Statics - TAM 210 & TAM 211

Lecture 22

March 7, 2018

Chap 7.2

Announcements

- No physical lecture. See these detailed post-lecture slides.
- Upcoming deadlines:
  - Quiz 4 (3/7-9)
    - Sign up at CBTF
    - Up thru and including Lecture 19 (Frames & Machines). Note that quiz and lecture material always builds on earlier fundamental concepts.
  - No class Friday March 9, enjoy EOH!
  - No Prof. H-W office hours on Friday March 9
  - Monday (3/12)
    - Mastering Engineering Tutorial 9
  - Tuesday (3/13)
    - PL HW 7
  - Quiz 5 (3/14-16)

Chapter 7: Internal Forces

Goals and Objectives

- Determine the internal loadings in members using the method of sections
- Generalize this procedure and formulate equations that describe the internal shear and bending moment throughout a member
- Be able to construct or identify shear and bending moment diagrams for beams when distributed loads, concentrated forces, and/or concentrated couple moments are applied

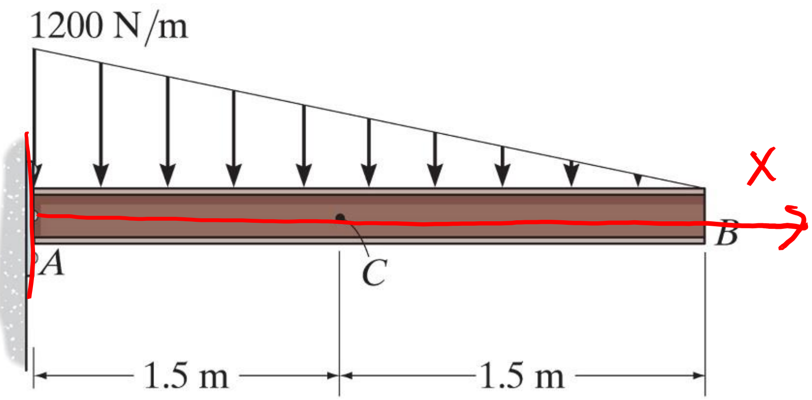
Recap: Shear Force and Bending Moment Diagrams

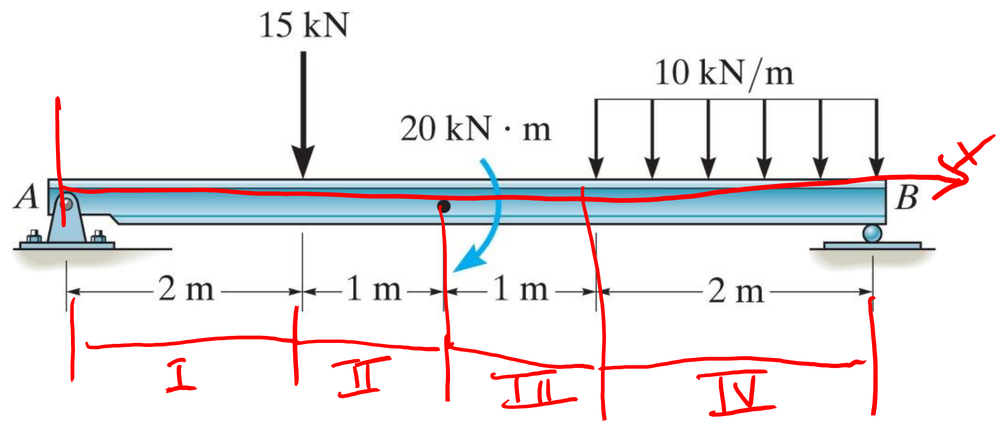
Goal: provide detailed knowledge of the variations of internal shear force and bending moments (V and M) throughout a beam when perpendicular distributed loads, concentrated forces, and/or concentrated couple moments are applied.

Normal forces (N) in such beams are zero, so we will not consider normal force diagrams.

Procedure

1. Find support reactions (free-body diagram of entire structure)
2. Specify coordinate x (start from left)
3. Divide the beam into sections according to loadings
4. Draw FBD of a section
5. Apply equations of equilibrium to derive V and M as functions of x: $V(x)$, $M(x)$

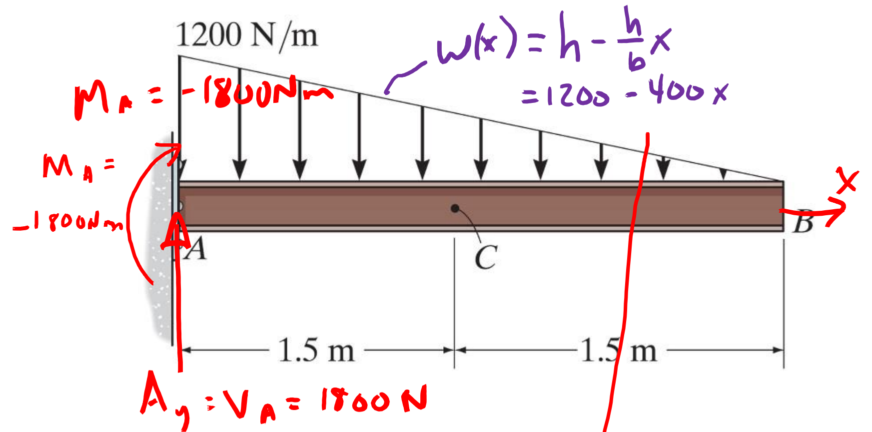




Recap: Draw the shear and bending moment diagrams for the beam.

