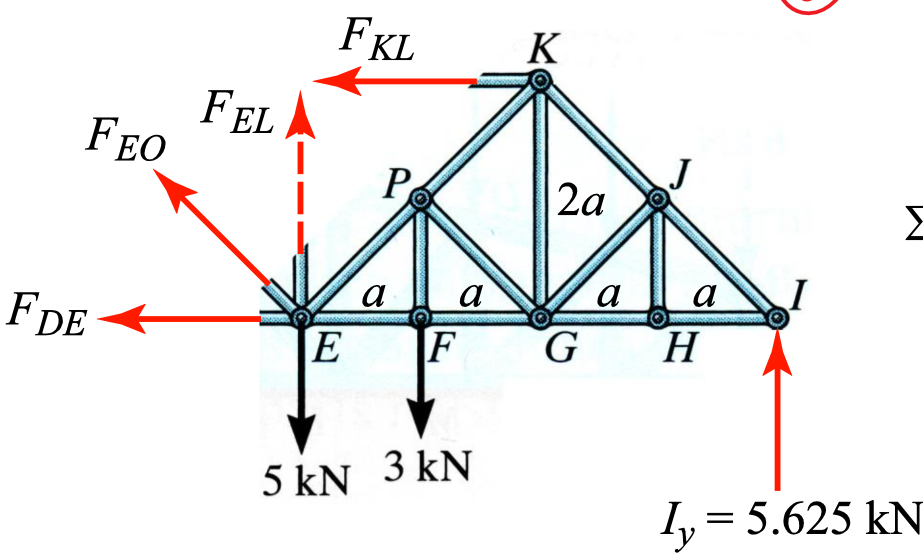
$$\Sigma M_A = 0: \quad -2a(2) - 3a(2) - 4a(5) - 5a(3) + 8aI_y = 0.$$

$$\boxed{A_x = 0, \quad A_y = 6.375 \text{ kN}, \quad I_y = 5.625 \text{ kN}.}$$



Normally, introducing four unknowns would make the problem intractable. However, *LE* is a *zero-force* member. Set *either* remaining section into equilibrium. Here, there is no real preference, but the right half will be fine

③



$$\Sigma F_x = 0: \quad -F_{DE} - \tfrac{1}{\sqrt{2}} F_{EO} - F_{KL} = 0,$$

$$\Sigma F_y = 0: \quad \tfrac{1}{\sqrt{2}} F_{EO} + 0 - 5 - 3 + I_y = 0,$$

$$\Sigma M_E = 0: \quad 0(5) + a(-3) + 4aI_y + 2aF_{KL} = 0.$$

$F_{DE} = +7.38$ kN (T),	$F_{EL} = 0$ (zero-force),
$F_{EO} = +3.36$ kN (T),	$F_{KL} = -9.75$ kN (C).

④ Final Result :