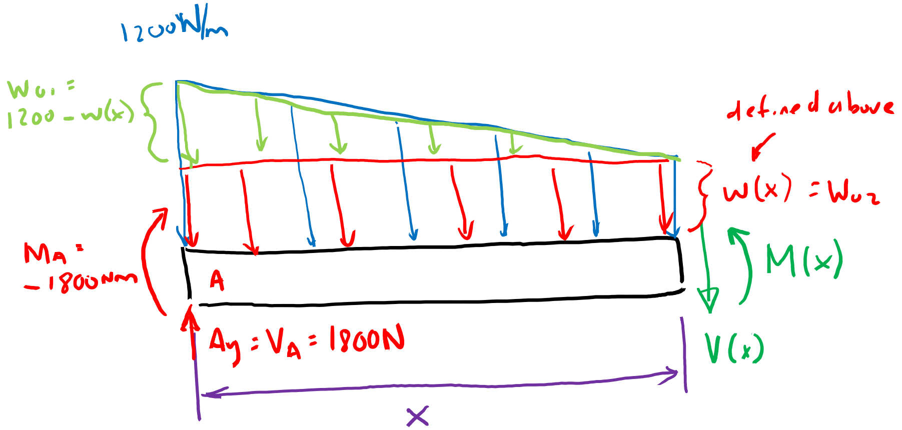
Detailed notes added to post-lecture version of Lecture 21

From previous example, we know that the support reactions are: $A_x = 0$, $A_y = 1800N \uparrow$, $M_A = -1800\ Nm$ ↺



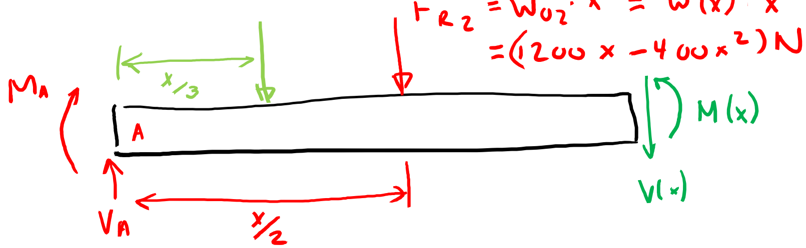
We are interested in finding V(x) & M(x) as these vary along the length of the beam.

So for any length x of the beam, we get the following generic FBD as a function of x.



$F_{R_1} = w_{0_1} \cdot \frac{L}{2} = [1200 - w(x)]\frac{x}{2} = (200x^2)N$

$F_{R_2} = w_{0_2} \cdot x = w(x) \cdot x$
$= (1200x - 400x^2)N$



$$\Sigma F_x: \; A_y - F_{R_1} - F_{R_2} - V(x) = 0$$

$$\boxed{V(x) = (200x^2 - 1200x + 1800)\,N}$$

Quadratic

Boundary conditions:

$V(x=0) = 1800N = A_y$

$V(x=L=3m) = 0\,N$

cf. $V(@C=1.5m) = 450N$ ✓ w/ previous example

$$+\circlearrowleft \Sigma M_A: \; -M_A - \left(\tfrac{x}{3}\right)F_{R_1} - \left(\tfrac{x}{2}\right)F_{R_2} - x \cdot V(x) + M(x) = 0$$

$$\boxed{M(x) = \left(\tfrac{200}{3}x^3 - 600x^2 + 1800x - 1800\right)\,Nm}$$

3rd Order Polynomial

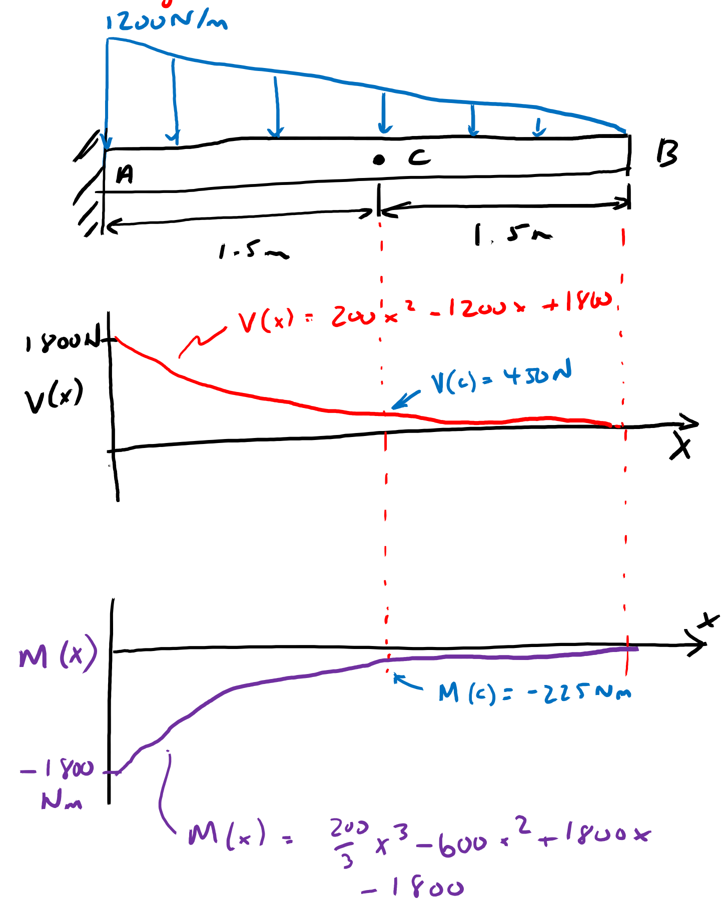
BC:

$M(0) = -1800\,Nm = M_A$

$M(L) = 0$

cf. $M(@C=1.5m) = -225\,Nm$ ✓ w/ previous

Draw Shear Force V(x) & Bending Moment M(x) diagrams



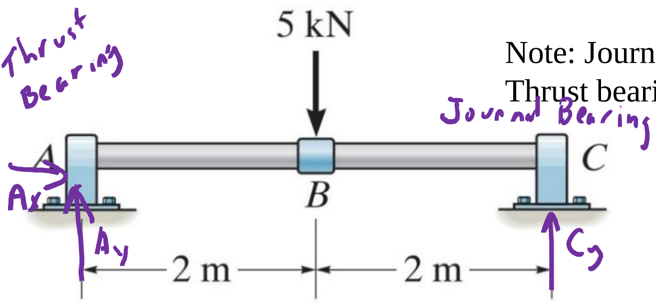
Note that since the applied load is a single distributed load along the entire length of the beam, then V(x) and M(x) are continuous functions. We will see (in Lecture 22) that V(x) and M(x) will be discontinuous functions when multiple loads are applied to a beam, and these discontinuities will happen at the transitions between loading regions.

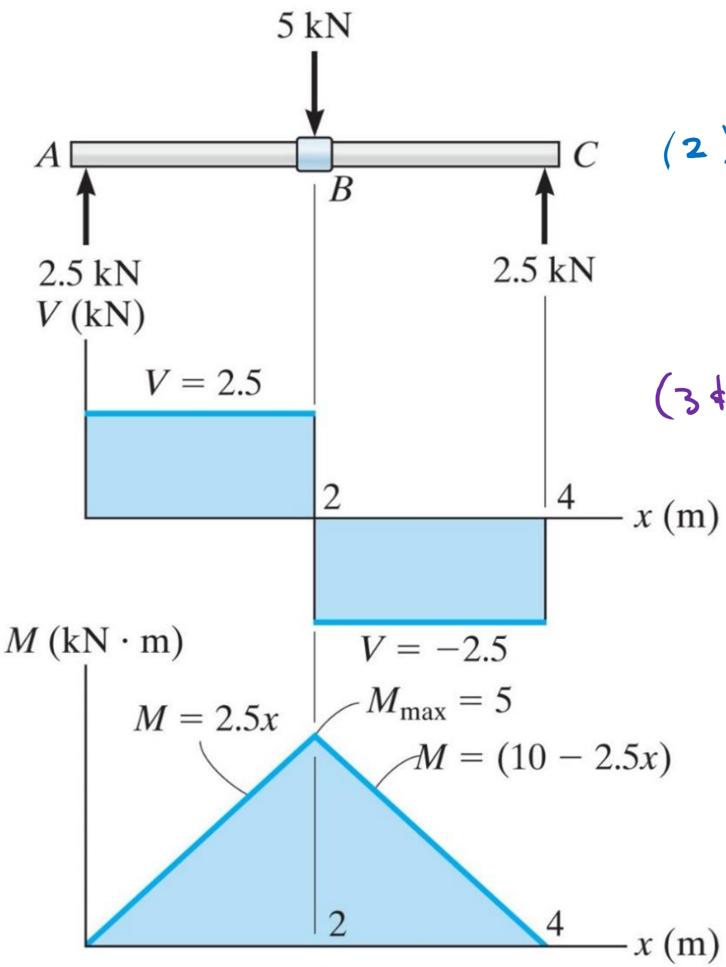
Explore and re-create the shear force and bending moment diagrams for the beam. A is thrust bearing & C is journal bearing.

Example: single concentrated load

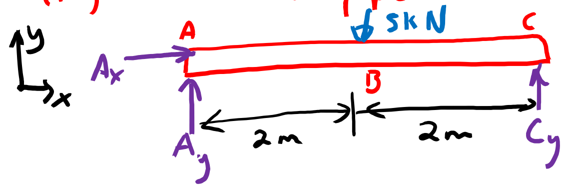
See Example 7.6 in text

Note: Journal bearings only have support reaction forces and moments on axes perpendicular to shaft. Thrust bearings are similar to journal bearings but with added support reaction force along axis of shaft





(1) Find support reactions :



ΣF_x : $\boxed{A_x = 0}$

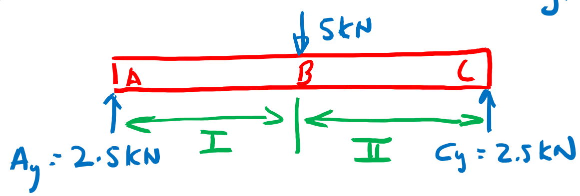
ΣF_y : $A_y + C_y - 5kN = 0$

$+\circlearrowleft \Sigma M_A$: $-(2m)5kN + (4m)C_y = 0$

$\rightarrow \boxed{C_y = 2.5kN}$

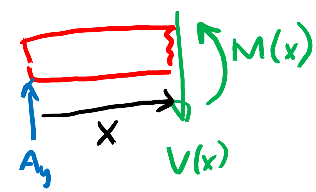
$\therefore \boxed{A_y = 2.5kN}$

(2) Divide beam into regions according to loadings.



(3&4) Draw FBD of a region. Use Eq of Eq to derive V(x) & M(x):

Region I :



ΣF_y : $A_y - V(x) = 0$

$\boxed{V(x) = A_y = 2.5kN}$

Constant, positive

$+\circlearrowleft \Sigma M_A$: $-x \cdot V(x) + M(x) = 0$ $\therefore$ $M(x) = x \cdot V(x)$

$\boxed{M(x) = x A_y = 2.5x \ kN \cdot m}$

Note: could chose ΣM_x where x is at cut on right side

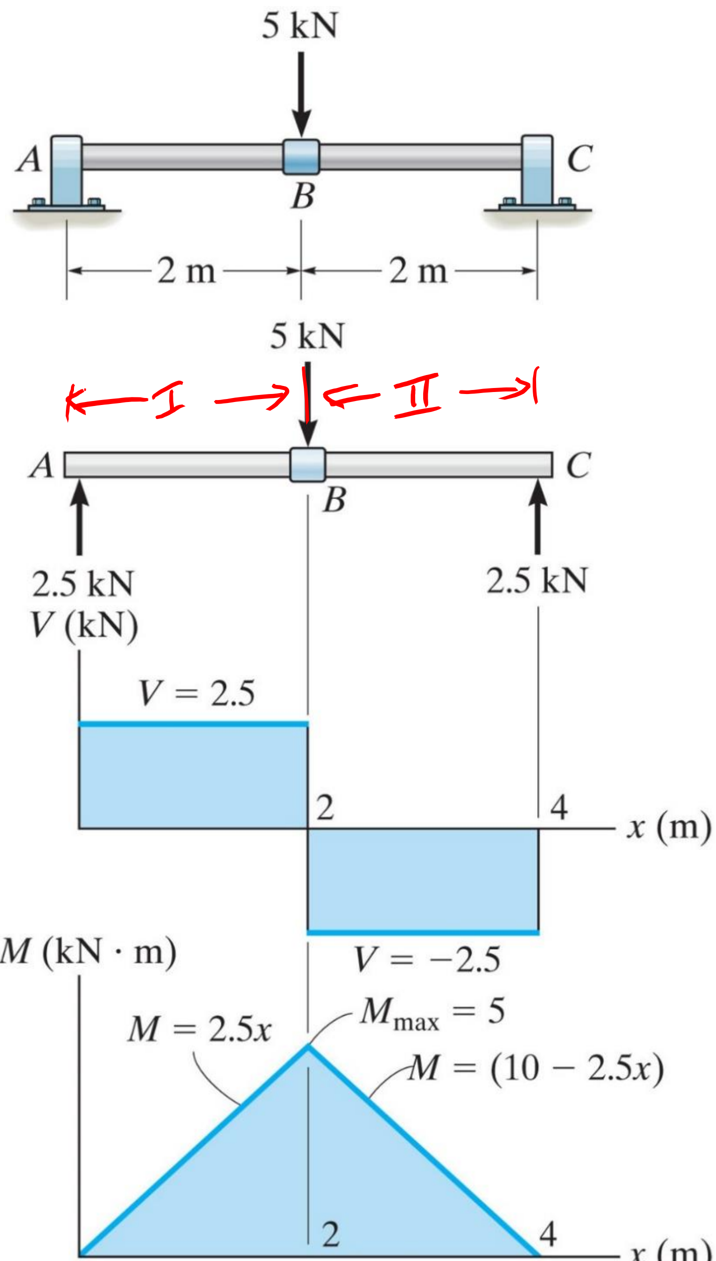
linear w/ slope A_y

Boundary Conditions : Compare results to plots to left

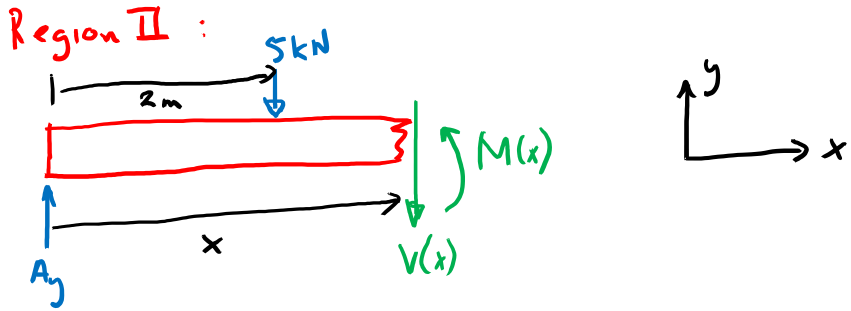
$x = 0$: $V(0) = 2.5kN$, $M(0) = 0$

$x = 2m^{(-)}$ {use (−) as immediately to left of 2m : $V(2m^{(-)}) = 2.5 \ kN$

$M(2m^{(-)}) = 5 \ kN \cdot m$



Region II :



ΣF_x : $A_y - V(x) - 5\,kN = 0$

$\boxed{V(x) = A_y - 5\ kN = -2.5\,kN}$

constant, negative

$+\circlearrowright \Sigma M_A$: $-(2m)5kN - x \cdot V(x) + M(x) = 0$

$M(x) = 10kN \cdot m + x \cdot V(x)$

$= 10\,kN \cdot m + x\,(A_y - 5\ kN)$

$\therefore \boxed{M(x) = (10 - 2.5x)\ kN\ m}$

linear w/ negative slope

cf. BC's :

$x = 2m^{(+)}$: $V(2m^{(+)}) = -2.5kN$, $M(2m^{(+)}) = 5kN \cdot m$

$x = 4m$: $V(4) = -2.5\,kN$, $M(4) = 0$

compare results to plots to left

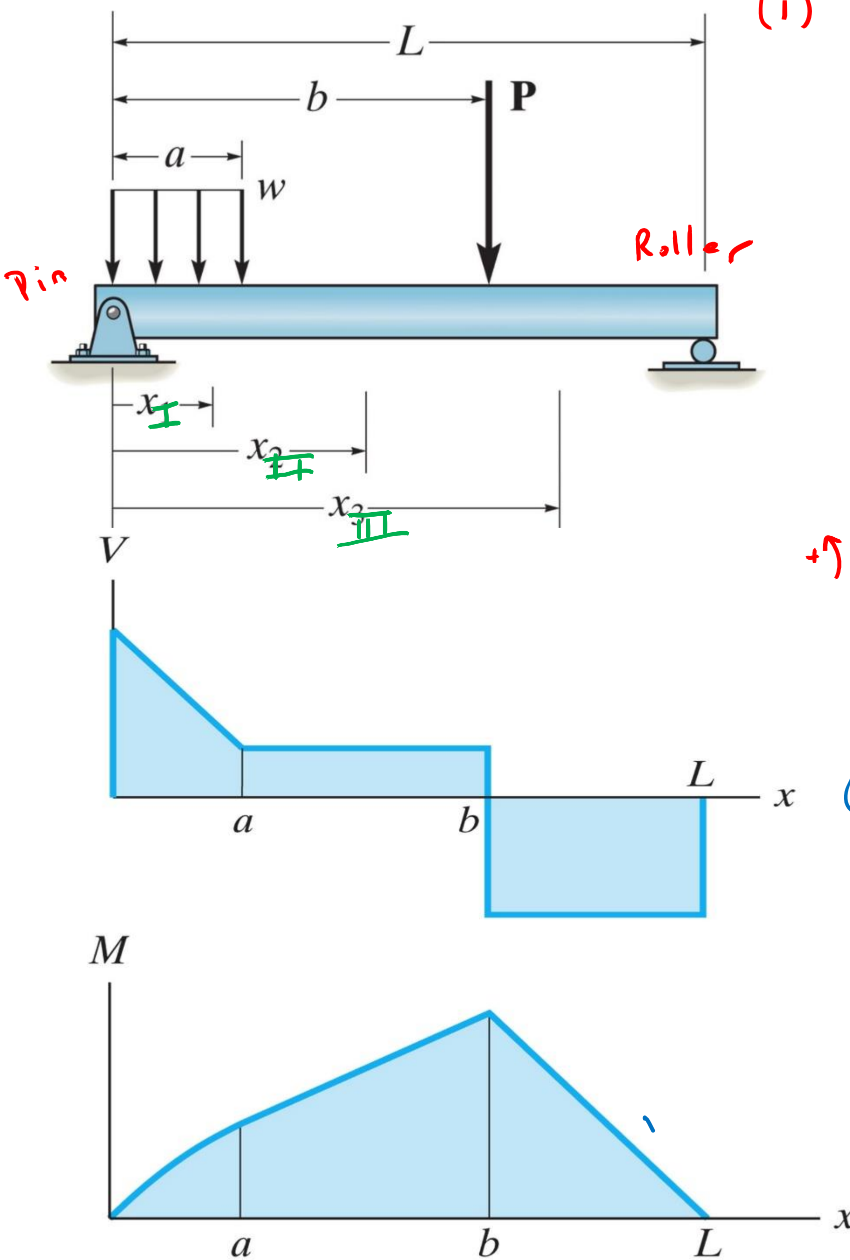
Note for <u>single concentrated load (P)</u>:

- $V(x)$ is <u>constant</u> within a region. $V(x)$ has a step change at location of load that is equivalent to magnitude and direction of applied load (e.g., $-P\hat{\jmath}$ or -5 kN).
- $M(x)$ is <u>linear</u>.

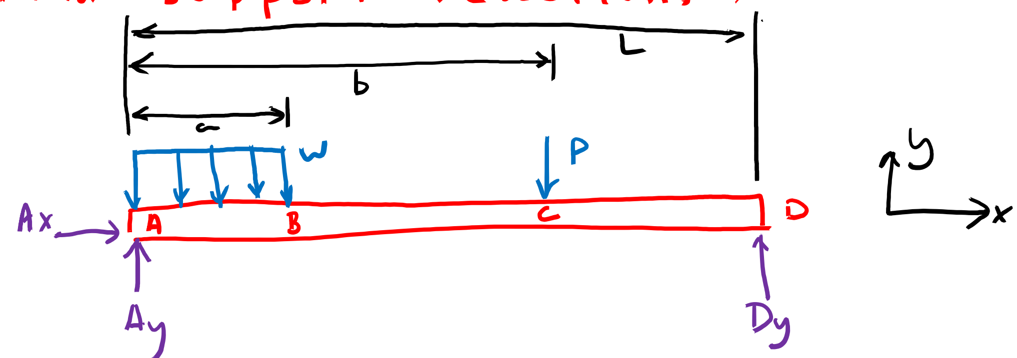
Also note that $V(x) = \frac{d}{dx}M(x)$, or slope of moment diagram

Explore and re-create the shear force and bending moment diagrams for the beam.

Example: single concentrated load, rectangular distributed load



(1) Find support reactions:



ΣF_x: $\boxed{A_x = 0}$

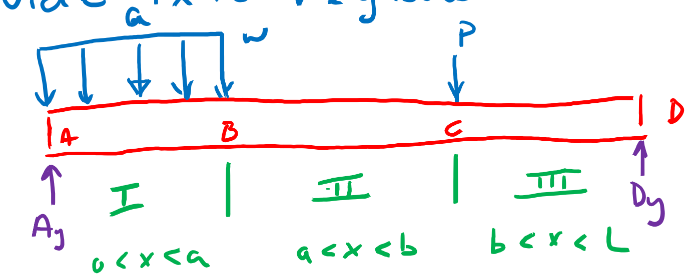
ΣF_y: $A_y + D_y - aw - P = 0$

$+\circlearrowleft \Sigma M_A$: $-\left(\frac{a}{2}\right)aw - b \cdot P + L \cdot D_y = 0$

$D_y = \frac{1}{L}\left(\frac{a^2 w}{2} + bP\right) \equiv \text{constant}$

$A_y = aw + P - D_y = aw\left(1 - \frac{a}{2L}\right) + P\left(1 - \frac{b}{L}\right) \equiv \text{constant}$

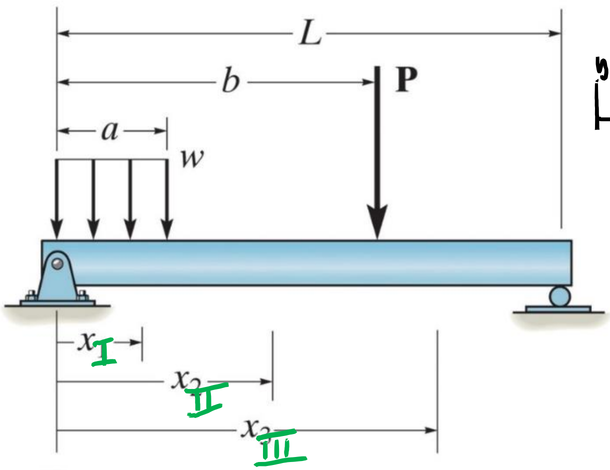
(2) Divide into regions

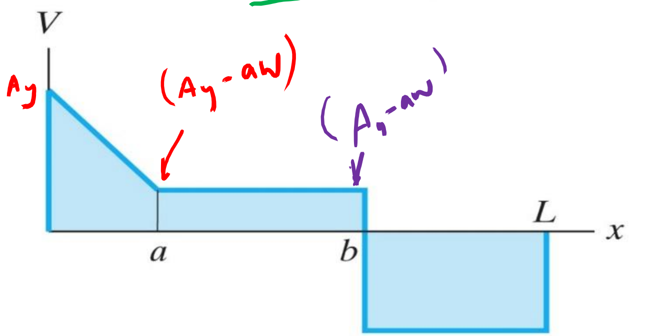


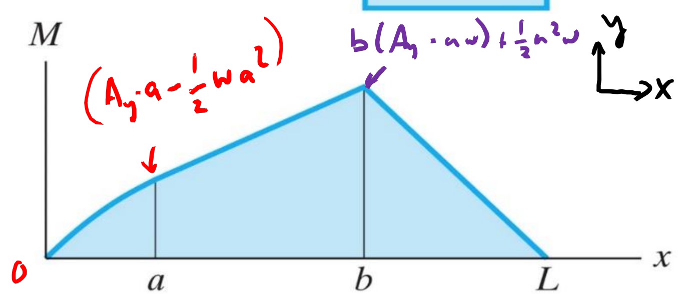
3 regions, 3 FBDs

Explore and re-create the shear force and bending moment diagrams for the beam.

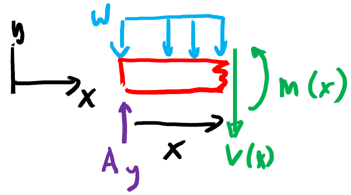
Example: single concentrated load, rectangular distributed load







Region I : $0 < x < a$



$\Sigma F_y : \; A_y - V(x) - x \cdot w = 0$

$$\boxed{V(x) = A_y - xw}$$ linear w/slope w

$+\circlearrowleft M_A : \; M(x) - \left(\frac{x}{2}\right)(xw) - x \cdot V(x) = 0$

$$M(x) = \frac{x^2 w}{2} + x\,(A_y - xw)$$

$$\therefore \; \boxed{M(x) = A_y x - \frac{1}{2} w x^2}$$ quadratic

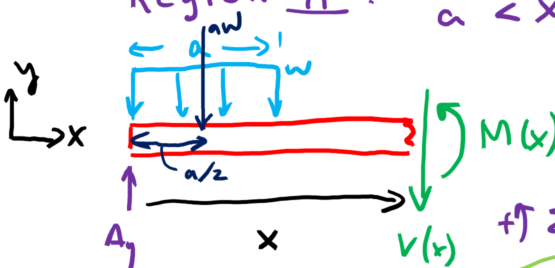
BC's:

$x = 0 : \; V(0) = A_y , \quad M(0) = 0$

$x = a^{(-)} : \; V(a) = A_y - aw , \quad M(a) = A_y \cdot a - \frac{1}{2} w a^2$

Compare to values labeled on V & M diagrams to left

Region II : $a < x < b$



$\Sigma F_y : \; A_y - aw - V(x) = 0$

$$\boxed{V(x) = A_y - aw}$$ constant

$+\circlearrowleft \Sigma M_x : \; M(x) - x \cdot A_y - \left(x - \frac{a}{2}\right) aw = 0$

$$\boxed{M(x) = x\,(A_y - aw) + \frac{1}{2} a^2 w}$$ linear w/slope $(A_y - aw) = V(x)$

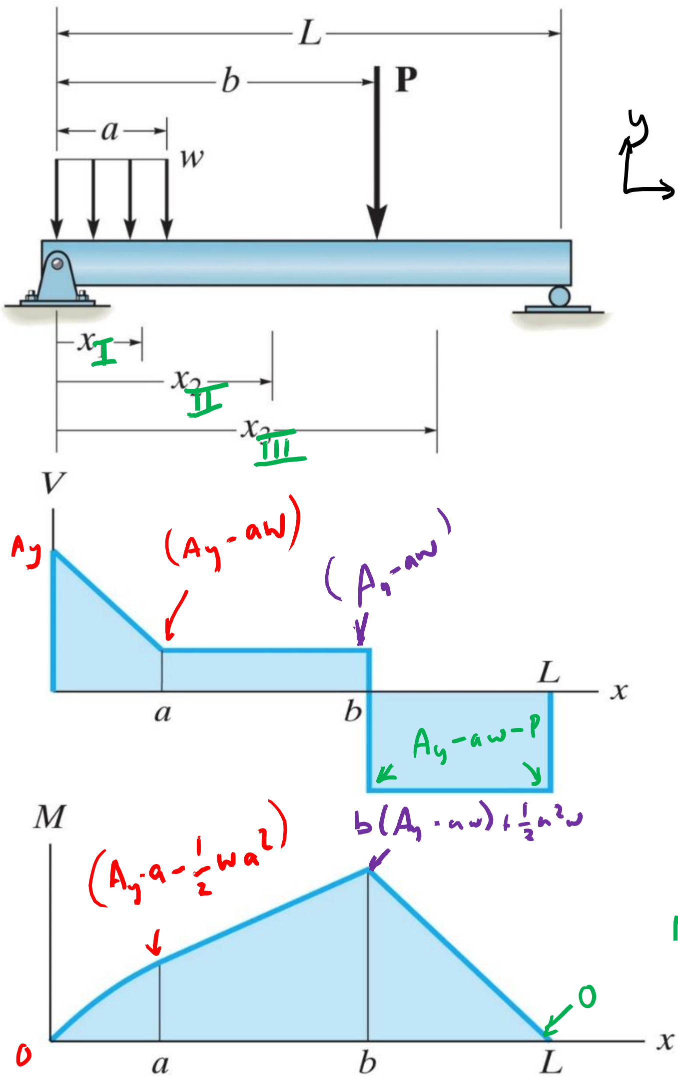
Note used pt x to sum moments

BC's: $x = a^{(+)} : \; V(a^{(+)}) = A_y - aw, \; M(a) = A_y a - \frac{1}{2} w a^2$

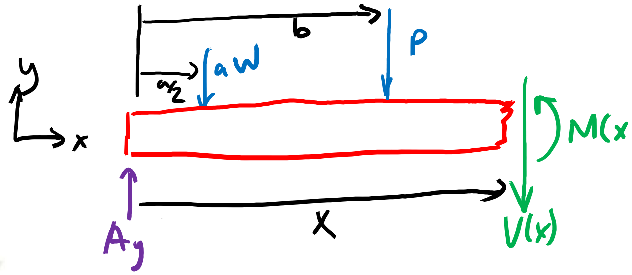
$x = b^{(-)} : \; V(b^{(-)}) = A_y - aw , \; M(b^{(-)}) = b(A_y - aw) + \frac{1}{2} a^2 w$

Explore and re-create the shear force and bending moment diagrams for the beam.

Example: single concentrated load, rectangular distributed load



Region III : $b < x < L$



$\Sigma F_y: A_y - aw - P - V(x) = 0$

$$\boxed{V(x) = A_y - aw - P}$$

constant

$+\circlearrowleft \Sigma M_x : M(x) - x A_y + (x - \frac{a}{2}) \cdot aw + (x - b) \cdot P = 0$

$$\boxed{M(x) = x \underbrace{(A_y - aw - P)}_{V(x)} + bP + \frac{1}{2} a^2 w}$$

linear w/ slope V(x)

BC's:

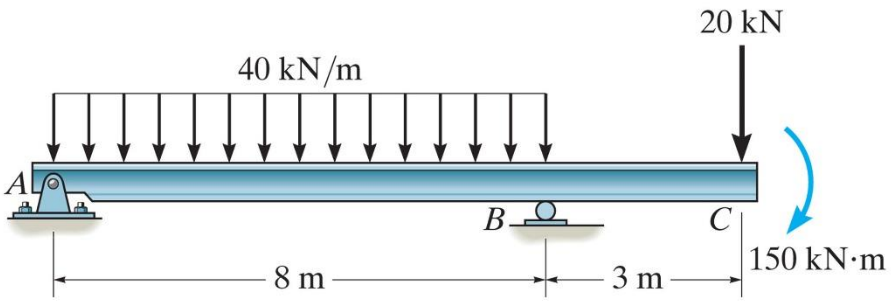
$x = b^{(+)}$: $V(b) = A_y - aw - P$, $M(b) = b(A_y - aw) + \frac{1}{2} a^2 w$

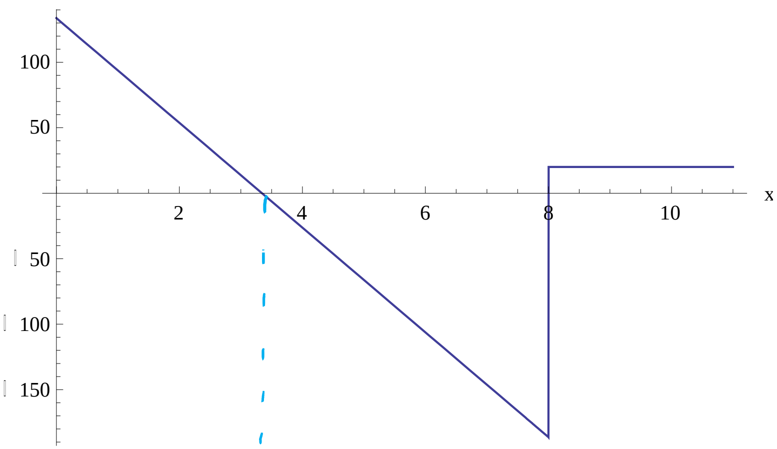
$x = L$: $V(L) = A_y - aw - P$, $M(L) = 0$

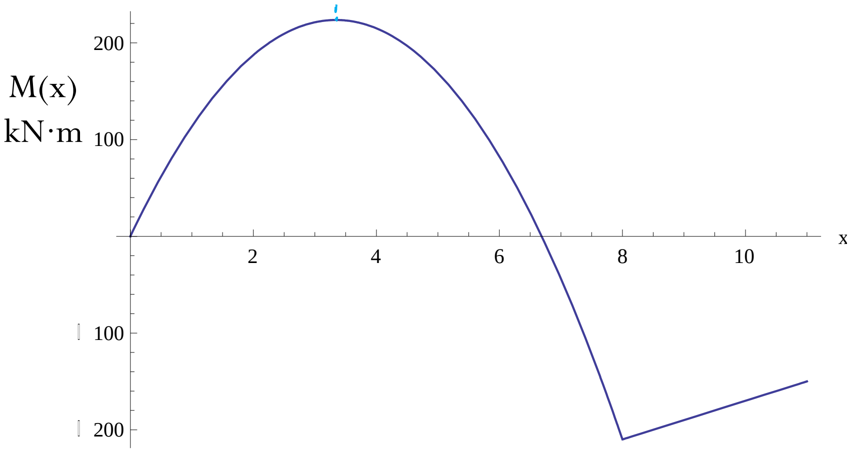
Note step change of $-P$ in $V(x)$ at $x = b$ due to concentrated load of P

Explore and re-create the shear force and bending moment diagrams for the beam.

Example: concentrated load, rectangular distributed load, concentrated couple moment







Draw the shear force and bending moment diagrams for the beam.

Example: concentrated load, rectangular distributed load, concentrated couple moment